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Preclusion for Radix Triangular Mesh

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Abstract

In this paper we find the values of $mp(T_n)$ and $mp_1(T_n)$ when $n(n+1) \equiv 0 \pmod 4$. Every minimum matching preclusion set in an n-triangular mesh network is trivial. Also, $mp(G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of G and $mp_1(G) \leq v_e(G)$.

Keywords Matching, perfect matching, conditional matching, matching preclusion number, radix triangular mesh.

1. Introduction

In this paper, we use only the finite simple graphs, i.e., without loops or multiple edges. Let G be a graph of order n, and also consider this n is even. A *matching* M of G is a collection of pairwise non-adjacent edges. A *perfect matching* in G is a collection of edges such that every vertex is incident with exactly one edge in this set. The *matching preclusion number* of graph G, denoted by mp(G), is the minimum number of edges whose deletion leaves the resulting graph without a perfect matching. We define mp(G) = 0 if G has no perfect matchings. The concept of matching preclusion was introduced by Birgham *et al.*[1] and further studied by Cheng and Liptak [2, 3] with special attention given to interconnection networks. In [4] Park also puts forward some results on perfect matching. In [1], the matching preclusion number was determined for three classes of graphs viz., the complete graphs, the complete

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bipartite graphs $K_{n,n}$ and the hypercubes. Hypercubes are classical in the area of interconnection networks and have generated a considerable amount of research including fault tolerant routings, strong connectivity properties, various Hamiltonian properties and some others also. In certain applications, every vertex requires a special partner at any given time and the matching preclusion number measures the robustness of this requirement in the event of edge failures as indicated in [1]. Hence in these interconnection networks, it is desirable to have the property that the only optimal matching preclusion sets are those whose elements are incident to a single vertex.

The following propositions are obvious.

Proposition 1.1

Let G be a graph with an even number of vertices. Then $mp(G)\leq\delta(G)$, where $\delta(G)$ is the minimum degree of G.

Proof

Deleting all edges incident to a single vertex will give a graph with no perfect matchings and the result follows.

In a distributed system, it is unlikely that, in the event of random edge failure, all edges at some vertex fail. So naturally we raise this question. What are the obstruction sets for a graph with edge failures to have a perfect matching subject to the condition that the faulty graph has no isolated vertices? This gives the following definition.

The *conditional matching preclusion number* of a graph G with an even number of vertices, denoted by $mp_1(G)$, is the minimum number of edges whose deletion leaves the resulting graph with no isolated vertices and without a perfect matching.**[7]**

We define $mp_1(G) = 0$ if G has no perfect matchings, and we will leave $mp_1(G)$ undefined if a conditional matching preclusion set does not exist. i,e we cannot delete edges to satisfy both conditions in the definition.

But consider the u-v-w path in the original graph and delete all the edges incident to either u or w but not v. Then the resulting graph has no perfect matchings.

Thus we define $v_e(G) = \min\{d_G(u)+d_G(w)-2-Y_G(u, w)$: there exist v $\in V(G)$ such that u-v-w is a path$\}$ where $d_G(.)$ is the degree function and $Y_G(u, w) = 1$ if u and w are adjacent $= 0$ otherwise.

Mirroring the above proposition we get the following result.

Proposition 1.2

Let G be a graph with an even number of vertices. Suppose that every vertex in G has degree at least 3. Then $mp_1(G) \leq v_e(G)$.

In [8] E. Cheng *et al.* proved so many results related to k-regular bipartite graph. There are many results related to perfect matching preclusion number and conditional matching preclusion number of different types of graphs and networks in [7,9,10].

In the next section we try to find out the mp(G) of radix triangular mesh networks.

2. Matching Preclusion for Radix n triangular Mesh

Definition 2.1

An a*b mesh $M_{a,b}$ is a set of vertices $V(M_{a,b}) =\{(x,y)/\ 1 \leq x \leq a, 1 \leq y \leq b\}$ where any two vertices (x_1,y_1) and (x_2,y_2) are connected by an edge if and only if $|x_1-x_2|+|y_1-y_2|=1$.[5]

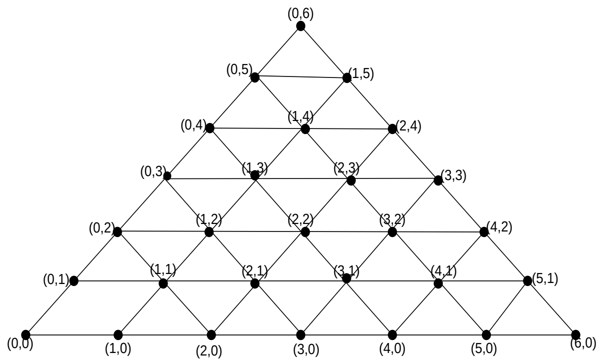
Definition 2.2

A pyramid of n levels, denoted by P_n, consists of a set of vertices $V(P_n) = \{ (k,x,y)/\ 0 \leq k \leq n\ 1 \leq x , \leq y \leq 2^k\}$. A vertex labelled (k,x,y) $\in V(P_n)$ is said to be a vertex at level k. All the vertices in level k form a $2^k \times 2^k$ mesh network. [6]

Definition 2.3

A radix n-triangular mesh network, denoted as T_n, consists of a set of vertices $V(Tn) =\{ (x,y)/\ 0 \leq x + y \leq n\}$ where any two vertices (x_1,y_1) and (x_2,y_2) are connected by an edge if and only if $|x_1-x_2|+|y_1-y_2| = n - 1$. The number of vertices and edges in T_n is equal to $n(n+1)/2$ and $3n(n-1)/2$ respectively. [6]

Example 2.4 T_6



Lemma 2.5

Any triangular mesh network T_n is Hamiltonian.

Proof

There are two cases to be considered.

Case 1 The network radix is odd.

Fig. 2 shows that how a Hamiltonian cycle can be constructed in a triangular mesh with an odd radix.

Case 2 The network radix is even

Fig. 3 shows how a Hamiltonian cycle can be constructed in a triangular mesh with an even radix.

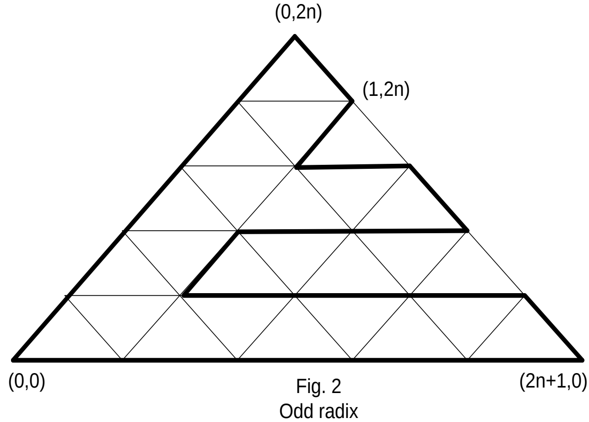


Fig. 2
Odd radix

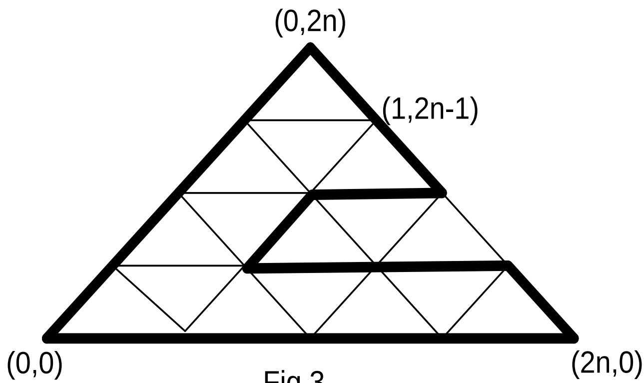


Fig.3
Even radix

Lemma 2.6

Let n be an integer and $n(n+1) \equiv 0(\bmod 4)$. Then T_n has exactly two edge disjoint perfect matchings.

Proof

Consider a Hamiltonian cycle in T_n. Take alternate edges in it. These will form two edge disjoint perfect matchings M_1 and M_2.

Let the edge (0, 0) and (1,0) $\in M_1$. Then (0, 0) and (0,1) $\in M_2$.

Suppose M_3 is another perfect matching in T_n. Then M_3 contains either {(0,0), (1,0)} or {(0,0), (0,1)}.

This implies that M_3 is not edge disjoint perfect matching from M_1 and M_2.

Proposition 2.7

For T_n, where $n > 3$ is an integer and $n(n+1) \equiv 0(\bmod 4)$, $v_e(G) = 3$.

Theorem 2.8

Let $n > 3$ be an integer and $n(n+1) \equiv 0(\bmod 4)$. Then the following statements hold.

(a) $mp(T_n) = 2$

(b) Every minimum matching preclusion set in T_n is trivial.

Proof

(a) By lemma 2.2, T_n has two edge disjoint perfect matchings M_1 and M_2.

$\therefore mp(T_n) > 1$

But $mp(T_n) \leq \delta = 2$ by prop 1.1

$\therefore mp(T_n) = 2$ □

Consider two perfect matchings M_1 and M_2 of T_n.

Assume that the edges r = (0,0),(1,0) is in M_1 and s = {(0,0), (0,1)} is in M_2.

Let F be a preclusion set in T_n.

Clearly $|F| = 2$.

T_n – F has no perfect matching. (∗)

Let F = $\{x, y\}$.

Case I

Let $x \in M_1$ and $y \in M_1$.

Then $M_2 \subseteq T_n - F$

$\therefore T_n - F$ has no perfect matching

A contradiction.

Similarly if x, y$\in M_2$, then we get a contradiction.

Case II

Let $x \in M_1$ and $y \in T_n - \{M_{1,} M_2\}$

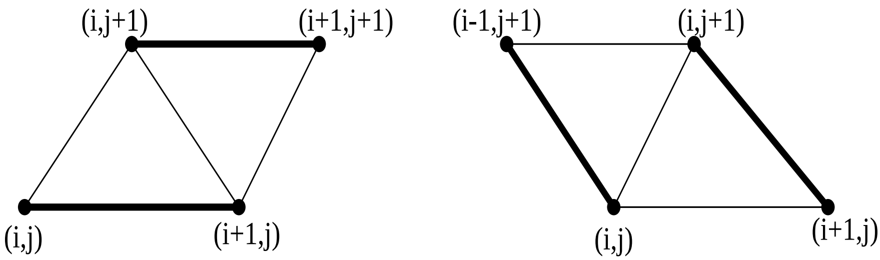
Then, $M_2 \subseteq T_n - F$, which again is a contradiction.

Case III

Let x = s, y $\in M_1$ and y $\neq$ r

Suppose y is an interior edge of T_n. Then y is a side of a parallelogram.

Clearly opposite sides of the parallelogram are also in M_1.



We've two types of parallelograms as shown in the above figure.

Suppose y = (i,j)(i+1,j) of the first diagram. Then (i,j+1)(i+1,j+1) $\in M_1$

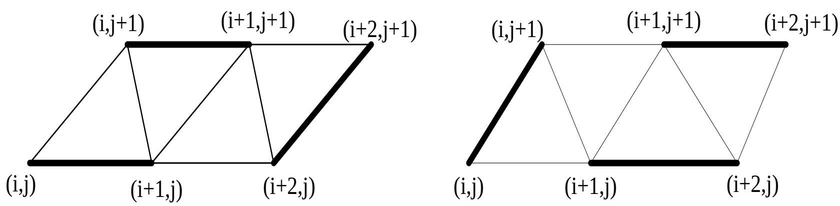
Now M_1 + {(i,j)(i,j+1), (i+1,j)(i+1,j+1)}- {y, (i,j+1)(i+1,j+1) } is a perfect matching in T_n - F

This is a contradiction.

Case IV

Let x = s, y $\in M_1$ and y $\neq$ r and y is a boundary edge of T_n. Then y may be a side of a parallelogram as in the previous case. Then we get a contradiction.

Suppose y is a side of a parallelogram as shown in the following figure.



Suppose y = (i,j)(i,j+1)

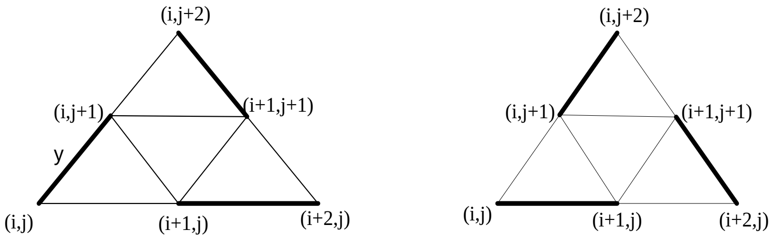
Then (i+1,j)(i+2,j),(i+1,j+1)(i+2,j+1) $\in M_1$

Now M_1 + {(i,j+1)(i+1,j+1), (i,j)(i+1,j)(i+2,j)(i+2,j+1)}- {y, (i+1,j)(i+2,j) ,(i+1,j+1)(i+2,j+1) } is a perfect matching in T_n - F

This is a contradiction.

Case V

Let x = s, y $\in M_1$ and y $\neq$ r and y is a boundary edge of T_n.



Suppose y = (i,j)(i,j+1)

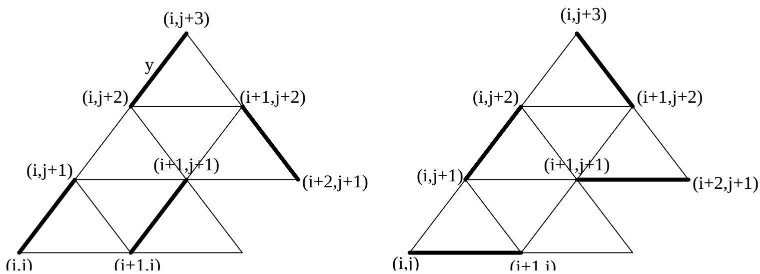
Then (i+1,j)(i+2,j),(i+1,j+1)(i,j+2) $\in M_1$

Now M_1 + {(i,j+1)(i,j+2), (i,j)(i+1,j), (i+1,j+1)(i+2,j)}- {y, (i,j+2)(i+1,j+1),(i+1,j) (i+2,j) } is a perfect matching in T_n - F

This is a contradiction.

Case VI

Suppose x = s, y $\in M_1$ and y $\neq$ r and also y in the boundary of T_n.



Suppose y = (i, j+2)(i,j+3)

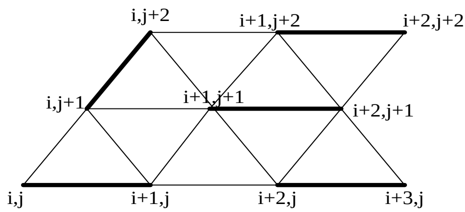
Then (i,j)(i,j+1), (i+1,j)(i+1,j+1), (i+2,j+1)(i+1,j+2) $\in M_1$

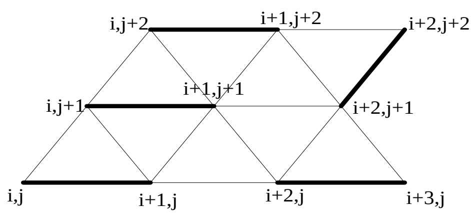
Now M_1 + {(i,j+3)(i+1,j+2), (i,j+1)(i,j+2),(i+1,j+1)(i+2,j+1), (i,j)(i+1,j)}- {y, (i,j)(i,j+1), (i+1,j)(i+1,j+1), (i+2,j+1)(i+1,j+2) } is a perfect matching in T_n - F

This is a contradiction.

Case VII

Suppose x = s, y $\in M_1$ and y $\neq$ r and also y in the boundary of T_n.





Suppose y = (i, j+1)(i,j+2)

Then (i,j)(i+1,j), (i+1,j+1)(i+2,j+1), (i+1,j+2)(i+2,j+2) $\in M_1$

Now M_1 + {(i,j)(i+1,j), (i+2,j)(i+3,j), (i,j+1)(i+1,j+1), (i+2,j+1)(i+2,j+2), (i,j+2)(i+1,j+2)}- {y, (i,j)(i+1,j), (i+1,j+1)(i+2,j+1), (i+1,j+2)(i+2,j+2) } is a perfect matching in T_n - F

This is a contradiction.

Hence the theorem ■

Theorem 2.9

Let n> 3 be an integer and n(n+1) ≡ 0(mod 4). Then the following statements hold true.

(a) $mp_1(T_n) = 3$

(b) Every conditional matching preclusion set in T_n is trivial.

Proof is similar to Theorem 2.4

References

[1] H. Sarbazi-Azad, M. Quld-Khaoua and L. Mackenzie,"Algorithemic constructions of Hamiltonians in pyramid networks," *Inf. Proc. Letters*, vol. 80, pp. 75-79, 2001.

[2] R. C. Birgham, F. Harry, E. C. Biolin and J. Yellen, "Perfect matching preclusion," *Congressus Numerantium,* vol. 174, pp. 185-192, 2005.

[3] E. Cheng and L. Liptak, "Matching preclusion for some interconnection networks," *Networks*, vol. 50, pp. 173-180, 2007.

[4] E. Cheng, L. Lensik, M. Lipman and L. Liptak, "Matching preclusion for alternating group graphs and their generalizations," *Int. J. Found. Comp. Sc.*, vol. 19, pp. 1413- 1437, 2008.

[5] J. Park, "Matching preclusion problem in restricted HL- graphs and recursive circulant G(2^m,4)," *J. KISS*, vol. 35, no. 2, pp. 60-65, 2008.

[6] E. Cheng, L. Lensik, M. Lipman and L. Liptak, "Conditional Matching preclusion sets," *Inf. Sc.*, vol. 179, pp. 1092- 1101, 2009.

[7] S. Razavi and H. S. Azad, "The triangular pyramid: Routing and topological properties." *Inf. Sc.,* vol. 180, pp. 2328-2339, 2010.

[8] E. Cheng, P. Hu, R. Jia and L. Liptak, "Matching preclusion and conditional matching preclusion for bipartite interconnection networks II, Cayley graphs generated by transposition trees and hyperstars", *Networks*, vol 59, no 4, pp. 357-364, 2012.

[9] E. Cheng and S. Padmanabhan, "Matching preclusion and conditional matching preclusion for crossed cubes," *Parallel Processing Letters*, vol.22, no.2, 2012.