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Hexagonal Difference Prime Labeling of Some Path Graphs

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Abstract

Hexagonal difference prime labeling of vertices of a graph is the labeling of the vertices of the graph with hexagonal numbers and the edges with absolute value of the difference of the labels of the incident vertices. The greatest common incidence number (*gcin*) of a vertex of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is 1, then the graph admits hexagonal difference prime labeling. Here we identify some path related graphs for hexagonal difference prime labeling.

Keywords: Graph labeling, hexagonal numbers, greatest common incidence number, path

Mathematics Subject Classification (2010): 05C10

1. Introduction

In this paper we deal with graphs that are connected, simple, finite and undirected. The symbol V(G) and E(G) denote the vertex set and edge set of a graph G, respectively. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)-graph.

A graph labeling is an assignment of integers to the vertices or edges or to both. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from [1] and [2]. In

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this paper, we investigate the hexagonal difference prime labeling of some path graphs.

Definition 1.1. *Let G be a graph with p vertices and q edges. The* ***greatest common incidence number****(gcin) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.*

Definition 1.2. *The* n^{th} ***hexagonal number*** *is* $n(2n-1)$*, where n is a positive integer. The hexagonal numbers are 1, 6, 15, 28, 45, 66, ...*

Definition 1.3. *Let G be a graph with p vertices and q edges. Let f: V(G)* $\rightarrow \{1, 6, 15, 28, \ldots, p(2p-1)\}$ *be a bijection given by* $f(v_i) = i(2i-1)$*, for every i from 1 to p. Let* $f^*_{hdpl}: E(G) \rightarrow \mathbb{N}$ *be a 1-1 mapping given by* $f^*_{hdpl}(uv) = |f(u)\text{-}f(v)|$*. The induced function* f^*_{hdpl} *is said to be* ***hexagonal difference prime labeling****, if the gcin of each vertex of degree at least 2, is one.*

Definition 1.4. *A graph which admits hexagonal difference prime labeling is called* ***hexagonal difference prime graph****.*

We now see some graphs that admit the hexagonal prime labeling.

2. Paths

Theorem 2.1. *The path* P_n *admits the hexagonal difference prime labeling.*

Proof. Let G = P_n. Let $v_1, v_2, \ldots, v_n$ be the vertices of G. Here |V(G)| = n and |E(G)| = $n-1$. Define a function $f: V \rightarrow \{1, 6, 15, 28, \ldots, n(2n-1)\}$ by $f(v_i) = i(2i-1)$, $i = 1, 2, \ldots, n$.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows:
$f^*_{hdpl}(v_i\, v_{i+1}) = (4i+1)$, i = 1, 2,..., n-1.
Clearly, f^*_{hdpl} is an injection. The *gcin* of (v_{i+1})=*gcd* of $\{f^*_{hdpl}(v_i, v_{i+1}), f^*_{hdpl}(v_{i+1}, v_{(i+2)})\}$ =*gcd* of {(4i+1), (4i+5)}=1, for i = 1, 2, ..., n-2.
So, the *gcin* of each vertex of degree greater than one is 1. Hence P_n, admits the hexagonal difference prime labeling. □

3. Ladder Graphs

Theorem 3.1. *The Ladder graph* $L_n = (P_n \square P_2)$ *admits the hexagonal difference prime labeling, when* $n \equiv 0 (mod 5)$.

Proof. Let G = L_n. Let $v_1, v_2, \ldots, v_{2n}$ be the vertices of G. Here |V(G)|= $2n$ and |E(G)| = $3n - 2$.
Define a function f: V → $\{1, 6, 15, 28, \ldots, 2n(4n-1)\}$ by
$f(v_i) = i(2i-1)$, $i = 1, 2, \ldots, 2n$.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows:
$f^*_{hdpl}(v_i v_{i+1}) = 4i+1$, i = 1,2,...,2n-1.
$f^*_{hdpl}(v_i v_{(2n-i+1)}) = (2n-i+1)(4n-2i+1) - i(2i-1)$, i = 1, 2, ..., n-1.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of (v_{i+1}) = 1, i = 1, 2,..., 2n-2. The *gcin* of (v_1) =*gcd* of $\{f^*_{hdpl}(v_1v_2), f^*_{hdpl}(v_1v_{2n})\}$ =*gcd* of $\{5, 8n^2\text{-}2n\text{-}1\}$= 1.
The *gcin* of (v_{2n}) =*gcd* of $\{f^*_{hdpl}(v_1v_2n), f^*_{hdpl}(v_{2n-1}v_{2n})\}$ =*gcd* of {8n-3, $8n^2$-2n-1}=*gcd* of {8n-3, n-1}=*gcd* of {5, n-1}=1.
So, the *gcin* of each vertex of degree greater than one is 1.
Hence L_n admits the hexagonal difference prime labeling. □

Example 3.2. *labeling of the ladder graph L_5*

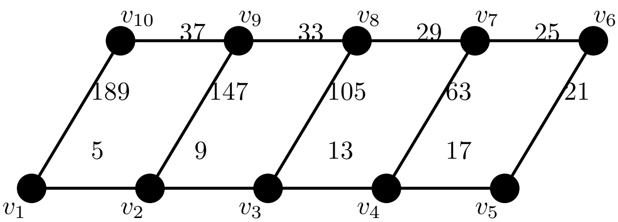

fig 1 : L 5

4. Middle Graphs

Theorem 4.1. *The middle graph of path P_n, admits the hexagonal difference prime labeling.*

Proof. Let G = $M(P_n)$. Let $v_1, v_2, \ldots, v_{2n-1}$ be the vertices of G.
Here |V(G)|= $2n-1$ and |E(G)| = $3n-4$.
Define f: V → {1, 6, 15, 28,..., (2n-1)(4n-3)} by $f(v_i) = i(2i-1)$, i = 1, 2, ..., 2n-1.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows:
$f^*_{hdpl}(v_i, v_{i+1})$= (4i+1), i = 1, 2, ..., 2n-2.
$f^*_{hdpl}(v_{2i}, v_{2i+2})$ = 16i+6, i = 1, 2, ..., n-2.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of (v_{i+1}) = 1, i = 1, 2, ..., 2n-3.

So, the *gcin* of each vertex of degree greater than one is 1. Hence $M(P_n)$ admits the hexagonal difference prime labeling. □

Example 4.2. *Labeling of the middle graph of P_5*

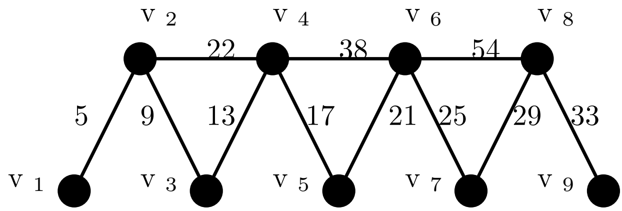

fig 2 : M (P 5)

5. Path Graphs

Theorem 5.1. *The total graph of path P_n admits the hexagonal difference prime labeling.*

Proof. Let $G = T(P_n)$. Let $v_1, v_2, \ldots, v_{2n-1}$ be the vertices of G.
Here $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$. Define a function f: $V \rightarrow \{1, 6, 15, 28, \ldots, (2n-1)(4n-3)\}$ by $f(v_i) = i(2i-1), i = 1, 2, \ldots, 2n-1$.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows: $f^*_{hdpl}(v_i v_{i+1}) = (4i+1)$, $i = 1, 2, \ldots, 2n-2$.
$f^*_{hdpl}(v_{2i} v_{2i+2}) = 16i+6$, $i = 1, 2, \ldots, n-2$.
$f^*_{hdpl}(v_{2i-1} v_{2i+1}) = 16i-2$, $i = 1, 2, \ldots, n-1$.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of $(v_{i+1}) = 1$, $i = 1, 2, \ldots, 2n-3$.
The *gcin* of (v_1) = *gcd* of $\{f^*_{hdpl}(v_1 v_2), f^*_{hdpl}(v_1 v_3)\}$= *gcd* of $\{5, 14\}= 1$.
The *gcin* of (v_{2n-1}) = *gcd* of $\{f^*_{hdpl}(v_{2n-3} v_{2n-1}), f^*_{hdpl}(v_{2n-1} v_{2n-2})\}$ = *gcd* of $\{8n-7, 16n-18\}$= *gcd* of $\{8n-7, 8n-11\}=1$.
So, the *gcin* of each vertex of degree greater than one is 1.
Hence $T(P_n)$ admits the hexagonal difference prime labeling. □

Example 5.2. *labeling of the total graph of P_5*

Theorem 5.3. *The graph P_n^2 admits the hexagonal difference prime labeling.*

Proof. Let $G = P_n^2$. Let $v_1, v_2, \ldots, v_n$ be the vertices of G.
Here $|V(G)| = n$ and $|E(G)| = 2n-3$. Define a function $f : V \rightarrow \{1, 6, 15, 28, \ldots, n(2n-1)\}$ by
$f(v_i) = i(2i-1)$, $i = 1, 2, \ldots, n$.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as

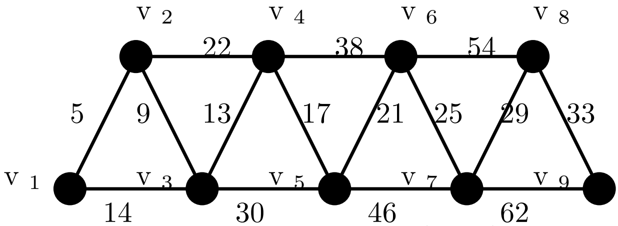

fig 3 : T (P_5)

follows:
$f^*_{hdpl}(v_iv_{i+1})= (4i+1)$, i = 1, 2, . . . , n-1.
$f^*_{hdpl}(v_iv_{(i+2)})= 8i+6$, i = 1, 2, . . . , n-2.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of (v_{i+1}) = 1, i = 1,2,. . . ,n-2
The *gcin* of (v_1) =*gcd* of $\{f^*_{hdpl}(v_1v_2), f^*_{hdpl}(v_1v_3)\}$=*gcd* of {5, 14}= 1.
The *gcin* of (v_n)=*gcd* of $\{f^*_{hdpl}(v_{(n-1)}v_n), f^*_{hdpl}(v_{(n-2)}v_n)\}$ =*gcd* of {4n-3, 8n-10}=*gcd* of {4n-3, 4n-7}= 1.
So, the *gcin* of each vertex of degree greater than one is 1.
Hence, P^2_n admits the hexagonal difference prime labeling. □

Theorem 5.4. *The Strong duplicate graph of path P_n, $SD(P_n)$, admits the hexagonal difference prime labeling, when n is not a multiple of 3.*

Proof. Let G = $SD(P_n)$. Let $v_1, v_2, \dots, v_{2n}$ be the vertices of G.
Here |V(G)| = $2n$ and |E(G)| = $3n - 2$.
Define a function f: V → {1, 6, 15, 28, . . . , 2n(4n-1)} by
$f(v_i) = i(2i-1)$, i = 1, 2, . . . , 2n.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows:
$f^*_{hdpl}(v_i, v_{i+1}) = (4i+1)$, i = 1, 2, . . . , 2n-1.
$f^*_{hdpl}(v_{(2i-1)}, v_{2i+2}) = 24i+3$, i = 1, 2, . . . , n-1.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of (v_{i+1}) = 1, i = 1, 2, . . . , 2n-2
The *gcin* of (v_1) =*gcd* of $\{f^*_{hdpl}(v_1v_2), f^*_{hdpl}(v_1v_4)\}$ =*gcd* of {5, 27}= 1.
The *gcin* of (v_{2n})=*gcd* of $\{f^*_{hdpl}(v_{2n-1}v_{2n}), f^*_{hdpl}(v_{(2n-3)}v_{2n})\}$ =*gcd* of {8n-3, 24n-21} =*gcd* of {8n-15, 8n-3}=*gcd* of {12, 8n-15}= 1.
So, the *gcin* of each vertex of degree greater than one is 1.
Hence $SD(P_n)$, admits the hexagonal difference prime labeling. □

Theorem 5.5. *The strong shadow graph of path P_n, admits the hexagonal difference prime labeling.*

Proof. Let G = $SD_2(P_n)$. Let $v_1, v_2, \dots, v_{2n}$ be the vertices of G.
Here |V(G)| = 2n and |E(G)| = $5n - 4$. Define a function f: V → {2, 6, 12, . . . ,2n(4n-1)} by

$f(v_i) = i(2i-1), i = 1, 2, \ldots, 2n$.
For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows:
$f^*_{hdpl}(v_i, v_{i+1})$= 4i+1,i = 1, 2, ..., 2n-1.
$f^*_{hdpl}(v_{(2i-1)}, v_{(2i+1)})$= 16i-2, i = 1, 2, ..., n-1.
$f^*_{hdpl}(v_{2i}, v_{2i+2})$ = 16i+6, i = 1, 2, ..., n-1.
$f^*_{hdpl}(v_{(2i-1)}, v_{2i+2})$ = 24i+3, = 1, 2, ..., n-1.
Clearly, f^*_{hdpl} is an injection.
The *gcin* of (v_{i+1}) = 1, i = 1, 2, ..., 2n-2.
The *gcin* of (v_1) = *gcd* of $\{f^*_{hdpl}(v_1v_2), f^*_{hdpl}(v_1v_3)\}$ =*gcd* of {5, 14} =1.
The *gcin* of (v_{2n}) =*gcd* of $\{f^*_{hdpl}(v_{2n}, v_{2n-1}), f^*_{hdpl}(v_{(2n-2)}\ v_{2n})\}$ =*gcd* of {8n-3, 16n-10}=*gcd* of {8n-7, 8n-3} =*gcd* of {4, 8n-7}=*gcd* of {1, 4}= 1.
So, the *gcin* of each vertex of degree greater than one is 1.
Hence $SD_2(P_n)$ admits the hexagonal difference prime labeling. □

6. Conclusion

In this paper, hexagonal difference prime labeling of vertices of ladder graphs, middle graphs and some path graphs are studied.

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