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MARAN'S THEOREM (NEW THEOREM) ON RIGHT-ANGLED TRIANGLE

* A.K. Maran

KEY WORDS

Geometry, Triangle and its related theorems, Pythagoras theorems I and II, Appollonius theorem and Euclid's theorems I and II. American mathematical society subject classification index.

Scope of Research

In Geometric, right-angled triangle is one of the two-dimensional plane having three sides with one of its angle is 90° and which is very important to solve problems related to Geometry and sometimes in other subject as well. Some fundamental concept /theorems of triangles are required to solve such problems and such theorems are (i) Pythagoras 1st theorem[1] (ii) Pythagoras 2nd theorem (iii) Appollonius theorem (iv) Euclid's 1st theorem[2] and (v) Euclid's 2nd theorem (Altitude theorem).[3] In addition to these, the author has attempted to develop a new theorem related to right-angled triangle (Maran's theorem of right-angled triangle). The new theorem have been discussed and proved with relevant examples.

* Assistant Executive Engineer (Vigilance) Ara. Kalai Maran, AEE(Vig), CSIR-H.Qrs, Anusandhan Bhavan, 2 Rafi Marg, New Delhi - 110001.

Maran's theorem on right-angled triangle

If a right-angled triangle ACB is divided into two parts viz., ADC and BDC by its altitude as its hypotenuse is its base (fig.1) and these two triangles are superimposed over triangle ABC as shown in figure.2

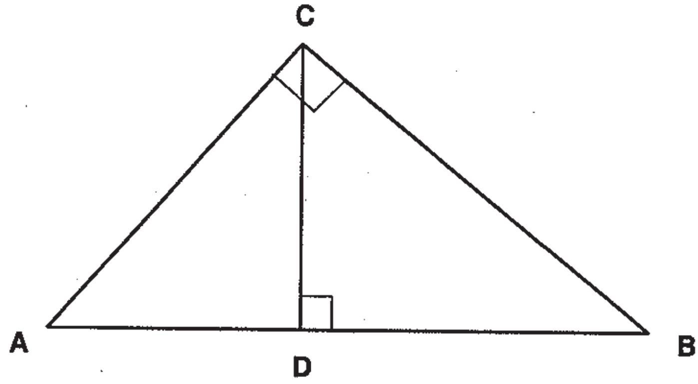

Fig.1: Typical example of Right angled Triangle ACB

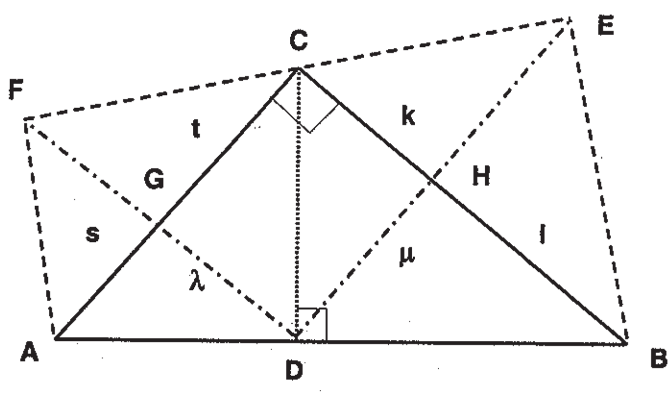

Fig.2: Superimposition of triangles over R.A.triangle ACB

Let, ACB is a right-angled triangle. m∠ACB = 90°. D is the point, which is projection of C on line AB i.e., line CD ⊥ to AB. Let CD is the altitude of the triangle, DG = λ, DH = μ, AG = s, CG = t, CH = k & BH = l. Therefore, m∠ADC = 90°, m∠BDC = 90°, ΔAFC ≡ ΔADC and ΔCEB ≡ ΔCDB.

Therefore, (i) The mathematical relation between a, b, λ and μ is

$$\lambda \times a = \mu \times b$$

(ii) The mathematical relation between k, l, s and t is

$$k \times t = l \times s$$

(iii) *ΔEDF is similar to ΔACB*

Example

See fig.2. In this figure, a right-angled triangle ACB, m∠ACB = 90°, a = 2√3, b = 2, m = 1, n = 3, λ = √3 / 2, μ = 3/2, k = √3 / 2, l = 3√3 / 2, s = 1/2 and t = 3/2. Prove the above all theorems.

(i) The first theorem is that: λ × a = μ × b and substitute the values a, b, λ & μ in LHS of the theorem,

$\lambda \times a = (\sqrt{3}/2) \times (2\sqrt{3}) = 3$ .... [1]

Substituting the values in RHS of the theorem,

$\mu \times b = (3/2) \times 2 = 3$ .... [2]

From [1] & [2], λ × a = μ × b and hence the above first theorem has been proved.

(ii) The second theorem is that: k × t = s × l and substitute the values k, t, s & l in LHS of the theorem,

$k \times t = (\sqrt{3}/2) \times (3/2) = (3\sqrt{3})/4$ .... [3]

Substituting the values in RHS of the theorem,

$\mu \times b = (3/2) \times 2 = 3$ .... [4]

From [3] & [4], k × t = s × l and hence the second theorem has been proved.

(iii) The third theorem is that: ΔEDF is similar to ΔACB

In triangle ACB, $\angle ACB = 180° - (\angle CAD + \angle CBD) = 90°$

Therefore, ACB is a right-angled triangle

$\angle CAD = Cos^{-1}(2/4) = Cos^{-1}(1/2) = 60°$ .... [5]

$\angle CBD = Cos^{-1}(2\sqrt{3}/4) = Cos^{-1}(\sqrt{3}/2) = 30°$ .... [6]

$\angle DFE = Cos^{-1}[(2\sqrt{3}/2) \div (2\sqrt{3})] = Cos^{-1}(1/2) = 60°$ .... [7]

$\angle DEF = Cos^{-1}[(2 \times 3/2) \div (2\sqrt{3})] = Cos^{-1}(3/2\sqrt{3}) = 30°$ .... [8]

In triangle FDE, $\angle FDE = 180° - (\angle DFE + \angle DEF)$

By substituting [7] & [8] in above equation, we get $\angle FDE = 90°$

From eqn [5] & [7], $\angle CAD = \angle DFE$. Similarly, from eqn [6] & [8], $\angle CBD = \angle DEF$

Therefore, ΔFDE is a right-angled triangle. ΔFDE is similar to ΔACB and hence the third theorem has also been proved.

References

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