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Nonlinear Analysis of the Non-Sinusoidal Temporal Periodic Modulation of Boundary Temperature and Non-Inertial Acceleration on Ferroconvection

Anthony Christy Melson*[a], Shazia P. A.[a], and G. N. Sekhara[a]
[a]Department of Mathematics, B.M.S. College of Engineering, Bengaluru, 560019, India.

Abstract

The paper reports the effects of different types of temporal periodic modulation of boundary temperature (TPMBT) on convective heat transfer in a rotating ferroliquid. The linear stability analysis yields the eigenvalue for the onset of convection. The amplitude equations for nonlinear analysis have been derived with the aid of the truncated double Fourier Series in its minimal mode. The system of amplitude equations which is a Lorenz-like model is a non-autonomous system due to TPMBT. The results for various parameters have been discussed for both terrestrial as well as the microgravity case. It is observed that increasing strength of the magnetic field advances the onset of convection but does not result in enhanced heat transfer. The Coriolis acceleration stabilizes the system and hence results in subdued heat transfer. It is also observed that choice of the waveform and frequency of modulation can be used to control heat transfer. Classical Lorenz model and the results of dielectric liquid can be obtained as a limiting case of the present study.

Keywords: Ferroliquids; Ferroconvection; Rotation; Modulation.

1 Introduction

It is well known that ferroliquids have been a preferred over magnetic fluids owing to its applications varying from mechanical and electrical devices to the medical field as reported by Kaiser and Miskolczy [1], Raj and Boulton [2], Raj and Moskowitz [3], Raj et al. [4], Raj and Chorney [5], Scherer and Figueiredo Neto [6], Sakellari [7], Laird [8] and references therein. These applications include convective rotatory systems in terrestrial gravity as well as microgravity environment.

The convective instablity in ferroliquids have been well documented starting with the work of Finlayson [9] followed by Lalas and Carmi [10], Berkovsky and Bashtovoi [11], Shliomis [12] to name a few. The effect of micropolarity on ferroconvction was reported by Abraham [13]. Some more studies on the onset of convection can be found in the works by Kaloni and Lou [14], Nanjundappa and Shivakumara [15], Prakash [16], Nanjundappa et al. [17], Laroze et al. [18], Sekhar et al [19], [20]. The effect of rotation on the onset of convection in ferroliquids have been investigated by Gupta and Gupta [21], Bhattacharyya and Abbas [22], Venkatasubramanian and Kaloni [23], Auernhammer and Brand [24], Kaloni and Lou [25], Laroze et al. [26] and Siddheshwar et al [27].

It is imperative to control convective instability in some of the applications and modulation of the boundary temperature or the body force or magnetic field or rotation is a natural choice. Siddheshwar and Abraham [28], Bajaj [29], Singh and Bajaj [30], [31], Singh et al. [32], Chandrashekara and Rajashree [33] have studied the effects of temperature modulation on the onset of convection. The effect of temperature modulation on convective mechanism of a rotating ferroliquid has been reported by

* anthonycm.maths@bmsce.ac.in

Sibanda and Noreldin [34] and Maruthamanikandan *et al.* [35]. The effects of g-jitter/rotational modulation/magnetic field modulation on convection have been investigated by Engler and Odenbach [36], [37], Neha Anam et al. [38], Jayalatha and Muchikel [39] and Meghana et al. [40]. Kanchana et al. [41] have studied the impact of higher modes on regular and chaotic convection in ferroliquids. Dev and Suthar [42] have investigated the effect of penetrative convection via internal heating on ferroconvection through linear stability as well as energy stability methods. Nandal [43] has studied the effect of internal heat source and mixed boundary conditions on ferroconvection with Langevin magnetization.

The mentioned literature deal mainly with the onset of convection. Very few works are available that deal with the nonlinear analaysis. The investigations on nonlinear analysis when available deal mainly with the modulations of the sinusoidal type only. The choice of temperature modulations can be a controlling mechanism as reported by Bhadauria [44]. Hence it is an endeavour to understand the combined effects of rotation and different forms of thermal modulation on heat transfer in ferroliquids.

2 Mathematical formulation

A Newtonian ferromagnetic liquid is confined between two horizontal, thermally conducting, non-deformable boundaries that are assumed to have vanishing tangential stresses. The lower boundary is maintained at higher temperature than upper boundary. The temperatures at the boundaries are further subjected to time-periodic non-sinusoidal modulations. This setup is rotated about the axis normal to the liquid layer. The schematic of the configuration is as shown in Figure 1.

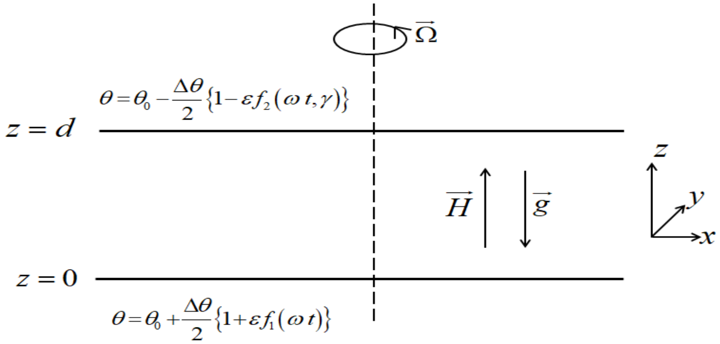

Figure 1: Schematic of the flow configuration.

The governing equations for the configuration mentioned above are as follows (Finlayson [9] and Auernhammer and Brand [24]):

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\rho_0 \left(\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} \right) = -\nabla P + \rho \vec{g} + \nabla \cdot \left(\vec{H}\vec{B} \right) + \mu \nabla^2 \vec{q} + 2\rho_0 \left(\vec{q} \times \vec{\Omega} \right), \tag{2}$$

$$\rho_0 C \left(\frac{\partial \theta}{\partial t} + \vec{q} \cdot \nabla \theta \right) = \kappa_1 \nabla^2 \theta, \tag{3}$$

$$\rho = \rho_0 [1 - \alpha(\theta - \theta_0)]. \tag{4}$$

It is important to note that ∇P incorporates the gradient of the fluid pressure and the centifugal force. The above system of equations are supplemented by the Maxwell's equations given by:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}), \tag{5}$$
$$\nabla \cdot \vec{B} = 0, \tag{6}$$
$$\nabla \times \vec{H} = 0. \tag{7}$$

The assumption that the magnetization is aligned with the magnetic field dictates that the $\vec{M} = M(H, \theta)\frac{\vec{H}}{H}$ where,

$$M(H,\theta) = M_0 + \chi_m(H - H_0) - K_m(\theta - \theta_0). \tag{8}$$

The boundary conditions accompanying the governing equations are given by

$$w = 0 \text{ and } \frac{\partial v}{\partial z} = 0 \qquad \text{at} \quad z = 0, d, \tag{9}$$
$$\left(\vec{B}_{internal} - \vec{B}_{external}\right).\hat{n} = 0 \qquad \text{at} \quad z = 0, d, \tag{10}$$
$$\left(\vec{H}_{internal} - \vec{H}_{external}\right) \times \hat{n} = 0 \quad \text{at} \quad z = 0, d, \tag{11}$$
$$\theta = \theta_0 + \frac{\Delta\theta}{2}[1 + \epsilon f_1(\omega t)] \qquad \text{at} \quad z = 0, \tag{12}$$
$$\theta = \theta_0 - \frac{\Delta\theta}{2}[1 - \epsilon f_2(\omega t, \gamma)] \quad \text{at} \quad z = d, \tag{13}$$

where $f_2(\omega t, 0) = f_1(\omega t)$ representing the rectangular, saw-tooth or traingular waveforms defined over $\left(0, \frac{\pi}{\omega}\right)$. The odd periodic extension of these waveforms over the interval $\left(0, \frac{2\pi}{\omega}\right)$ enables the use of Fourier series of sine's and hence expressed as:

$$f(\omega t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t), \tag{14}$$

where $b_n = \frac{2}{n\pi}[1 - (-1)^n]$ for the square wave, $b_n = \frac{2}{n\pi}(-1)^{n+1}$ for the sawtooth wave and $b_n = \frac{8}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$.

Conduction phase

The quiescent state being motionless is characterized by $\vec{q}_b = 0$ where b represents the basic state. The temporal modulation of the boundary temperature enables that the basic temperature to be expressed through a steady as well a transient component given by:

$$\theta_b = \theta_0 + \Delta\theta\left(\frac{1}{2} - \frac{z}{d}\right) + \epsilon\theta_t(z,t). \tag{15}$$

The transient part can be expressed as:

$$\theta_t(z,t) = \sum_{n=1}^{\infty} Re\left[\left\{a(\lambda)\exp\left(\frac{\lambda z}{d}\right) + a(-\lambda)\exp\left(\frac{-\lambda z}{d}\right)\right\} b_n i \exp(-in\omega t)\right]$$

where $a(\lambda) = \frac{\Delta\theta}{2}\left\{\frac{\exp(-i\gamma)-\exp(-\lambda)}{\exp(\lambda)-\exp(-\lambda)}\right\}$. The solution of the other state variables in the basic state can be expressed as:

$$\rho_b(z,t) = \rho_0[1-\alpha\{\theta_b(z,t)-\theta_0\}], \quad (17)$$

$$M_b(z,t) = M_0 - \frac{K_m\{\theta_b(z,t)-\theta_0\}}{1+\chi_m}, \quad (18)$$

$$H_b(z,t) = H_0 + \frac{K_m\{\theta_b(z,t)-\theta_0\}}{1+\chi_m}, \quad (19)$$

$$H_0^{external} = M_0 + H_0. \quad (20)$$

Convection Phase

The convection phase is analsysed by superposing infinitesimally small disturbances (f') on the basic state (f_b) to represent the transition from the condution phase to the convection phase. Hence all the field varaibles (f) are expressed as $f = f_b + f'$. The magnetization vector yields:

$$\vec{M}' = \left(\frac{M_0}{H_0}H_1', \frac{M_0}{H_0}H_2', \chi_m H_3' - K_m\theta'\right). \quad (21)$$

Equations (1) – (8) reduces to:

$$\nabla\cdot\vec{q}' = 0, \quad (22)$$

$$\rho_0\left(\frac{\partial\vec{q}'}{\partial t} + \vec{q}'\cdot\nabla\vec{q}'\right) = -\nabla P' + \rho'\vec{g} + \mu\nabla^2\vec{q}' + B_0\frac{\partial\vec{H}'}{\partial z} + (\vec{B}'.\nabla)\vec{H}_b' + (\vec{B}'.\nabla)\vec{H}' + 2\rho_0(\vec{q}'\times\vec{\Omega}), \quad (23)$$

$$\rho' = -\rho_0\alpha\theta, \quad (24)$$

$$\frac{\partial\theta'}{\partial t} = \frac{\Delta\theta}{d}w' - \vec{q}'\cdot\nabla\theta' + \kappa\nabla^2\theta', \quad (25)$$

$$\left(1+\frac{M_0}{H_0}\right)\left(\frac{\partial H_1'}{\partial x} + \frac{\partial H_2'}{\partial y}\right) + (1+\chi_m)\frac{\partial H_3'}{\partial z} - K_m\frac{\partial\theta'}{\partial z} = 0, \quad (26)$$

by assuming $K_m\Delta\theta \ll (1+\chi_m)H_0$.

The geometry considered indicates that all the field vaiables are invariant along y direction and convection cells are essentially two-dimensional rolls. Further, a stream function ψ' which agrees with equation (22) is introduced to facilitate the representation of the two-dimensional rolls as $\vec{q}' = [\nabla\times\psi'(x,z)\hat{j}] + v'(x,z)\hat{j}$. Equation (7) also introduces the scalar magentic potential ϕ'. Operating $-\hat{j}.[\nabla\times \text{equation}(23)]$, dropping the primes and following the standard non-dimensional procedure in accordance with the Buckingham-Π theorem yeilds:

$$\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2\psi = R\frac{\partial\theta}{\partial x} + RM_1\left[\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial\theta}{\partial x}\right]\frac{\partial\theta_b}{\partial z} + \nabla^4\psi - \sqrt{Ta}\frac{\partial v}{\partial z} + RM_1 J\left(\frac{\frac{\partial\phi}{\partial z},\theta}{x,z}\right) - \frac{1}{Pr}J\left(\frac{\psi,\nabla^2\psi}{x,z}\right). \quad (27)$$

The additional equation for v can be extracted from equation (23) as:

$$\frac{1}{Pr}\frac{\partial v}{\partial t} = \nabla^2 v + \sqrt{Ta}\frac{\partial\psi}{\partial x} + \frac{1}{Pr}J\left(\frac{v,\psi}{x,z}\right). \quad (28)$$

Equations (25) and (26) reduces to:

$$\frac{\partial\theta}{\partial t} = \frac{\partial\psi}{\partial z} + \nabla^2\theta + J\left(\frac{\theta,\psi}{x,z}\right), \quad (29)$$

$$M_3\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} - \frac{\partial\theta}{\partial z} = 0. \quad (30)$$

The dimensionless boundary conditions can now be expressed as:

$$\psi = \frac{\partial^2\psi}{\partial z^2} = \frac{\partial v}{\partial z} = \theta = \frac{\partial\phi}{\partial z} = 0 \text{ at } z = 0, 1. \quad (31)$$

The dimensionless numbers appearing in equations (27) - (30) are Pr, the Prandtl number, R, Rayleigh Number which can percieved as the dimensionless temperature difference, Ta, the Taylor number which is a measure of the Coriollis acceleration, M_1, the magnetic buoyancy parameter which depends heavily on the strength of the magnetic field, and M_3, the magnetization parameter which is a measure of departure of magnetization from linearity.

Linear Analysis

The linear stability analysis is adopted to investigate the effect of parameter on the onset of convection. The study involves dropping of the nonlinear terms from equations (27) - (30). The resultant system of equations is an eigen boundary value problem where R is the eigenvalue. The solution of the linearised equations satisfying equation (31) is of the form:

$$\psi(x,z,t) = A_{11}e^{\sigma t}\sin kx \sin\pi z, \quad (32)$$

$$\theta(x,z,t) = B_{11}\cos kx \sin\pi z, \quad (33)$$

$$v(x,z,t) = C_{11}\sin kx \sin\pi z, \quad (34)$$

$$\phi(x,z,t) = D_{11}\cos kx \sin\pi z, \quad (35)$$

where $\sigma = \sigma_r + i\omega$ is the complex frequency and k is the wavenumber. The use of equations (32) - (35) in the linearized eqautions results in a residual system which is integrated over the convective cell $(x,z) \in \left(0,\frac{2\pi}{k}\right) \times (0,1)$ after multiplying by correct orthogonal functions yields a system of linear homogeous equations in the amplitudes A_{11}, B_{11} and C_{11}. The absense of the time derivative in equation (30) results in D_{11} to be expressed in terms of B_{11}. The condition for the existence of the non-trivial solution of the homogeneous equation results in

$$\begin{vmatrix} \eta^2\left(\eta^2 + \frac{\sigma}{Pr}\right) & -Rk\left(1 + M_1 - \frac{M_1\pi^2}{\xi^2}\right) & \pi\sqrt{Ta} \\ \pi\sqrt{Ta} & 0 & -\left(\eta^2 + \frac{\sigma}{Pr}\right) \\ k & -(\eta^2 + \sigma) & 0 \end{vmatrix} = 0, \quad (36)$$

where $\eta^2 = \pi^2 + k^2$ and $\xi^2 = \pi^2 + M_3k^2$. Solving for the eigenvalue R, equation

(36) simplifies to

$$R = \frac{\xi^2(\eta^2+\sigma)\left\{\eta^2\left(\eta^2 Pr+\sigma\right)^2 + \pi^2 Ta\, Pr^2\right\}}{k^2 Pr\{\xi^2(1+M_1) - M_1\pi^2\}(\eta^2 Pr+\sigma)}. \quad (37)$$

It is customary to consider $\sigma_r = 0$ for marginal stability and further equation (37) can be expressed as $R = Re(R) + i\omega N$. The eigenvalue for the onset of convection is purely real. This results in the possiblity of either $\omega = 0$, which indicates the preference of instabilty to be through stationary mode or $N = 0$ which open up the possibility of oscillatory mode instability. Solving $N = 0$ for ω yields:

$$\frac{\omega^2}{Pr} = \frac{\pi^2 Ta(1-Pr)}{\eta^2(1+Pr)} - \eta^4 \quad . \tag{38}$$

It is necessary for ω to be real and this results in $Pr \in (0,1)$ and

$$Ta > \frac{(1+Pr)\eta^6}{(1-Pr)\pi^2}. \tag{39}$$

The eigenvalue for the stationary instability, R^S can be expresssed as:

$$R^S = \frac{\xi^2(\eta^6+\pi^2 Ta)}{k^2(\xi^2(1+M_1)-M_1\pi^2)}. \tag{40}$$

The critical values can be obtained by minimizing R^S with respect to the wave number k. The results of the usual viscous liquid (non-ferroliquid) can be obtained by setting $M_1 = 0$.

Convection in ferroliquids can set in even in the absence of gravity or the microgravity due to strong magnetic fields as established by Finlayson [9]. Setting $M_1 \to \infty$ and $R \to 0$ but $RM_1 = Rm$ to be finite; results in the eigenvalue Rm^S for stationary mode in microgravity environment given by:

$$Rm^S = \frac{\xi^2(\eta^6+\pi^2 Ta)}{M_3 k^4}. \tag{41}$$

Nonlinear Analysis

The nonlinear analysis involves solving the system of equations (27) – (30) subjected to the boundary conditions given by equation (30). The nature of the problem dictates a periodic solution. Hence a truncated double Fourier series with time-dependent amplitude is considered as given below.

$$\psi(x,z,t) = A_{11}\sin kx \sin \pi z \tag{42}$$
$$\theta(x,z,t) = B_{11}\cos kx \sin \pi z + B_{02}\sin 2\pi z \tag{43}$$
$$v(x,z,t) = C_{11}\sin kx \sin \pi z \tag{44}$$
$$\phi(x,z,t) = D_{11}\cos kx \sin \pi z + D_{02}\sin 2\pi z \tag{45}$$

The substitution of equations (31) - (34) in equations (27) - (30) and integrating over the convective cell given by $(x,z) \in \left(0,\frac{2\pi}{k}\right) \times (0,1)$, after multiplying by correct orthogonal functions yields a magnetic Lorenz model for convection in rotating ferroliquids. The amplitudes are further scaled using $X_1 = \frac{k\pi}{\sqrt{2}\eta^2}A_{11}, X_2 = \frac{\pi r}{\sqrt{2}}B_{11}, X_3 = -\pi r B_{02}, X_4 = \frac{k}{\sqrt{2Ta}}C_{11}$ and $\tau = \eta^2 t$ to obtain:

$$\frac{1}{Pr}\frac{dX_1}{d\tau} = -X_1 + X_2\left(1+\frac{\pi^2 Ta}{\eta^6}\right)\left[1 - M_{13}\left\{\frac{X_3}{\pi r} + 2\frac{\epsilon}{\pi}I(\tau)\right\}\right] - \frac{\pi^2 Ta}{\eta^6}X_4, \tag{46}$$

$$\frac{dX_2}{d\tau} = r[1 - 2\epsilon I(\tau)]X_1 - X_2 - X_1 X_3, \tag{47}$$

$$\frac{dX_3}{d\tau} = -bX_3 + X_1 X_2, \tag{48}$$

$$\frac{dX_4}{d\tau} = Pr[X_1 - X_4], \tag{49}$$

where $M_{13} = \frac{\pi M_1 M_3 k^2}{\pi^2 + k^2 M_3 (1+M_1)}$, $r = \frac{R}{R^S(Ta=0)}$, $I(\tau) = \int_0^1 F(z,\tau)\sin^2 \pi z \, dz$, and $b = \frac{4\pi^2}{\eta^2}$.

The system of equations (46) - (49) can be termed as the magnetic Lorenz model for a rotating ferroliquid which are solved subject to the initial conditions considered in the study are:

$$(X_1(0), X_2(0), X_3(0), X_4(0)) = (3, 3, 3, 3). \tag{50}$$

The choice of the inital values is from the unmouldated case where it is observed that the choice of intial value only changes the intercept by $Nu(\tau)$ on the vertical axis but converge to the same value with increasing τ. The main objective of the nonlinear analysis is to quantify heat transfer at the hotter boundary. It can be expressed throught the Nusselt number: a ratio of convective heat transfer to conductive heat transfer given by

$$Nu(\tau) = \frac{\frac{k_c}{2\pi}\int_0^{2\pi/k_c}\left[\left.\frac{d\theta_b}{dz}\right|_{\epsilon=0} + \frac{\partial\theta\prime}{\partial z}\right]_{z=0}}{\frac{k_c}{2\pi}\int_0^{2\pi/k_c}\left[\frac{d\theta_b}{dz}\right]_{\epsilon=0,z=0,}} = 1 + \frac{2X_3(\tau)}{r}. \tag{51}$$

The Lorenz-like model derived above is a dynamical system and hence the Nusselt number number is expected to dispaly an oscillatory behaviour. This could jeopardise the analysis of the impact of various parameters on heat transfer. This can be overcome through time averaging given by

$$\overline{Nu(\tau)} = \frac{1}{\tau_n}\int_0^{\tau_n} Nu(\tau)\, d\tau, \tag{52}$$

where $\tau_n = \frac{2\pi\eta^2}{\omega}$.

3 Results and Discussion

A linear as well nonlinear stablity analysis has been performed to investigate the heat transfer in rotating ferroliquid with time-periodically modulated thermal boundaries. The linear stability analysis reveals the effect of parameters on the onset of convection and the nonlinear stability identifies the effect of various parameters on the heat transfer at the hotter boundary.

Linear Stablity

The results are obtained are expressed in equations (39) – (41). Even though equation (39) is mathematically valid, there are no known commercially available ferroliquid with the Prandtl number $Pr \in (0,1)$. Hence, the possibility of oscillatory instabillity in rotating ferroliquids is ruled out. These results are in agreement with that of Auernhammer and Brand [24] and Melson et al. [45]. The critical values of the stationary Rayeigh number,, R_c^S (or Rm_c^S) can be obtained by minimizing equation (40) and (41) with respect to k. The critical values are represted in the table below.

Table 1: Critical values for the marginal stability in a rotating ferroliquid under terrestrial gravity.

M_1	Ta	$M_3=1$		$M_3=1.5$		$M_3=2$	
		k_c	R_c^S	k_c	R_c^S	k_c	R_c^S
1	10	2.5604	493.8912	2.5528	460.9519	2.53543	439.9223
	10^2	2.8704	577.6951	2.8518	540.1532	2.82962	516.9282
	10^3	3.9449	1049.1648	3.9075	992.9137	3.8791	960.6049
5	10	2.9300	223.5740	2.8450	193.8179	2.7792	177.2055
	10^2	3.1846	251.1261	3.0975	219.7269	3.0332	202.3904
	10^3	4.1538	413.4637	4.0656	373.6843	4.0073	352.3653
10	10	3.0359	131.2564	2.9219	111.6309	2.8407	100.9877
	10^2	3.2724	146.0242	3.1613	125.5999	3.0841	114.6041
	10^3	4.2077	234.7018	4.1045	209.7054	4.0382	196.5195

It can be observed from Table (1) that M_1 and M_3 have a destabilizing effect on the system as increasing M_1 or M_3 results in decreasing values of R_c^S which indicates an advanced onset of convection. But M_1 or M_3 have contrasting effects on the wave number. Increasing M_1 results in an increase in the wavenumber indicating a reduction in the convective cell but increasing M_3 results in a decrease in the wavenumber indicating an enlargment of the convective cell. The same is depicted through figure (2). The Coriollis force represented by the Taylor number Ta has a stabilizing effect on system as there is a delay in the onset of convection indicated by increasing value of R_c^S with increasing Ta. But this stabilization is slower than that of the non-ferroliquid, the results of which can be obtained by setting $M_1=0$. The same can be observed in figure (3).

Table 2: Critical values for the marginal stability in a rotating ferroliquid under microgravity.

Ta	$M_3=1$		$M_3=1.5$		$M_3=2$	
	k_c	Rm_c^S	k_c	Rm_c^S	k_c	Rm_c^S
10	3.1710	1578.2656	3.0181	1311.1878	2.9168	1170.9897
10^2	3.3848	1736.3133	3.2411	1462.2317	3.1471	1318.9000
10^3	4.2756	2710.4618	4.1527	2387.7056	4.0761	2220.6301

The critical values for the onset of convection in the microgravity environment is documented in Table (2). It is observed that increasing M_3 or Ta results in delayed onset of convection thereby stabilzing the system.

Nonlinear Stability

The nonlinear analysis of the effects of Coriolis acceleration and temporal modualtion of the boundary has been made. The abridged Fourier series in the minimal mode yields a non-autonomous system of temporal differential equations, contrary to the unmodulated case which yields an autonomous system of equations. Equations (46) – (49) recuces to the classical Lorenz model under the limiting case. The equations for microgravity environment can obtained as a limiting case. These nonlinear equations are solved numerically as they are analytically intractable. The 'ode' solver in Scilab an open source softawre is used for the purpose. The 'ode' solver from the LSODA package makes a dynamic choice between stiff and non-stiff methods by testing for stiffness at each step. This numerical solution is further utilised to compute Nusselt number to quantify heat transfer using equation (51). The time averaging given by equation (52) is through a numerical quadrature via the Simpson's $\frac{3}{8}$ rule. The results of temporally averaged Nusselt number as a function of various parameters are plotted to smooth out the oscillatory behaviour of a typical dynamical system.

The parameter domain has been briefed by Laroze et al. [18]. The fixed values of the paramters in the obtainining the results are $Pr=10$, $r=5$, $M_1=5$, $M_3=1.5$, $Ta=100$, $\omega=40$, $\epsilon=0.2$, with $k=\frac{\pi}{\sqrt{2}}$ in the case of terrestrial gravity and $k=\pi$ in the microgravity environment.

We now discuss the effect of each of the parameters on heat transfer for the terrestrial gravity case followed by the microgravity case.

Terrestrial gravity

Figures (4) - (10) are the plots of $\overline{Nu(\tau)}$ as functions of various parameters in the terrestrial gravity environment. Figure (4) shows the impact of Prandtl number Pr on It is interesting to note from figures (4) – (8) that there is no distinction between an unmodulated case and the case in which the modulation at both the boundaries are in-sync. But the heat transfer is enhanced upon introduction of a phase difference γ for moderate values of ω, and reduced heat transfer during out-of-sync modulation for very high values of ω. This indicates the importance of phase difference compared to the other modulation parameters.

The figures (4) – (10) also show the influence of different waveforms used in TPMBT. The square wave form of TPMBT with a phase difference is an ideal choice among the three forms for enhancing the heat transfer and the saw-tooth wave is the least favourable waveform at moderate values of ω. At very high values of ω, the order is reversed.

Microgravity case

Figures (11)-(16) are the plots of $\overline{Nu(\tau)}$ as functions of various parameters in the microgravity environment. Figure (11) shows the influence of Pr on heat transfer. Even in the case of microgravity, the heat transfer reduces with increase in the values of Pr but quantitatively the amount of heat transfer is slightly lower than that of the terrestrial gravity case. Figure (12) indicates that an increase in the value of scaled magnetic Rayleigh number r results in an increase in the values of $\overline{Nu(\tau)}$, in other words, the increase in temperature difference results in enhanced heat transfer. Figure (13) shows the plot of $\overline{Nu(\tau)}$ as function of M_3 in the microgravity environment. Unlike the terrestrial gravity case, increasing M_3 results in enhanced heat transfer in the microgravity situation. Figure (14) shows the effect of Coriolis acceleration on heat transfer. The declining curve is indicative of the fact that heat transfer reduces in a rotating ferroliquid with increasing angular velocity, in other words due to an increase in Ta. Figure (15) shows the effect of modulation frequency on $\overline{Nu(\tau)}$. The figure indicates that increasing the modulation frequency results in a very slight increase in heat transfer for small values of ω and then we see a reduced heat transfer with further increase in ω. At high values of ω, in-sync modulation results in higher heat transfer when compared to the asynchronous modulation. Figure (16) shows the effect of amplitude of modulation on heat transfer. The graph shows that the $\overline{Nu(\tau)}$ curve increases with increasing values of ϵ, which indicates that increasing the amplitude of modulation results in enhanced heat transfer.

Similar to the terrestrial gravity case, the in-phase modulation has no impact on the heat transfer. The effect of different waveforms with asynchronous TPMBT in the microgravity case is similar to that of the terrestrial gravity case. Further, one can see from figures (11)-(15) that the microgravity results in slightly reduced heat transfer when compared to the terrestrial gravity case.

An analogy between the ferroliquids and dielectric fluids is well established and reported by Siddheshwar et al. [27] and reference there in. The results of the dielectric fluid can be obtained as a limiting case of the present study by setting $M_1 = L$, the electric number and $M_3 = 1$.

4 Conclusion

The linear and non-linear analysis of the ferro-convection under the influence of Coriolis acceleration when the boundary temperatures are subjected to temporal periodic modulation has been investigated. The expressions for the eigenvalue for the marginal stability and the magnetic Lorenz model have been derived for both terrestrial gravity and microgravity environments. The following inferences are drawn from the results of the study.

- Increasing magnetic field as well as the departure of the mgnetization from linearity destabilizes the ferroliquid by advancing the onset of convection.
- Increasing rotation rates results in a stabilized system by delaying the onset of convection.
- There is a reduction in the heat transfer with increasing the values of the Prandtl number Pr.
- Increasing the temperature difference between the boundaries which is manifested through a corresponding increase in r results in enhanced heat transfer.
- Increasing values of M_1 results in diminished heat transfer in the presence/absence of temperature modulation.
- Increasing values of M_3 has similar effect on heat transfer as M_1 in the case of terrestrial gravity only. In the microgravity situation, the buoyancy forces are negligible and the heat transfer due to M_3 is antagonistic to that of the terrestrial gravity environment.

- The increasing values of Ta results in the reduced heat transfer.
- TPMBT has no influence on heat transfer when the modulation is in-phase i.e., when $\gamma=0$. The phase difference γ plays an important role in controlling the heat transfer. The phase difference results in higher heat transfer when compared to the unmodulated/in-phase modulation for moderate values of ω and diminished heat transfer at very high values of ω.
- The non-sinusoidal forms of modulation with phase difference can be used to obtain varied levels of heat transfer. The study reveals that

$$\overline{Nu(\tau)}_{square} > \overline{Nu(\tau)}_{triangular} > \overline{Nu(\tau)}_{saw\text{-}tooth}$$

  for moderate values of ω. The above order is reversed at very high values of ω.
- At a certain value of ω, the heat transfer due to different wave forms and in-sync and out-of-sync thermal modulation converge.

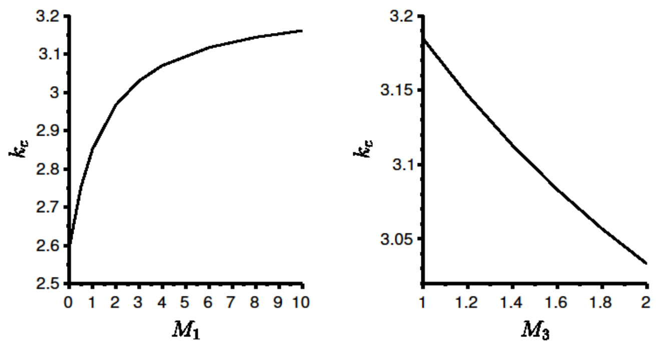

Figure 2: a) Plot of wave number vs M_1 when $M_3 = 1.5$ and $Ta = 100$.
b) Plot of wave number vs M_3 when $M_1 = 5$ and $Ta = 100$.

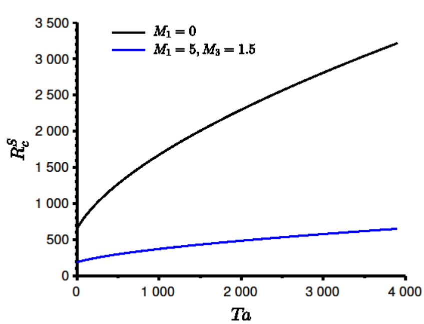

Figure 3: Plot of R_c^S vs Ta.

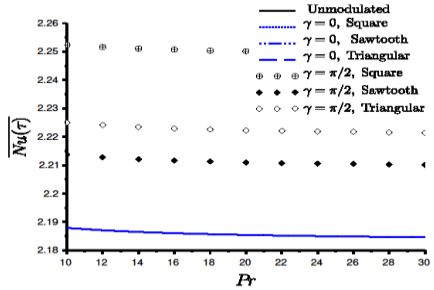

Figure 4: Variation of $\overline{Nu(\tau)}$ with Pr under terrestrial gravity.

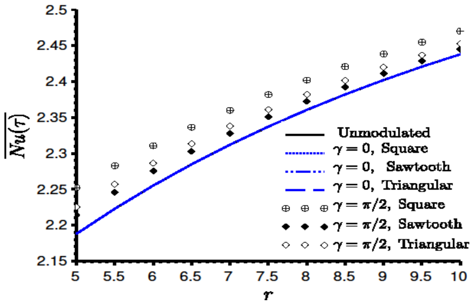

Figure 5: Variation of $\overline{Nu(\tau)}$ with r under terrestrial gravity.

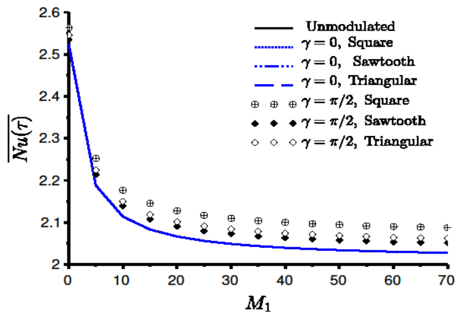

Figure 6: Variation of $\overline{Nu(\tau)}$ with M_1 under terrestrial gravity.

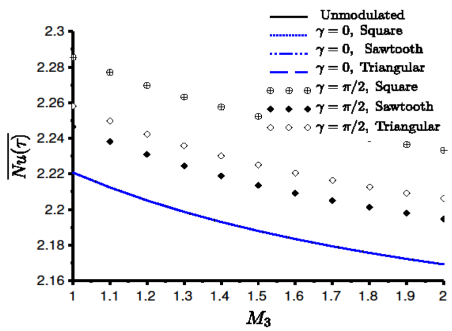

Figure 7: Variation of $\overline{Nu(\tau)}$ with M_3 under terrestrial gravity.

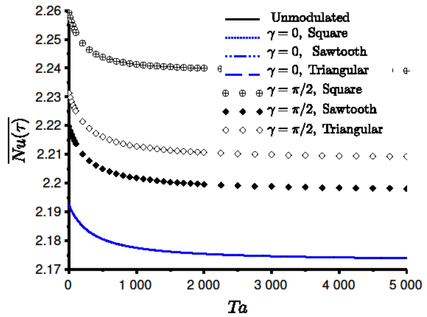

Figure 8: Variation of $\overline{Nu(\tau)}$ with Ta under terrestrial gravity.

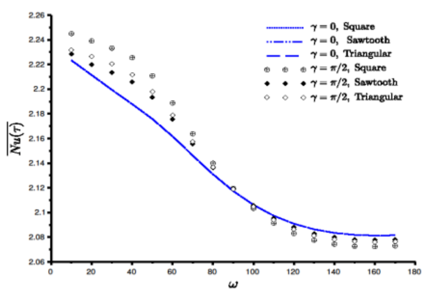

Figure 9: Variation of $\overline{Nu(\tau)}$ with ω under terrestrial gravity.

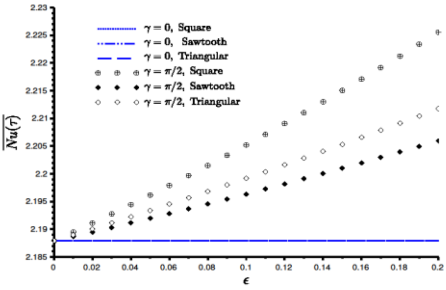

Figure 10: Variation of $\overline{Nu(\tau)}$ with ϵ under terrestrial gravity.

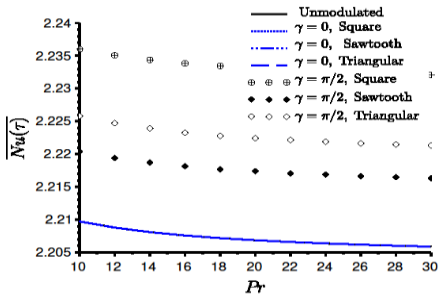

Figure 11: Variation of $\overline{Nu(\tau)}$ with Pr under microgravity.

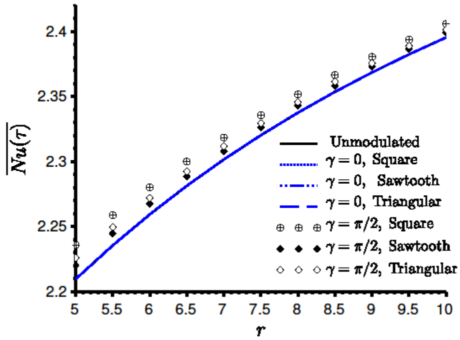

Figure 9: Variation of $\overline{Nu(\tau)}$ with r under microgravity.

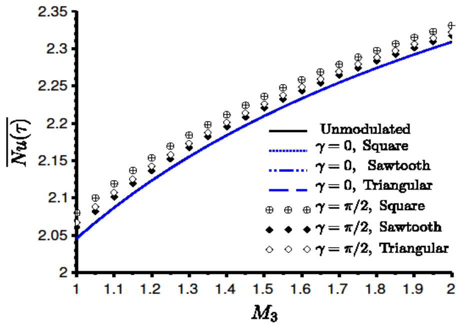

Figure 10: Variation of $\overline{Nu(\tau)}$ with M_3 under microgravity.

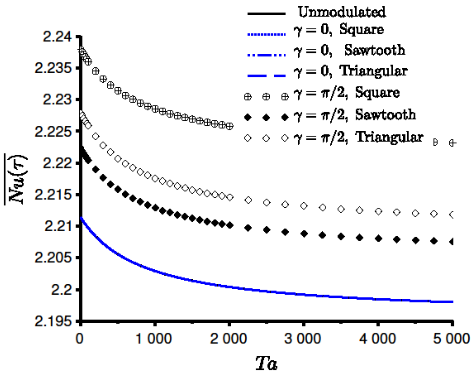

Figure 11: Variation of $\overline{Nu(\tau)}$ with Ta under microgravity.

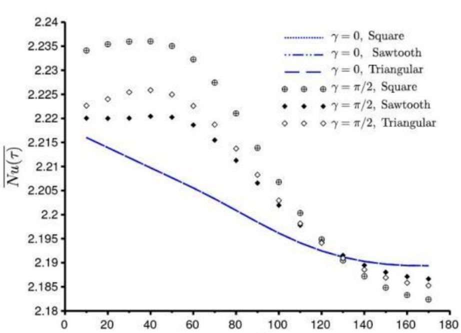

Figure 15: Variation of $\overline{Nu(\tau)}$ with ω under microgravity.

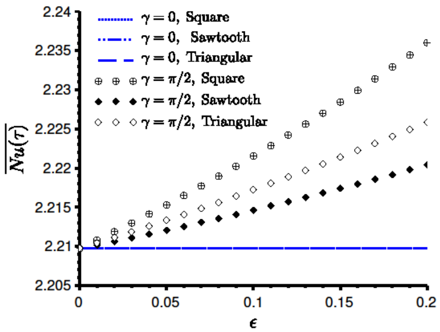

Figure 16: Variation of $\overline{Nu(\tau)}$ with ϵ under microgravity.

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Data Availability

The authors declare that no data was generated.