CU Journal of Non-Linear Fluid Mechanics
Vol. 01, No. 02, 2025
https://doi.org/10.11615/cujnlfm.01205-5

Derivation of the Ginzburg-Landau Equation and Estimation of the Heat Transfer in a Rayleigh-Bénard Convection of a Micropolar fluid with Time Periodic Body Force

Anirudh P[a], M S Jagadeesh Kumar *[a], and Nur Aisyah Abdul Fataf[b]

[a] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.
[b] Centre for Defence Foundation Studies, Universiti Pertahanan Nasional Malaysia, Kuala Lumpur 57000, Malaysia

Abstract

This study examines the behavior of a micropolar fluid in a Rayleigh–Bénard configuration under a time-varying gravitational force. A scaled fourth-order Lorenz model is employed to describe weakly nonlinear convection. The model conserves energy and retains all the essential characteristics of the classical Lorenz system. The scaled Rayleigh number and the Ginzburg-Landau equation are derived using the Venezian method. The graphs showing the variation of the correction Rayleigh number with the modulation frequency for different parameter combinations are plotted, and it is found that the system supports supercritical motion. Furthermore, an analytical expression for the time-average Nusselt number is obtained and plotted for various values of the parameters, and it is found that the presence of micropolar fluid generally promotes the heat transfer.

Keywords: Rayleigh-Bénard Convection; Micropolar fluid; Gravity modulation; Scaled Lorenz model; Time-average Nusselt number

1 Introduction

Over the past several decades, numerous research works have been conducted on the Rayleigh-Bénard convection (RBC) due to its unique features and many industrial applications. The RBC was first experimentally studied by Bénard [1], later, the theoretical explanation was given by Rayleigh [2]. Chadrasekar [3], in his book, gave a detailed mathematical explanation for RBC, which motivated many researchers to study this field.

The RBC has been extensively investigated under a wide range of physical conditions, including the inclusion of additional solutes, imposed magnetic fields, Coriolis effects, and porous media. In the past few decades, among these extended configurations, modulated convection has gained significant attention as a means to explore the influence of time-dependent forcing on convective behavior. In the classical RBC configuration, the control parameters such as gravitational acceleration, thermal boundary conditions, system geometry, and others are assumed to be constant in time. However, in many natural and engineered systems, these parameters are not steady but instead vary periodically or quasi-periodically with time. The introduction of such time-dependence is

* ✉ msjagadeesh@vit.ac.in

generally referred to as modulation in RBC. These forms of modulation influence not only when convection begins, but also the system's stability properties, the evolution of its flow patterns, and the efficiency of heat transfer. In mathematical terms, the Boussinesq framework then contains coefficients that vary periodically in time, converting the stability question into one of parametric excitation. Floquet analysis is therefore the natural means of identifying stability limits. Depending on the imposed modulation amplitude and frequency, the system may respond through resonant amplification, delayed or accelerated destabilization, or subharmonic behaviour, each of which can shift the critical Rayleigh number and reshape the emerging convection patterns. Within the RBC literature, several forms of modulation have been examined:

1. Gravity modulation - where the effective gravitational acceleration is time - dependent directly modifying the buoyancy term.

2. Temperature modulation - in which the imposed temperature or heat flux at the boundaries varies periodically, altering the thermal forcing of the base state.

3. Internal-heating modulation, where the volumetric heat input fluctuates in time, adding unsteady forcing within the fluid layer.

4. Multi-frequency or waveform-based modulation, involving multiple input frequencies or non-sinusoidal shapes such as square or sawtooth waves, which produce richer bifurcation behaviour.

Among these, gravity modulation stands out because it acts directly on the buoyancy force, causing the Rayleigh number itself to oscillate. Such forcing can postpone or hasten the onset of convection, switch the character of the instability between steady and oscillatory modes, and significantly influence the heat-transport rate once convection develops. Strong, low-frequency modulation typically creates pronounced resonance regions, whereas high-frequency forcing tends to stabilize the conductive state by averaging out the buoyancy variations. Some of the practical applications of gravity modulation include the low-frequency vibrations have been used in attempts to control the shape and position of the solid-liquid interface during Bridgman growth, which was studied by Liu et al.[4], and Zharikov et al. [5] studied the effect of controlled gravity variation in the crystal growth. Gresho and Sani [6] were the pioneers in investigating the effects of time-dependent gravity in RBC. They demonstrated that the periodic modulation of the gravitational field can either destabilize or stabilize a thermally stratified fluid layer depending on the modulation frequency and amplitude. Earlier three-dimensional simulations by Biringen and Peltier [7], along with two-dimensional studies by Clever et al. [8], showed that gravitational vibration can trigger oscillatory and subharmonic modes. Using a Ginzburg–Landau equation (GLE), Siddheshwar [9] demonstrated that parametric resonance governs the amplitude dynamics of modulated convection. Subsequent work by Siddheshwar and Kanchana [10] highlighted that the modulation waveform, whether sinusoidal or piecewise linear, plays an important role in determining the onset of instability, with non-sinusoidal forms generally providing greater stabilization. These ideas were expanded further by Siddheshwar et al. [11] and Pranesh et al. [12], who examined multi-diffusive systems and reported that modulation can create stability "windows" by simultaneously altering thermal and solutal gradients. More recent analyses by Francis et al. [13], Kiran [14], Jakhar et al. [15], Bixapathi and Babu [16] and Nagaraj et al. [17] considered a range of boundary conditions and found that rigid boundaries tend to strengthen the stabilizing influence of modulation; they also showed that gravity modulation can significantly reduce both convective amplitudes and heat transfer in weakly nonlinear regimes. Taken together, these studies underline the role of time-periodic gravity as a viable control parameter for convection in Newtonian fluids.

In many engineering applications, the working medium contains dispersed micro-scaled particles that modify its rheology and transport behaviour. Such suspensions often depart from Newtonian characteristics because suspended particles may rotate and translate independently of the bulk motion. To account for these microstructural effects, Eringen [18] proposed the micropolar fluid (MPF) model, which augments classical continuum theory by introducing additional variables, such as the microrotation vector and the micro-inertia tensor. These modifications lead to a non-symmetric stress tensor and introduce couple stresses into the angular momentum balance. Łukaszewicz [19] later offered a detailed mathematical development of MPFs and discussed their relevance in porous media flows, lubrication, and particle-laden systems. Because the microstructure can influence how long thermal energy remains in the fluid, MPFs have been widely investigated in connection with convective stability and heat transfer. Although much of the early work on convection focused on Newtonian fluids, the growing industrial relevance of MPFs has motivated analyses using linear, energy, and weakly nonlinear frameworks. The interaction between micropolar parmeters and gravity modulation has been explored in several contexts, particularly in thermomagnetic and electrically conducting fluids. Siddheshwar and Pranesh [20] investigated the combined effects of time-dependent boundary temperature and gravity modulation in weakly conducting magneto-convection, showing that the phase difference between the modulations can either suppress or enhance instability. In a related study, Siddheshwar and Pranesh [21] analysed electrically conducting micropolar fluids and found that the interplay between spin inertia, magnetic effects, and oscillatory buoyancy can significantly modify the critical Rayleigh number and alter the emerging flow patterns. Pranesh et al. [22] carried this work further by conducting linear and weakly nonlinear analyses of micropolar RBC under combined gravity modulation and electric fields, demonstrating that modulation frequency and amplitude affect both the stability threshold and the resulting roll amplitudes. Yekasi et al. [23] later included internal heat generation and reported that microstructural effects can markedly delay the initiation of convection while reducing the associated heat transport. Overall, these studies show that micropolar parameters combined with gravity modulation leads to a rich set of parametric-resonance phenomena that can be used to control convective stability in electrically conducting and particle-laden fluids.

The goal of the present study is to develop the GLE governing the weakly nonlinear behaviour of convection in a gravity-modulated MPF and to use this formulation to evaluate heat transport through the time-averaged Nusselt number. In Section 2, the scaled Lorenz model is considered from Siddheshwar et al. [24], and a perturbation scheme is applied to obtain the expressions for the Rayleigh number and the GLE. The analytical expression for the Nusselt number is then obtained as a function of the gravity modulation and micropolar parameters. Section 3 examines how the corrected Rayleigh number varies with modulation frequency and whether the resulting motion is supercritical or subcritical. Section 4 summarizes the main theoretical insights, emphasizing the effects of gravity modulation and MPF parameters on and the associated heat transfer behaviour.

2 Mathematical Formulation

2.1 The scaled Lorenz Model

Siddheshwar et al. [24] formulated a hexa-modal-Lorenz model for a MPF in such a way that it preserves all the essential characteristics of the third-order Lorenz model originally proposed by Lorenz [25]. In the present problem, two additional assumptions were introduced to the Lorenz model:

1. The gravitational acceleration is treated as a time-dependent function.

2. The micropolar parameter N_5 is neglected due to its negligible influence on the system dynamics, as supported by the findings of Siddheshwar et al. [24] and Anirudh and Kumar [26].

By considering the above assumptions, we obtain the following fourth-order scaled Lorenz model, given by:

$$\frac{dA}{d\tau} = Pr((1+\varepsilon_1\delta sin(\Omega^*\tau))C - A - N_1B), \tag{1}$$

$$\frac{dB}{d\tau} = -\frac{Pr}{M_1}(N_1A + M_2B), \tag{2}$$

$$\frac{dC}{d\tau} = rA - C - AD, \tag{3}$$

$$\frac{dD}{d\tau} = -bD + AC, \tag{4}$$

where

$$\left.\begin{aligned} &K^2 = \pi^2(1+\alpha^2), \tau = K^2t, Pr = Pr(1+N_1), b = \tfrac{4\pi^2}{K^2}, r = \tfrac{\pi^2\alpha^2 rR_a}{K^6(1+N_1)},\\ &M_1 = N_2K^2(1+N_1)^2, M_2 = (1+N_1)(N_3K^2+2N_1), M_3 = K^2(1+N_1),\\ &R_a = \left(\tfrac{K^6}{\pi^2\alpha^2}\right)\left(\tfrac{N_1(N_3K^2+N_1)+N_3K^2+2N_1}{N_3K^2+2N_1}\right), \Omega^* = \tfrac{\Omega}{K^2}. \end{aligned}\right\} \tag{5}$$

2.2 Linear Stability Analysis

The expression for the correction Rayleigh and the scaled Rayleigh number are obtained using Venezian [27] approach, through which we study the onset of convection. The amplitudes A, B, C, and $\tilde{r}$ are expanded in the form:

$$\left.\begin{aligned} A &= A_0 + \varepsilon_1 A_1 + \varepsilon_1^2 A_2 + \dots \\ B &= B_0 + \varepsilon_1 B_1 + \varepsilon_1^2 B_2 + \dots \\ C &= C_0 + \varepsilon_1 C_1 + \varepsilon_1^2 C_2 + \dots \\ \tilde{r} &= \tilde{r}_0 + \varepsilon_1 \tilde{r}_1 + \varepsilon_1^2 \tilde{r}_2 + \dots \end{aligned}\right\}. \tag{6}$$

Using equation (6) in the linearized version of the equations (1)-(4) and equating the same powers of ε_1 on both the side of the resulting equation, we get

$$O(\varepsilon_1^0) : J_1\phi_0 = [0,0,0]^{Tr}, \tag{7}$$

$$O(\varepsilon_1^1) : J_1\phi_1 = [R_{21}, R_{22}, R_{23}]^{Tr}, \tag{8}$$

$$O(\varepsilon_1^2) : J_1\phi_2 = [R_{31}, R_{32}, R_{33}]^{Tr}, \tag{9}$$

where

$$J_1(\tau) = \begin{bmatrix} -\frac{1}{\tilde{Pr}}\frac{d}{d\tau} - 1 & -N_1 & 1 \\ -N_1 & -\frac{M2}{\tilde{Pr}}\frac{d}{d\tau} - M_2 & 0 \\ 1 & 0 & -\frac{1}{\tilde{r0}}\left(\frac{d}{d\tau} - 1\right) \end{bmatrix}, \tag{10}$$

$$\phi_i = \begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix}, i = 0, 1, 2, \tag{11}$$

$$\left.\begin{aligned} R_{21} &= -\delta\overline{C_0 sin(\Omega^*\tau)}, R_{22} = 0, R_{23} = -\frac{\tilde{r}_1 A_0}{\tilde{r}_0}, \\ R_{31} &= -\delta\overline{C_1 sin(\Omega^*\tau)}, R_{32} = 0, R_{33} = \frac{-\tilde{r}1A_1 - \tilde{r}_2 A_0}{\tilde{r}_0}, \end{aligned}\right\} \tag{12}$$

where overline on the $C_0\, sin(\Omega^*\tau)$ and $C_1\, sin(\Omega^*\tau)$ represents the time-average in $\left[0, \frac{2\pi}{\Omega^*}\right]$.

Solving the equation (7), we get the following solutions:

$$\phi_0 = \begin{bmatrix} -\frac{M2}{N_1}B_0 \\ B_0 \\ -\frac{M2\tilde{r}_0}{N_1}B_0 \end{bmatrix}. \tag{13}$$

Using the conditions from the equation (13), the expression for the $\tilde{r}_0$ is given by,

$$\tilde{r}_0 = \frac{M_2 - N_1^2}{M_2}. \tag{14}$$

Solving the equation (8), we get the following solutions:

$$\phi_1 = \frac{\delta M_2 \tilde{r}_0 \tilde{Pr}^2 sin(\Omega^* \tau)(Y_5 - iY_6)}{Y_5^2 + Y_6^2} \begin{bmatrix} -\dfrac{(Y_1 - iY_2)}{N_1 \tilde{Pr}} B_0 \\ B_0 \\ -\dfrac{\tilde{r}_0 (Y_3 + iY_4)}{N_1 \tilde{Pr}} B_0 \end{bmatrix}, \tag{15}$$

where

$$\left.\begin{aligned} Y_1 &= M_2 \tilde{Pr}, \\ Y_2 &= \Omega^* M_1, \\ Y_3 &= \frac{Y_1 + \Omega^* Y_2}{1 + \Omega^{*2}}, \\ Y_4 &= \frac{Y_1 \Omega^* - Y_2}{1 + \Omega^{*2}}, \\ Y_5 &= -\Omega^* Y_2 + Y_1 \tilde{Pr} - N_1^2 \tilde{Pr}^2 - \tilde{r}_0 \tilde{Pr} Y_3, \\ Y_6 &= -\Omega^* Y_1 - Y_2 \tilde{Pr} - \tilde{r}_0 \tilde{Pr} Y_4 \end{aligned}\right\}. \tag{16}$$

The summation form of the Fredholm-solvability condition is defined as:

$$\sum_{j=1}^{3} R_{ij} \hat{\phi} = 0, i = 2, 3. \tag{17}$$

Substituing equations (12) and (15) in equation (17), we obtain,

$$\tilde{r}_1 = 0 \tag{18}$$

and

$$\tilde{r}_2 = -\frac{\delta^2 \tilde{r}_0 \tilde{Pr}(Y_3 Y_5 + Y_4 Y_6)}{2(Y_5^2 + Y_6^2)}. \tag{19}$$

Thus the scaled Rayleigh number is given by,

$$\tilde{r} = \frac{M_2 - N_1^2}{M_2} - \varepsilon_1^2 \left(\frac{\delta^2 \tilde{r}_0 \tilde{Pr}(Y_3 Y_5 + Y_4 Y_6)}{2(Y_5^2 + Y_6^2)} \right). \tag{20}$$

Now we proceed to derive the Ginzburg-Landau equation(GLE) in the next section.

2.3 Derivation of Ginzburg-Landau Equation

To derive the GLE, we again use the Venezian approach [27]. Let us consider the following regular perturbations given by:

$$\left.\begin{aligned} A &= \varepsilon_2 A_1 + \varepsilon_2^2 A_2 + \varepsilon_2^3 A_3 \ldots \\ B &= \varepsilon_2 B_1 + \varepsilon_2^2 B_2 + \varepsilon_2^3 B_3 + \ldots \\ C &= \varepsilon_2 C_1 + \varepsilon_2^2 C_2 + \varepsilon_2^3 C_3 + \ldots \\ D &= \varepsilon_2 D_1 + \varepsilon_2^2 D_2 + \varepsilon_2^3 D_3 + \ldots \\ \tilde{r} &= \tilde{r}_0 + \varepsilon_2 \tilde{r}_1 + \varepsilon_2^2 \tilde{r}_2 + \ldots \end{aligned}\right\}, \tag{21}$$

where ε_2 is a small amplitude and ε_1=$O(\varepsilon_2^2)$.

Substituting equation (21) and assuming $\tilde{\tau} = \varepsilon_2^2\tau$ and $\varepsilon_1 = \varepsilon_2^2$ in the Lorenz model (1)-(4) and considering the like powers of ε_2, we get:

$$O(\varepsilon_2) : J_2 v_1 = [0,0,0,0]^{Tr}, \tag{22}$$

$$O(\varepsilon_2^2) : J_2 v_2 = [0,0,0,-A_1C_1]^{Tr}, \tag{23}$$

$$O(\varepsilon_2^2) : J_2 v_3 = \left[\frac{1}{\tilde{Pr}}\frac{dA_1}{d\tilde{\tau}} - \delta C_1 sin(\tilde{\Omega\tau}), \frac{M2}{\tilde{Pr}}\frac{dB_1}{d\tilde{\tau}}, \frac{1}{\tilde{r_0}}\frac{dC_1}{d\tilde{\tau}} - \frac{\tilde{r_2}A_1}{\tilde{r_0}} + \frac{A_1D_2}{\tilde{r_0}}, \frac{dD_1}{d\tilde{\tau}} \right. \\ \left. -A1C_2 - A_2C_1\right]^{Tr}, \tag{24}$$

where $\tilde{\Omega} = \dfrac{\Omega^*}{\varepsilon_2^2}$,

$$J_2 = \begin{bmatrix} -1 & -N_1 & 1 & 0 \\ -N_1 & -M_2 & 0 & 0 \\ 1 & 0 & -\dfrac{1}{\tilde{r_0}} & 0 \\ 0 & 0 & 0 & -b \end{bmatrix}, \tag{25}$$

$$v_i = \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix}, i = 1,2,3. \tag{26}$$

Using the solvability condition, we obtain the GLE of the form:

$$\frac{d}{d\tilde{\tau}}A_1(\tilde{\tau}) = \left(\frac{P_2 sin(\tilde{\Omega}\tilde{\tau}) + \tilde{r_2}}{P_1}\right)A_1(\tilde{\tau}) - \frac{P_3}{P_1}A_1^3(\tilde{\tau}), \tag{27}$$

where

$$\left.\begin{aligned} P_1 &= \frac{1}{\tilde{Pr}} + \frac{N_1^2 M_1}{M2^2\tilde{Pr}} + \tilde{r_0}, \\ P_2 &= \tilde{r_0}\delta, P_3 = \frac{\tilde{r_0}}{b} \end{aligned}\right\} \tag{28}$$

The equation (27) is solved numerically with the initial condtion $A_1(0) = 1$.

To investigate the effect of heat transfer, the expresion for the Nusselt number is derived in the next section.

2.4 Heat Transport: The Nusselt number

The Nusselt number, $Nu(\tau)$, is defined as:

$$Nu(\tilde{\tau}) = \frac{\left.\frac{\alpha}{2}\int_0^{\frac{2}{\alpha}}(1-z+T)\,dx\right|_{z=0}}{\left.\frac{\alpha}{2}\int_0^{\frac{2}{\alpha}}(1-z)\,dx\right|_{z=0}}. \tag{29}$$

Simplifying the equation (29) and obtaining the expression for $D(\tilde{\tau})$ in terms of A_1 by using euqations (23) and (27), we obtain the expression for the Nusselt number given by,

$$Nu(\tilde{\tau}) = 1 + \frac{2\tilde{r0}}{\tilde{r}_0 + \varepsilon_2^2\tilde{r}_2}A_1^2. \tag{30}$$

The average Nusselt number, in the interval $\left[0, \frac{2\pi}{\tilde{\Omega}}\right]$ is given by,

$$\overline{Nu(\tilde{\tau})} = \frac{\tilde{\Omega}}{2\pi}\int_0^{\frac{2\pi}{\tilde{\Omega}}} Nu(\tilde{\tau})d\tilde{\tau}. \tag{31}$$

We next discuss the results and then finally the conclusion.

3 Results and Discussion

This study focuses on formulating the GLE corresponding to the RBC system subjected to gravity modulation in a MPF framework. Furthermore, an expression for the time-averaged Nusselt number is derived to quantify the heat transfer behavior of the system. The physical significance of the MPF parameters involved in the analysis are as follows: The coupling parameter N_1 denotes the concentration of micron-sized suspended particles, where an increase in N_1 enhances system stability, with its values confined to $0 \leq N_1 \leq 0.9$. The inertia parameter N_2 accounts for the inertial effects of suspended particles; higher values of N_2 contribute to system stabilization, with $0 \leq N_2 \leq O_1$. The couple-stress parameter N_3 measures the resistance to fluid motion arising from couple stresses. An increase in N_3 intensifies this resistance, elevating the temperature and thereby promoting system instability. The parameter N_3 extends the classical Newtonian fluid framework by including the influence of couple stresses and body couples, and it satisfies $0 \leq N_3 \leq O_2$, where O_1 and O_2 are positive real constants.

By considering the accelerated gravity condition and neglecting the micropolar heat conduction parameter N_5, the governing equations, as formulated by Siddheshwar et al. [24], the modified Lorenz model is expressed through equations (1)–(4). Further, by employing the method proposed by Venezian [27], the scaled Rayleigh number $\tilde{r}$ obtained as given in equation (20), is obtained. Figures (1)–(5) illustrate the variation of $\tilde{r}_{2_c}$ with respect to the modulation frequency Ω^* for different values of Pr, N_1, N_2, and

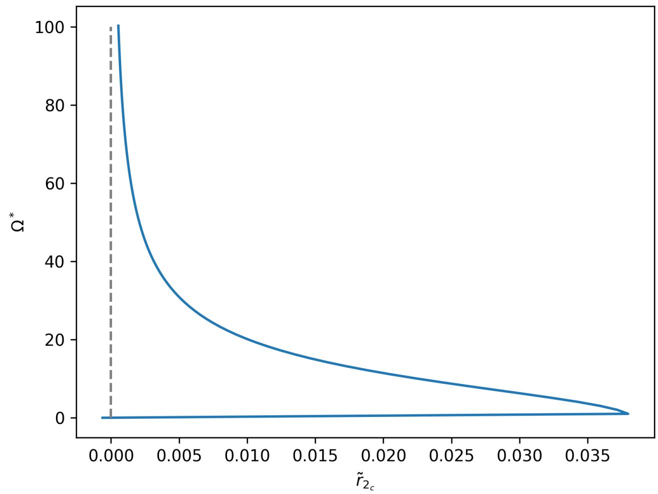

Figure 1: The graph of $\tilde{r_2}$ vs Ω^* for $\delta = 1$, $Pr = 10$, $N_1 = 0.1$, $N_2 = 1.0$, and $N_3 = 1.0$.

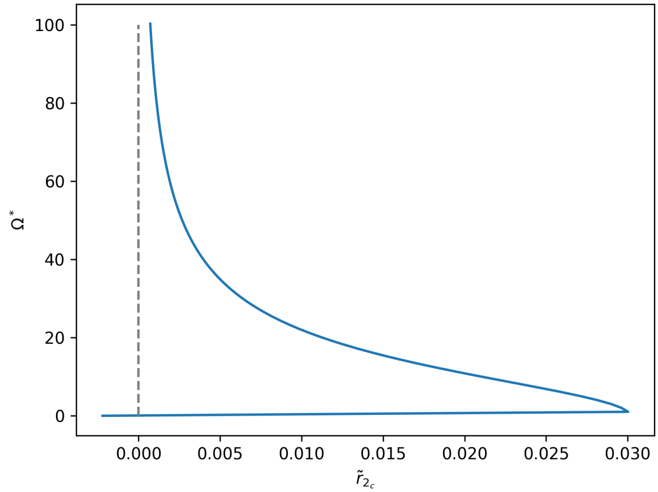

Figure 2: The graph of $\tilde{r_2}$ vs Ω^* for $\delta = 1$, $Pr = 10$, $N_1 = 0.5$, $N_2 = 1.0$, and $N_3 = 1.0$.

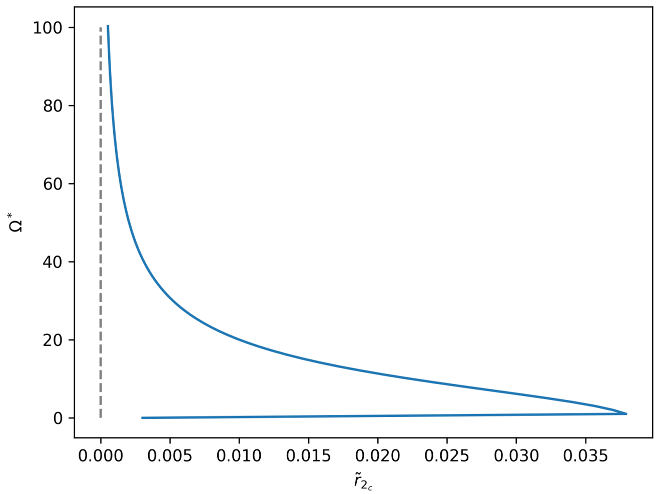

Figure 3: The graph of $\tilde{r_2}$ vs Ω^* for $\delta = 1$, $Pr = 10$, $N_1 = 0.1$, $N_2 = 1.0$, and $N_3 = 0.1$.

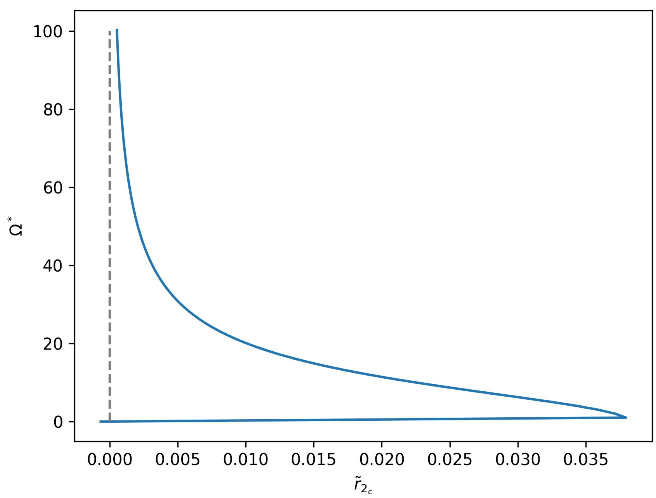

Figure 4: The graph of $\tilde{r_2}$ vs Ω^* for $\delta = 1$, $Pr = 10$, $N_1 = 0.1$, $N_2 = 0.1$, and $N_3 = 1.0$.

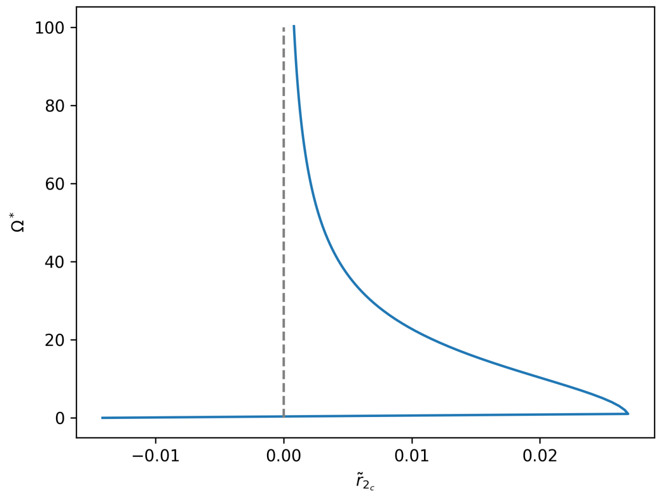

Figure 5: The graph of $\tilde{r_2}$ vs Ω^* for $\delta = 1$, $Pr = 15$, $N_1 = 0.1$, $N_2 = 1.0$, and $N_3 = 1.0$.

N_3. The results clearly indicate that the MPF parameters promote supercritical motion within the system.

Using the same analytical framework proposed by Venezian [27], the GLE for the present system is presented in equation (27). The expression for the time-averaged Nusselt number is given in equation (30), expressed in terms of the amplitude parameter A_1. The numerical value of A_1 is obtained by solving equation (27) numerically with the initial condition $A_1(0) = 1$. By substituting the values of the $Nu(\tau)$ into equation (31) and performing integration, the corresponding values of $\overline{Nu}$, are determined for various values of parameters and presented in the Figures (6)–(9). It is observed from figures (6) and (7) that an increase in Pr and N_1 leads to a rise in $\overline{Nu}$, indicating enhanced heat transfer with increasing Pr and N_1. In contrast, figure (8) presents that increasing N_2 hinders the heat transfer rate. The parameter N_3 exhibits a dual behavior: for smaller values of N_3, $\overline{Nu}$ increases, while for higher values of N_3, it remains nearly constant.

The gravity modulation in RBC systems with MPF has significant practical and theoretical applications in various areas of fluid dynamics and heat transfer. It serves as an effective mechanism for controlling convective instabilities and enhancing or suppressing heat transport in thermally driven flows. Such modulation techniques are particularly relevant in aerospace and microgravity environments, where variations in artificial gravity can affect thermal management and material processing. Additionally, gravity-modulated convection models are employed in geophysical and astrophysical studies to simulate oscillatory buoyancy effects in planetary and stellar interiors. In engineering contexts, controlled gravity modulation can be utilized in thermal control systems, chemical reactors, and crystal growth processes to optimize fluid mixing and improve uniformity in

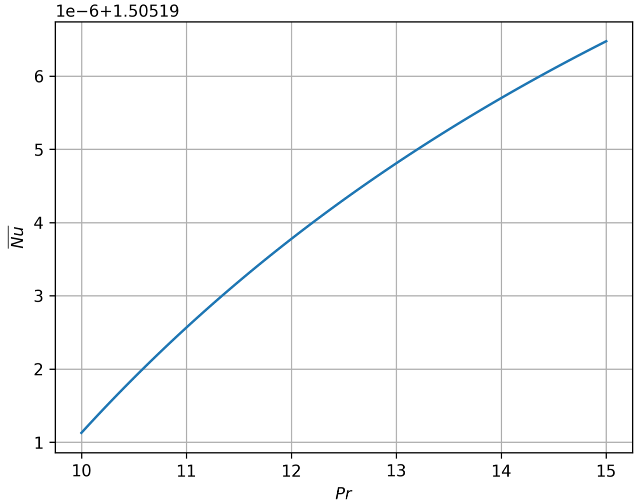

Figure 6: The graph of $\overline{Nu}$ vs Pr for $N_1 = 0.1$, $N_2 = 1.0$, $N_3 = 1.0$, $\delta = 1$, $\tilde{\Omega} = 10$.

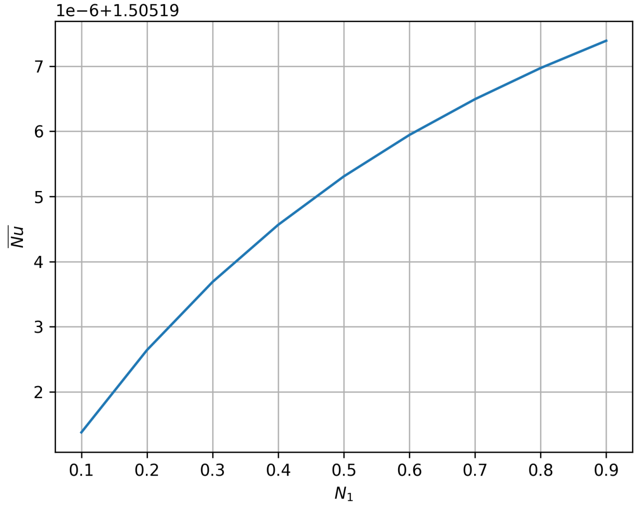

Figure 7: The graph of $\overline{Nu}$ vs N_1 for $Pr = 10$, $N_2 = 1.0$, $N_3 = 1.0$, $\delta = 1$, $\tilde{\Omega} = 10$.

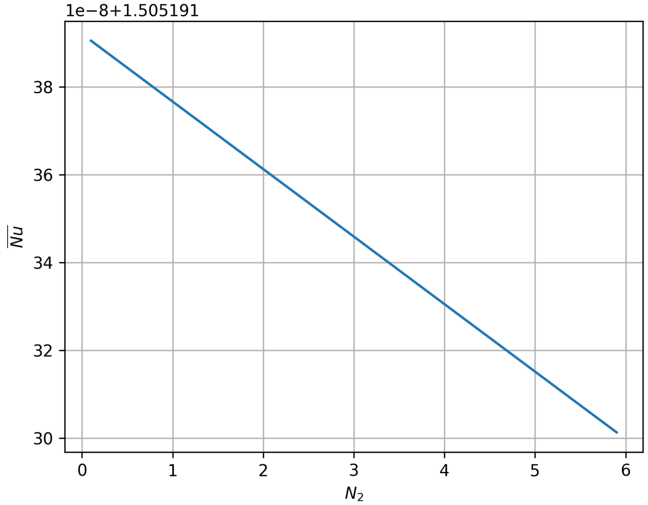

Figure 8: The graph of $\overline{Nu}$ vs N_2 for $Pr = 10$, $N_1 = 0.1$, $N_3 = 1.0$, $\delta = 1$, $\tilde{\Omega} = 10$.

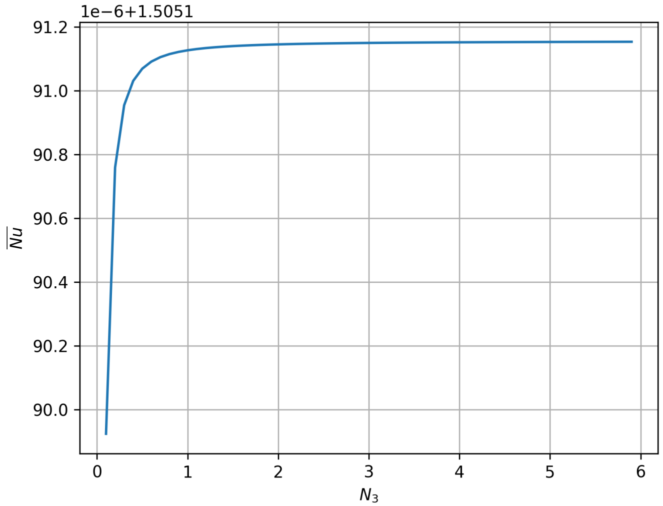

Figure 9: The graph of $\overline{Nu}$ vs N_3 for $Pr = 10$, $N_1 = 0.1$, $N_2 = 1.0$, $\delta = 1$, $\tilde{\Omega} = 10$.

heat distribution. The present problem can be further extended by including additional conditions, such as magnetic field, rotation, porous media, additional solutes and others.

4 Conclusion

The key findings of the study are:

1. The Ginzburg-Landau equation is derived for the Rayleigh-Bénard convection system of a micropolar fluid with gravity modulation using the Venezian method.
2. The parameters Pr, N_1, N_2, and N_3 are found to promote supercritical motion in the system.
3. Increase in the parameters Pr and N_1, advances the heat transfer.
4. Increase in the parameter N_2 is to suppress the heat transfer.
5. The heat transfer initially increases with N_3, followed by saturation at larger values, indicating a dual effect of the parameter.

References

[1] Bénard H., "Les tourbillons cellulaires dans une nappe liquide", *Revue Générale des Sciences Pures et Appliquées*, vol. 11, pp. 1261–1271, 1900.

[2] Rayleigh L., "On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side", *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 32, (192), pp. 529–546, 1916.

[3] Chandrasekhar S., *Hydrodynamic and Hydromagnetic Stability* (Dover Books on Physics Series). New York: Dover Publications, 1981, ISBN: 9780486640716.

[4] Liu W.-S., Wolf M. F., Elwell D, and Feigelson R. S., "Low frequency vibrational stirring: A new method for rapidly mixing solutions and melts during growth", *Journal of crystal growth*, vol. 82, (4), pp. 589–597, 1987.

[5] Zharikov E. V., Prihod'ko L. V., and Storozhev N. R., "Fluid flow formation resulting from forced vibration of a growing crystal", *Journal of crystal growth*, vol. 99, (1-4), pp. 910–914, 1990.

[6] Gresho P. M. and Sani R. L., "The effects of gravity modulation on the stability of a heated fluid layer", *Journal of Fluid Mechanics*, vol. 40, (4), 783–806, 1970. DOI: 10.1017/S0022112070000447.

[7] Biringen S and Peltier L., "Numerical simulation of 3-d bénard convection with gravitational modulation", *Physics of Fluids A: Fluid Dynamics*, vol. 2, (5), pp. 754–764, 1990.

[8] Clever R., Schubert G., and Busse F. H., "Two-dimensional oscillatory convection in a gravitationally modulated fluid layer", *Journal of Fluid Mechanics*, vol. 253, 663–680, 1993. DOI: 10.1017/S0022112093001946.

[9] Siddheshwar P. G., "A series solution for the Ginzburg-Landau equation with a time-periodic coefficient", *Applied Mathematics*, vol. 1, (06), p. 542, 2010.

[10] Siddheshwar P. G. and Kanchana C, "Effect of trigonometric sine, square and triangular wave-type time-periodic gravity-aligned oscillations on Rayleigh–Bénard convection in Newtonian liquids and Newtonian nanoliquids", *Meccanica*, vol. 54, (3), pp. 451–469, 2019.

[11] Siddheshwar P. G., Revathi B. R., and Kanchana C, "Effect of gravity modulation on linear, weakly-nonlinear and local-nonlinear stability analyses of stationary double-diffusive convection in a dielectric liquid", *Meccanica*, vol. 55, (10), pp. 2003–2019, 2020.

[12] Pranesh S., Siddheshwar P. G., Zhao Y., and Mathew A., "Linear and nonlinear triple diffusive convection in the presence of sinusoidal/non-sinusoidal gravity modulation: A comparative study", *Mechanics Research Communications*, vol. 113, p. 103 694, 2021.

[13] Francis R., Narayana M., and Siddheshwar P. G., "Gravity-modulated Rayleigh–Bénard convection in a Newtonian liquid bounded by rigid–free boundaries: a comparative study with other boundary conditions", *Journal of Engineering Mathematics*, vol. 139, (1), p. 5, 2023.

[14] Kiran P., "Gravity modulation effect on weakly nonlinear thermal convection in a fluid layer bounded by rigid boundaries", *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 24, (3), pp. 867–881, 2023. DOI: doi:10.1515/ijnsns-2021-0054.

[15] Jakhar A., Anurag, and Kumar A., "Weakly nonlinear instability analysis in triple-diffusive convection under gravity modulation in the presence of an internal heat generator", *Heat Transfer*, vol. 54, (1), pp. 167–183, 2025. DOI: https://doi.org/10.1002/htj.23170. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/htj.23170.

[16] Bixapathi S. and Babu A. B., "Analyzing the effects of internal heating and chemical reactions in a porous layer due to a vertical oblique magnetic field and gravity modulation", *Physics of Fluids*, vol. 37, (1), p. 014 119, Jan. 2025, ISSN: 1070-6631. DOI: 10.1063/5.0250097. eprint: https://pubs.aip.org/aip/pof/article-pdf/doi/10.1063/5.0250097/20343016/014119_1_5.0250097.pdf.

[17] Nagaraj S. B., Subramaniam P. V., and Bandaru N. R., "Rayleigh-Bénard convection with gravity modulation bounded by rigid isothermal boundaries", *AIP Conference Proceedings*, vol. 3283, (1), p. 040 006, Apr. 2025, ISSN: 0094-243X. DOI: 10.1063/5.0265353. eprint: https://pubs.aip.org/aip/acp/article-pdf/doi/10.1063/5.0265353/20475463/040006_1_5.0265353.pdf.

[18] Eringen A. C., "Theory of micropolar fluids", *Journal of Mathematics and Mechanics*, vol. 16, (1), pp. 1–18, 1966.

[19] Lukaszewicz G., *Micropolar fluids: Theory and Applications*. New York: Springer Science & Business Media, 1999.

[20] Siddheshwar P. G. and Pranesh S., "Effect of temperature/gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum", *Journal of Magnetism and Magnetic Materials*, vol. 192, (1), pp. 159–176, 1999, ISSN: 0304-8853. DOI: https://doi.org/10.1016/S0304-8853(98)00384-9.

[21] Siddheshwar P. and Pranesh S., "Effect of temperature/gravity modulation on the onset of magneto-convection in electrically conducting fluids with internal angular momentum", *Journal of Magnetism and Magnetic Materials*, vol. 219, (2), pp. 153–162, 2000, ISSN: 0304-8853. DOI: https://doi.org/10.1016/S0304-8853(00)00438-8.

[22] Pranesh S, Tarannum S., and Joseph T. V., "Linear and Weakly Non-Linear Analyses of Gravity Modulation and Electric Field on the onset of Rayleigh-Bénard Convection in a Micropolar Fluid", *Journal of Advances in Mathematics*, vol. 9, (3), 2014.

[23] Yekasi V., Pranesh S, and Bathul S., "Effects of gravity modulation and internal heat generation on the onset of Rayleigh-Bénard convection in a micropolar fluid", *Journal of Advances in Mathematics*, vol. 12, (06), 2016.

[24] Siddheshwar P. G., Anirudh P., Jagadeesh Kumar M. S., and Idris R., "Study of chaos in Rayleigh–Bénard convection of a micropolar fluid", *International Communications in Heat and Mass Transfer*, vol. 165, p. 108 967, 2025, ISSN: 0735-1933. DOI: https://doi.org/10.1016/j.icheatmasstransfer.2025.108967.

[25] Lorenz E. N., "Deterministic nonperiodic flow", *Journal of atmospheric sciences*, vol. 20, (2), pp. 130–141, 1963.

[26] Anirudh P and Kumar M. J., "Study of regular and chaotic dynamics in double diffusive Rayleigh-Bénard convection of a micropolar fluid", *Results in Engineering*, vol. 28, p. 107 609, 2025, ISSN: 2590-1230. DOI: https://doi.org/10.1016/j.rineng.2025.107609.

[27] Venezian G., "Effect of modulation on the onset of thermal convection", *Journal of Fluid Mechanics*, vol. 35, pp. 243 –254, 1969.