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Visualisation of Collage Grammar to Cell Works: ET0L Mode and Part Sensitive Mode

Ann Susa Thomas*

Abstract

Images are an important aspect of human life as one remembers pictures better than words. Informally, a two-dimensional string is called a picture. A two-dimensional language (or picture language) is a set of pictures. Picture generation and analysis has become a widely investigated field in Theoretical Computer Science and in Mathematics. Collage grammars are studied as devices that generate pictures by rewriting based on hyperedge replacement. A cell-work is a finite set of cells where each cell (being a three dimensional entity) is surrounded by one or more faces. This paper focuses on how cell work languages can be captured by collage grammar in ET0L and Part Sensitive modes.

Keywords: Formal Languages, Collage Grammar, Cell Work, ET0L Mode, Part Sensitive Mode, Production Rule, Hyperedge Replacement

Mathematics Subject Classification (2010): 03D05, 18B20, 68Q45, 68Q70, 68Q80

1. Introduction

From the early stage of development of 'formal language theory,' picture languages have attracted the attention of researchers who tried to give interesting frameworks for generating two and higher dimensional pictures such as arrays, trees, graphs etc. Syntactic methods that give rise to models for image or picture generation have been motivated by various problems that arise in the framework of pattern recognition and image processing and by different applications such as character recognition, pictorial information system design, and so

*Department of Mathematics, St. Thomas College, Kozhencherry, Pathanamthitta, Kerala; annsthomas11@gmail.com

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on. Graph grammars was introduced by A Rosenfeld as a formulation of some problems in pattern recognition and image processing. Graph grammar is similar to a string grammar in the sense that the grammar consists of finite sets of labels for nodes and edges, an axiom and a finite set of productions. Each production shows how a resulting graph can be derived from the previous one by a rewrite step. Collage grammars are graph grammars studied as devices that generate pictures by rewriting based on hyperedge replacement.

A collage [3] consists of a set of geometrical parts, a sequence of pin points, and a set of hyperedges each coming with a non terminal label and a sequence of attachment points. It specifies a picture by the overlay of all its parts. A cell work [6] is a finite set of cells. Each cell has one or more faces. Each face is surrounded by a boundary consisting of a finite, circular sequence of edges which meet at the vertices. Faces cannot intersect without forming an edge, although there can be faces without edges (in the case of a cell shaped as sphere or torus). A face is said to be incident with vertices and edges on its boundary. Each edge has one or two vertices associated with it. Cell work languages consist of cells which multiply in a certain pattern. The study of syntactic methods of picture generation aims at using concepts and results from formal language and automata theory [5] for picture generation. Collage grammars have been introduced [2] as devices that generate pictures by rewriting based on hyperedge replacement, that is, the replacement of a hyperedge by a collage if its pin points meet the attachment points.

2. Definitions

Now we see the major definitions.

Definition 2.1. *A **collage** (in $\mathbb{R}^n$)* [[2], [3], [4]] *is a pair (PART, pin) where PART $\subseteq \mathcal{P}(\mathbb{R}^n)$ is a set of parts, each part $\in$ PART being a set of points in $\mathbb{R}^n$, and pin $\in (\mathbb{R}^n)^*$ is a sequence of pin points. The class of all collages is denoted by C.*

In this paper each collage is a cell work.

Definition 2.2. *Let N be a set of labels. A **(hyperedge-)decorated collage** (over N)* [[2], [3], [4]] *is a construct C=(PART, E, att, lab, pin) where (PART, pin) is a collage, called the collage underlying C, E is a set of hyperedges, att: E $\to (\mathbb{R}^n)^*$ is called attachment mapping and lab: E $\to$ N is called label mapping. The class of all decorated collages over N is denoted by C(N).*

Informally, hyperedges act as place holders for (decorated) collages.

Definition 2.3. ***Hyperedge replacement*** [4] *is a construction where some hyperedges are removed, the associated collages are transformed in such a way that the images of the pin points match the points attached to the corresponding hyperedges and then the transformed collages are added.*

Definition 2.4. *Let $C \in C(N)$ and t: $\mathbb{R}^n \rightarrow \mathbb{R}^n$ be a mapping which will be referred to as a transformation. The transformation of C* [[2], [3], [4]] *by t yields the decorated collage, $t(C) = (t(PART_C), E_C, att, lab_C, pin)$ with $att(e) = t(att_C(e))\ \forall e \in E_C$ and $pin_{t(C)} = t\ (pin_C)$. The transformation transforms points and parts according to t. The set of hyperedges is not changed, labels remain unchanged.*

Definition 2.5. *Let N be a set of labels. A* ***production*** *(over N)* [[2], [3], [4]] *is a pair $p=(A, R)$ with $A \in N$ and $R \in C(N)$. A is called the left hand side of p and is denoted by lhs(p). R is called the right hand side and is denoted by rhs(p). A production $p= (A, R)$ is also denoted by $A::=_p R$.*

Definition 2.6. *A* ***collage grammar*** [[2], [3], [4]] *is a system $CG = (N, P, Z)$ where N is a finite set of non terminals, P is a finite set of productions (over N) with finite right hand sides and $Z \in C(N)$ is a finite decorated collage called the axiom.*

Definition 2.7. ***Context free collage grammar*** *is a collage grammar whose production p is such that the lhs(p) is a member of N and rhs(p) is a decorated collage or simply a collage.*

Definition 2.8. *Let $C, C' \in C(N)$. Then C* ***directly derives*** *C' if C' is isomorphic to the transformed C where the transformed C is such that the images of the pin points match the points attached to the corresponding hyperedges, and the transformed decorated collages are added. A direct derivation is denoted by $C \Longrightarrow C'$.*

Definition 2.9. *A* ***collage language*** [[2], [3], [4]] *generated by CG consists of all collages which can be derived from Z by applying productions of P and is denoted by $L(CG)=\{C \in C \mid Z \overset{*}{\underset{P}{\Longrightarrow}} C\}$.*

Definition 2.10. *An* ***ET0L collage grammar*** [2] *is a system $G = (N, T, Z)$ where T is a finite set of tables, every table $P \in T$ being a finite set of productions over N such that $\{lhs(p) \mid p \in P\} = N$, and $(N, \cup T, Z)$ is a collage grammar.*

Definition 2.11. *The* ***ET0L collage language*** *consists of all collages which can be derived from Z such that in each derivation step all hyperedges are replaced by productions of one production set $P \in T$.*

In the case of context free collage grammar in each derivation step, different productions may be used for different hyperedges. ET0L mode of rewriting makes use of only one production set in each derivation.

3. ET0L Cell Work captured by Collage Grammar

Let us now try to visualize how ET0L cell work languages can be captured by collage grammar through an example.

Example 3.1. *Let CG = ({S}, {{p}, {q}}, Z) be the collage grammar with the axiom and the productions as shown in figure 1.*

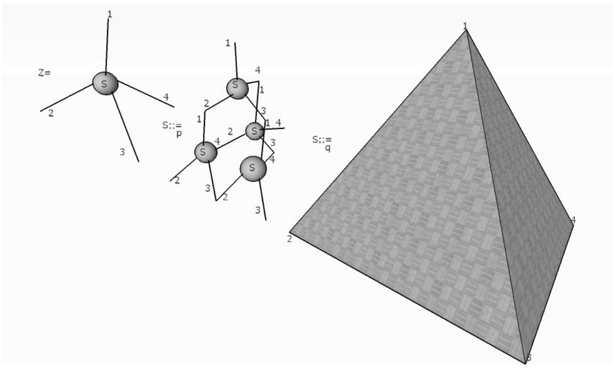

Figure 1: The Collage Grammar with the axiom and the productions

The hyperedge S is indicated as a sphere carrying four tentacles with attachment points named 1, 2, 3 and 4 as shown in figure 1. (It is to be noted that the attachment points are the end tips of the tentacles.) When the production rules are applied, the associated cells are transformed in such a way that the images of the pin points match the points attached to the corresponding hyperedges, and the transformed decorated cells are added. The transformed decorated collages replacing hyperedges are fully embedded into the resulting decorated collage, but their pin points lose their status.

4. Applying Production Rules

Starting from the axiom Z, if the production rule q is applied, the hyperedge labeled S is replaced in such a way that the attached points of S match the pin points of the triangular pyramid [1], a cell and the numbers are dropped as shown in figure 2. On the other hand, in the first derivation if the production rule p is applied, the hyperedge will be replaced by four hyperedges labeled S. To obtain a collage, now each of the four hyperedges will be replaced by a triangular pyramid in the second derivation (applying production rule q) after appropriate transformations such that images of the pin points match the points attached to the corresponding hyperedges as shown in figure

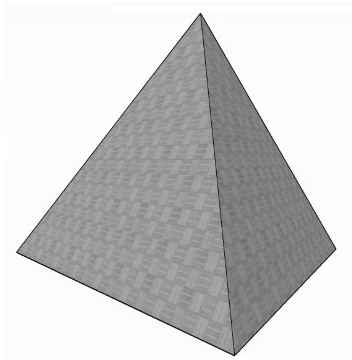

Figure 2: Triangular Pyramid

3.

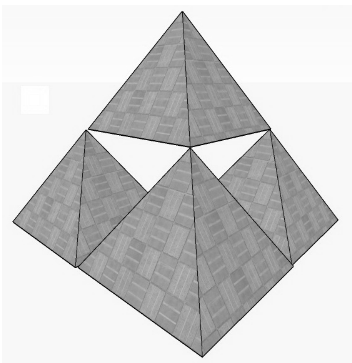

Figure 3: Hyperedges replaced by a Triangular Pyramid each

Assume that the production rule p is applied in the first derivation, followed by the production rule p again in the second derivation replacing each hyperedge by four hyperedges. In the third derivation apply the production rule q. The resultant cell work would be as in figure 4.

Continuing in this manner, if the production rule p is applied in n-1 derivations on each hyperedge starting from the axiom and then replace all the hyperedges simultaneously using production rule q, the resultant cell work would consist of 4^n cells as shown in figure 5.

The cell work language $L(CG)=\{C\in\mathcal{C} \mid Z \overset{*}{\underset{P}{\Longrightarrow}} C\}$ generated by ET0L collage grammar consists of Sierpinski tetrahedrons (3D version of Sierpinski triangle) using the two production rule sets. The size of

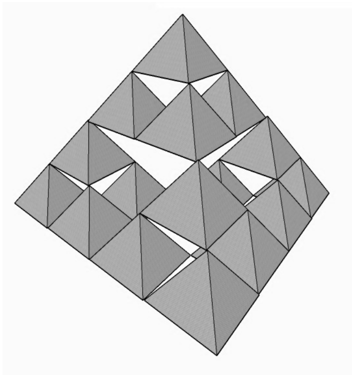

Figure 4: Cell Work

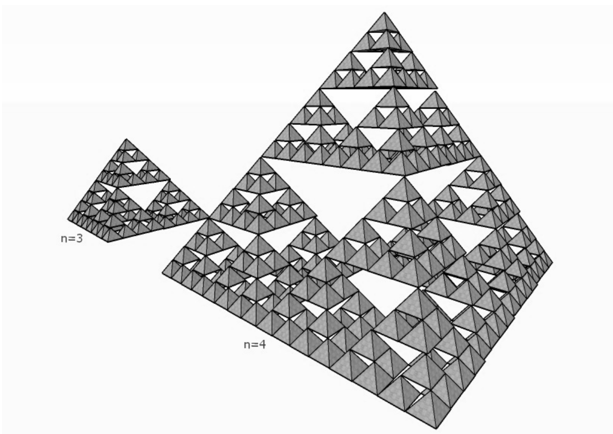

Figure 5: ET0L Cell Works

these collages grows with the factor 4 if the triangular pyramid in the right-hand side of q is considered to be one part.

If the production rules were not partitioned into tables, the resultant collage language would be of context free mode and hence many other collages would be derivable from the axiom.

5. Part-Sensitivity

As understood from the theory of formal languages the context sensitive grammar has production rules such that in a derivation a production rule can only be applied if certain symbols (specified in the production) are present in the neighbourhood. A context-sensitive version of collage grammars also works in a similar manner. Even though hyperedge replacement does not affect the rest of the collages, a production can only be applied if certain items (specified in the production) are present in the neighbourhood of the replaced hyperdege. In context sensitive collage grammars [2] hyperedges can move around and change in size. As a result, its size and location of items of their context carries information. This is why we shall only discuss this type of context-sensitivity, which we call part-sensitivity.

Definition 5.1. *A part-sensitive collage grammar [2] is a system* $G = (N, P, Z)$ *where N is the finite set of non terminals,* $Z \in \mathcal{C}(N)$ *is the axiom, and P is a finite set of part-sensitive productions over N. The collage language generated by G is* $L(CG) = \{C \in \mathcal{C} \mid Z \overset{*}{\underset{P}{\Longrightarrow}} C\}$.

Let us try to understand how part-sensitive cell work languages can be captured by collage grammar through an example.

Consider the part-sensitive collage grammar G = ({A, S}, P, Z) where P consists of the following six productions as shown in figures 6 to 10. Note that each cell considered is of the same size.

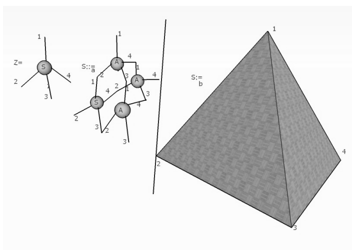

Figure 6: Production rules a and b

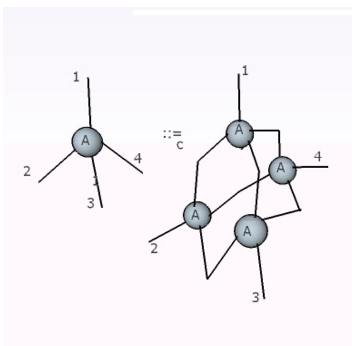

Figure 7: Production rule c

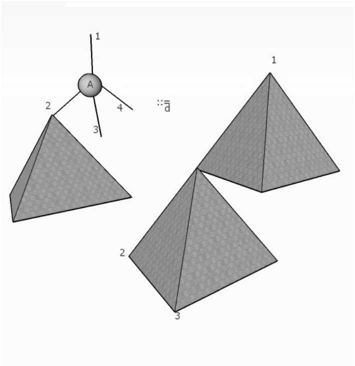

Figure 8: Production rule d

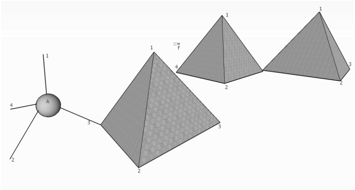

Figure 9: Production rule f

6. Deriving a Cell Work using Part-Sensitive Production Rules

Let us now derive a cell work using the part-sensitive production rules.

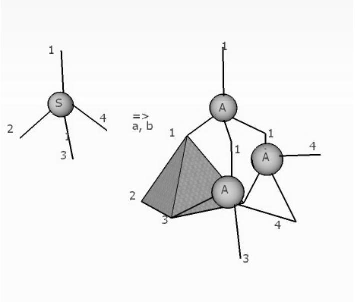

Figure 10: Decorated Cell Work 1

The cell work in figure 14 could be derived in a finite number of steps as shown in figures 11, 12 and 13 successively. This may not be the case always. Let us go for another derivation.

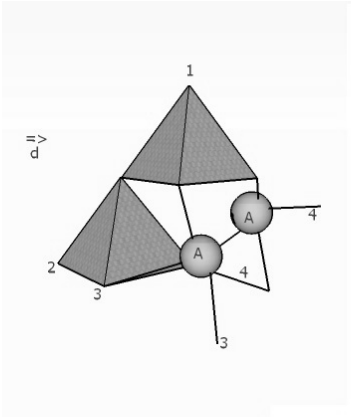

Figure 11: Decorated Cell Work 2

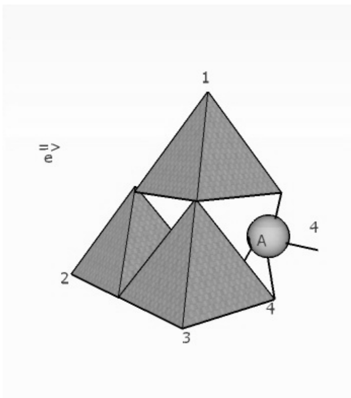

Figure 12: Decorated Cell Work 3

Starting from the axiom and applying the production rules a, b, c respectively we arrive at the decorated collage as in figure 15.

7. Acknowledgment

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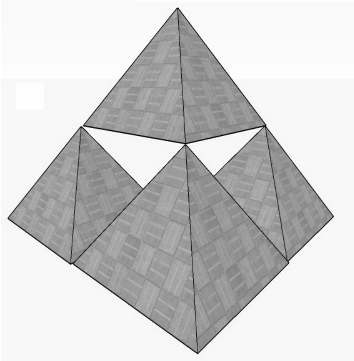

Figure 13: Decorated Cell Work 4

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