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NEW THEOREM ON PERPENDICULAR BISECTORS OF FOCAL RADII OF AN ELLIPSE

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ABSTRACT

Ellipse[1] is one of the conic sections, it is sort of elongated circle. It is the locus of a point that moves in such a way that the ratio of its distance from a fixed point to its distance from a fixed line equals to constant 'e'. According to Keplar's law of planetary motion,[2] the ellipse is very important in the field of Astronomy. The author has developed a new theorem for a new property of ellipse.

Key words: *Ellipse, Focal radius, Perpendicular bisector, Major axis and Minor axis*

New theorem on Ellipse

"If two focal radii[3] (pair) are drawn through both foci[4] from a point anywhere on peripheral of an ellipse (except extremities of major axis), then the perpendicular bisectors[5] of these two focal radii meets at line of minor axis[6]."

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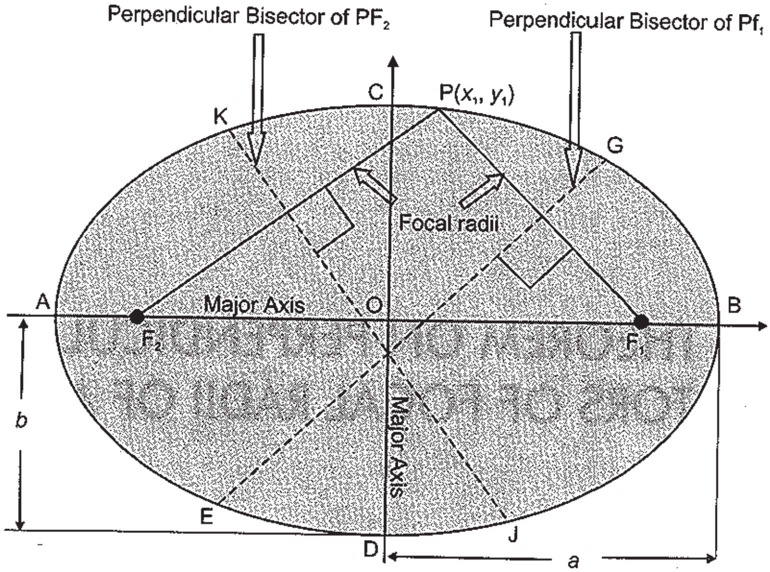

Figure 1 : Ellipse

Figure1 is the typical diagram of an ellipse. In this figure, 'O' is the centre of ellipse, F_1 & F_2 are the foci, A, B & C, D are the extremities of major and minor axis respectively, 'P' is the point anywhere on periphery of the ellipse (except extremities of major axis), PF_1 & PF_2 are the pair focal radii through F_1 & F_2 respectively. EG & JK are the perpendicular bisectors to PF_1 & PF_2 respectively. According to the theorem EG and JK meets at line of Minor axis.

Proof of the theorem

We know that the co-ordinates of points F_1 and F_2 are $\left(\sqrt{a^2-b^2},0\right)$ and $\left(-\sqrt{a^2-b^2},0\right)$ respectively. Let, co-ordinates of point P be (x_1, y_1). Therefore, the

Co-ordinate of mid point of line PF_1 is $\left(\dfrac{x_1+\sqrt{a^2-b^2}}{2},\dfrac{y_1}{2}\right)$ (1)

and co-ordinate of mid point of line PF_2 is $\left(\dfrac{x_1-\sqrt{a^2-b^2}}{2},\dfrac{y_1}{2}\right)$ (2)

We know that the slope of the line PF_1 is $\frac{y_2-y_1}{x_2-x_2} = \frac{y_1}{x_1-\sqrt{a^2-b^2}}$

Similarly, slope of the line PF_1 is $\frac{y_2-y_1}{x_2-x_1} = \frac{y_1}{x_1+\sqrt{a^2-b^2}}$

We already knew that "Two non-vertical lines are perpendicular if and only if m_1 and m_2 such that $m_1 \times m_2 = -1$" where m_1 and m_2 are slopes of any two lines perpendicular to each other.

So, we know that the equation of straight line through point (x_1, y_1) with slope 'm' is

$$y - y_1 = m(x - x_1)$$

Therefore, equation of perpendicular bisector of line PF_1 is given by

$$2yy_1 - y_1^2 = 2x\sqrt{a^2-b^2} - a^2 + b^2 - 2xx_1 + x_1^2 \quad (3)$$

Similarly, equation of perpendicular bisector of line PF_2 is given by

$$2yy_1 - y_1^2 = -2x\sqrt{a^2-b^2} - a^2 + b^2 - 2xx_1 + x_1^2 \quad (4)$$

Equating equations [3] and [4], we get

$$4x\sqrt{a^2-b^2} = 0$$

Therefore, $x = 0$ and it is nothing but y-axis (line of minor axis). Hence the theorem is proved.

Conclusion

The theorem is very useful for those doing research or further study in the field of ellipse or geometry, since this is also one of the important properties of an ellipse.

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