Mapana J Sci, **13**, 3 (2014), 31-50
ISSN 0975-3303 | doi:10.12723/mjs.30.3

Shooting Method to Study Mixed Convection Past a Vertical Heated Plate with Variable Fluid Properties and Internal Heat Generation

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Abstract

Study of Mixed Convection past a vertical heated plate embedded in a sparsely packed porous medium with internal heat generation and variable fluid properties like permeability, porosity and thermal conductivity has been carried out numerically. In this analysis, the governing highly non-linear coupled partial differential equations are transformed into a system of ordinary differential equations with the help of similarity transformations and solved them numerically by using the shooting algorithm with Runge-Kutta-Fehlberg scheme and Newton Raphson method to obtain velocity, temperature and concentration distributions. The features of fluid flow, heat and mass transfer characteristics are analyzed by plotting the graphs and the physical aspects are discussed in detail to interpret the effect of various significant parameters of the problem. The results obtained show that the impact of buoyancy ratio parameter, Prandtl number Pr, Schmidt number Sc and other parameters plays an important role in the fluid flow through porous medium. The obtained results are compared with previously published work of

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the problem and they are found to be in very good agreement.

Keywords: Mixed Convection, porous medium, Internal heat generation, boundary layer.

1. Introduction

Mixed Convection is the flow situation where both free and forced convection effects are of comparable order. Study of such flow situations finds application in several industrial and technical processes such as electronic devices cooling, solar central receives exposed to winds, nuclear reactors cooled during emergency shut down and heat exchangers placed in a low-velocity environment. Internal heat generation enhances melting and impedes freezing. This phenomenon occurs in nuclear, geologic, cryogenic and material processing applications. The study of heat generation or absorption effects in moving fluids is important in problem dealing with chemical reactions and these concerned with dissociating fluids such as fluids undergoing exothermic or endothermic chemical reaction. Boundary layer flows with internal heat generation past a infinite vertical plate continues to receive considerable attention because of its many practical applications in a broad spectrum of Engineering systems like geothermal reservoirs, cooling of nuclear reactors, thermal insulation, combustion chamber, rocket engine, etc. Many investigations have been made for natural convection over a vertical plate with internal heat generation. Vajravelu and Hadjinicolaon [1] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. However the investigations have been made for mixed convection heat and Mass transfer with internal heat generation also. Chamkha and Kaled [2] investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption. Results of Seddeek [3] showed that the particle deposition rates were strongly influenced by thermophoresis and buoyancy force, particularly for opposing flow and hot surfaces in his study of the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation absorption. Alam et al [4] studied numerically combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Double-diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects was studied by Mohamed [5]. Ferdows [6] studied the effect of an exponential form of internal heat generation and variable viscosity in double diffusion problem of MHD from a porous boundary past a continuously moving semi-infinite vertical porous plate.

In some industrial applications, such as fixed-bed catalytic reactors, packed bed heat exchangers and drying, the value of the porosity is maximum at the wall and minimum away from the wall, so the porosity of the porous medium should be taken as non – uniform. Chandrasekhara et al ([7],[8]) has incorporated the variable permeability to study the flow past and through a porous medium and have shown that the variation of porosity and permeability has greater influence on velocity distribution and on heat transfer. Nevertheless, the inertia effects become important in a sparsely packed porous medium and hence their effect on mixed convection problems needs to be investigated. Mohammadein and El-shaer [9] studied mixed convective flow past a semi-infinite vertical plate embedded in a porous medium incorporating the variable permeability in Darcy's model. Nalinakshi et al [10] found numerical solutions for heat transfer from a vertical heated plate embedded in a Newtonian fluid sparsely packed porous medium considering the variable fluid properties with the influence of inertial parameter. Numerical Study of Double Diffusive Mixed Convection with variable Fluid properties was analyzed by Nalinakshi et al [11].

The main objective of the present investigation is to study systematically and numerically mixed convection heat and mass transfer for a Newtonian fluid flow past a semi infinite vertical heated plate embedded in a sparsely packed porous medium incorporating the variable porosity, permeability and thermal conductivity in the presence of Internal heat generation (IHG). To achieve this objective our plan of work is, in the analysis highly coupled non-linear partial differential equations governing the physical system are first reduced by a similarity transformations to the ordinary differential equations and then the resultant boundary value problem is converted into the system of seven simultaneous equations of first-order for seven unknowns. These equations are solved numerically by shooting technique by Runge-Kutta Methods to obtain horizontal velocity, temperature and concentration profiles for various physical parameters. The computed results here verify the accuracy of the method used under the limiting conditions which agree well with the existing ones.

2. Mathematical Formulation

We consider two-dimensional, laminar, steady- state boundary layer flows of an incompressible fluid past a semi-infinite vertical heated plate embedded in a sparsely packed Newtonian fluid saturated porous medium of variable porosity, permeability and thermal conductivity with Internal heat source. The x-coordinate is measured along the plate from its leading edge, and y-coordinate normal to it. Let U_0 be the Velocity of the fluid in the upward direction and the gravitational field, g, is acting in the downward direction. The plate is maintained at a uniform temperature T_w and at uniform concentration C_w which is always greater than the free stream values existing far from the plate (i.e. $T_w > T_\infty$ and $C_w > C_\infty$).

Considering the theory of boundary layer effect for sparsely packed porous medium with high porosity ε (but less than unity), with the assumptions: (a) the Bousinesque approximation is valid i.e., density is constant everywhere in the momentum equation except in the buoyancy force (b) permeability, porosity, thermal resistance are functions of the vertical coordinate y, and (c) local thermal equilibrium exists between fluid and solid phase, the governing basic equations for the conservation of mass, momentum, energy and species concentration for steady, viscous, incompressible, Newtonian fluid flow can be written as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta_T\left(T-T_\infty\right)-g\beta_C\left(C-C_\infty\right)+\frac{\bar{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\mu}{\rho}\frac{\varepsilon(y)}{k(y)}\left(U_o-u\right), \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+q'''+\frac{\bar{\mu}}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\gamma_m\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

where $\vec{q}=(u,v)$, u and v are the velocity components along the x and y directions, respectively. ρ is the density of the fluid, $\vec{g}$ is the acceleration due to gravity, U_0 is the free stream velocity, T is the temperature of the fluid, C is the concentration of the fluid, $\bar{\mu}$ is the effective viscosity of the fluid, μ is the fluid viscosity, C_p is the specific heat at constant pressure, β_T is the coefficient of volume expansion, β_C is the volumetric coefficient of expansion with species concentration, q''' is the exponential form of Internal Heat Generation (IHG). Equation (2) is the well-known Darcy-Brinkman equation which was first proposed by Brinkman [12] to include the boundary layer effect in the momentum equation. $k(y)$ is the variable permeability of the porous medium, $\varepsilon(y)$ is the variable porosity of the saturated porous medium, $\alpha(y)$ is the variable effective thermal diffusivity of the medium and γ_m is the effective solutal diffusivity of the medium.

To determine the flow field the above governing equations need to be solved subject to the boundary conditions. The different types of rigid surfaces boundary conditions have been stated to describe flow characteristics at the boundary, near the plate and far away from the plate embedded in a sparsely packed porous medium. Beavers and Joseph [13] studies, was one of the first attempts to study the fluid flow boundary conditions at the interface region. The following are the boundary conditions on velocity and temperature fields:

$$u = 0 \;,\quad v = 0, \qquad T = T_w \quad C = C_w \quad \text{at } y = 0 \tag{5}$$

$$u = U_o \;,\quad v = 0, \qquad T = T_\infty \quad C = C_\infty \quad \text{as } y \to \infty \tag{6}$$

Equations (1) – (4) are highly nonlinear partial differential equations, in order to solve them the following dimensionless variables f , $\theta, \phi,$ and q''' and as well as the similarity variable η are introduced

(Mohammadein and El-shaer [9]):

$$\eta = \left(\frac{y}{x}\right)\left(\frac{U_o x}{v}\right)^{1/2},\; \psi = \sqrt{vU_o x}\, f(\eta),\;\; \theta = \frac{T - T_\infty}{T_w - T_\infty},\;\; \phi = \frac{C - C_\infty}{C_w - C_\infty}\;,$$

$$q''' = \frac{U_o (T_w - T_\infty)}{2x} e^{-\eta}, \tag{7}$$

where v is the kinematic viscosity of the fluid, $\psi = \psi(x, y)$ is the stream function defined by $u = \frac{\partial \psi}{\partial y},\; \mathrm{v} = -\frac{\partial \psi}{\partial \mathrm{x}}$ is such that the continuity equation (1) is satisfied automatically and the velocity components are given by

$$u = U_o f'(\eta),\; \mathrm{v} = -\frac{1}{2}\sqrt{\frac{vU_o}{x}}\big(f(\eta) - \eta\; f'(\eta)\big), \tag{8}$$

the variable permeability k(η), the variable porosity ε(η) and the variable effective thermal diffusivity α(η) are given by, following the study made by Chandrasekhara and Namboodiri [8]

$$k(\eta) = k_o (1 + de^{-\eta}), \tag{9}$$

$$\varepsilon(\eta) = \varepsilon_o (1 + d^* e^{-\eta}), \tag{10}$$

$$\alpha(\eta) = \alpha_o [\varepsilon_o (1 + d^* e^{-\eta}) + \sigma^* \{1 - \varepsilon_o (1 + d^* e^{-\eta})\}], \tag{11}$$

where k_o, ε_o, and α_o are the permeability, porosity and diffusivity at the edge of the boundary layer respectively, σ^* is the ratio of the thermal conductivity of solid to the conductivity of the fluid, d and $d^*$are treated as fixed constants having the values 3.0 and 1.5, respectively, for variable permeability (VP) and $d = d^* = 0$ for uniform permeability (UP).

Equations (2), (3) & (4) are transformed into ordinary differential equations by substituting the dimensionless variables introduced in equations (7) to (11), the simplified local similarity equations are

$$f''' = -\frac{1}{2} ff'' - \frac{Gr}{\mathrm{Re}^2}(\theta - N\phi) - \frac{\alpha^*}{\sigma \mathrm{Re}} \left(\frac{1 + d^* e^{-\eta}}{1 + de^{-\eta}} \right) (1 - f'), \tag{12}$$

$$\theta'' = \frac{-\frac{1}{2}\mathrm{Pr}\, f\, \theta' - \mathrm{Pr}\, E\ f''^2 - \frac{1}{2}\mathrm{Pr}\ e^{-\eta} - \varepsilon_o d^* e^{-\eta}(\sigma^* - 1)\ \theta'}{\varepsilon_o + \sigma^*(1 - \varepsilon_o) + \varepsilon_o d^* e^{-\eta}(1 - \sigma^*)}, \tag{13}$$

$$\phi'' = -\frac{1}{2} Sc\, \mathrm{f}\phi', \tag{14}$$

where, $\mathrm{Pr} = \bar{\mu}/\rho\alpha_o$ is the Prandtl number, $Sc = \bar{\mu}/\rho\gamma_o$ is the Schmidt number, $\alpha^* = \mu/\bar{\mu}$ is the ratio of viscosities, $N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ is the Buoyancy ratio, $E = U_o^2/C_p(T_w - T_\infty)$ is the Eckert number, $\sigma = k_o/x^2\varepsilon_o$ is the local permeability parameter, $\mathrm{Re} = U_o x/v$ is the local Reynolds number and $Gr_T = g\beta_T(T_w - T_\infty)x^3/v^2$ is the thermal Grashof number, $Gr_C = g\beta_C(C_w - C_\infty)x^3/v^2$ is the solutal Grashof number. $\frac{Gr}{\mathrm{Re}^2}$ is the mixed convection parameter and $Gr_T = Gr_C$.

The transformed boundary conditions are:

$$f = 0,\ \ \mathrm{f}' = 0,\ \ \theta = 1,\ \ \phi = 1 \qquad \text{at } \eta = 0, \tag{15}$$

$$f' = 1,\ \ \theta = 0,\ \ \phi = 0 \qquad \text{as } \eta \to \infty, \tag{16}$$

once the velocity, temperature and concentration distributions are known, the skin friction and rate of heat and mass transfer can be calculated respectively by

$$\tau = -f''(0)/\sqrt{\mathrm{Re}}, \quad Nu = -\sqrt{\mathrm{Re}}\,\theta'(0) \text{ and } Sh = -\sqrt{\mathrm{Re}}\,\phi'(0), \tag{17}$$

where τ is the skin friction , Nu is the Nusselt number and Sh is the Sherwood number.

3. Numerical Method

The boundary value problems arising due to vertical heated plate are highly coupled non linear equations which are difficult to solve analytically, hence numerical method by shooting technique is employed. Equations (12)-(14) with the boundary conditions (15) and (16) have been solved by using the Newton-Raphson shooting technique along with Runge-Kutta fourth order integration scheme. Equations (12)-(14) constitute a highly non-linear coupled boundary value problem of third and second order respectively, they are transformed into system of simultaneous equations of first order. Further they are transformed into initial value problem by applying shooting technique. The obtained initial value problem is then solved by employing Runge-Kutta fourth order integration scheme. The method is illustrated as given below:

1. Decision on ∞
2. Converting BVP to IVP by choosing suitable initial condition for $f, \theta \,\&\, \phi$
3. $f''(0), \theta'(0) \,\&\, \phi'(0)$ required for the solution of initial value problem are chosen by the classical, explicit Runge-Kutta method of fourth order.

The decision on an appropriate '∞' for the problem depends on the proper parameter values chosen. In view of this, for each parameter combination, the appropriate value of '∞' has to be decided. The algorithm for the shooting method with Runge-Kutta fourth order approximating is used.

Initially, we chose guess values as $f''(0) = P$, $\theta'(0) = \mathrm{Q}$ and $\phi'(0) = \mathrm{R}.$. The process of obtaining P, Q & R accurately involves iteration process and can be calculated, repeating the same calculation we get another improved value, but these chosen guess values are not the most accurate values and hence there is a need to redefine. The better guess can be obtained by using the Newton-Raphson method. We solve the equations (12)-(14) with initial conditions

$$f(0)=0,\ f'(0)=0,\ \ f''(0)=P,$$

$$\theta(0)=1,\ \ \theta'(0)=Q,\ \ \phi(0)=1,\ \ \phi'(0)=\mathrm{R} \tag{18}$$

Due to crude choice of $f''(0), \theta'(0)\ \&\ \phi'(0)$, the solution at '∞' does not match with those given in the problem using the classical explicit Runge-Kutta method of fourth order. Thus, the coupled nonlinear boundary value problem (BVP) of third-order in f and second-order in θ and ϕ has been reduced to a system of seven simultaneous equations of first-order for seven unknowns as follows (see Vajravelu [14]):

$$f = f_1,$$

$$\frac{df_1}{d\eta} = f_2,$$

$$\frac{df_2}{d\eta} = f_3,\quad \frac{df_3}{d\eta} = -\frac{1}{2} f_1 f_3 - \frac{Gr}{\mathrm{Re}^2}\left(f_4 - Nf_6\right) - \frac{\alpha^*(1+d^* e^{-\eta})}{\sigma \mathrm{Re}(1+de^{-\eta})}(1-f_2),$$

$$\theta = f_4,$$

$$\frac{df_4}{d\eta} = f_5,$$

$$\frac{df_5}{d\eta} = -\frac{(1/2)\ \mathrm{Pr}\, f_1 f_5 + \mathrm{Pr}\, E\ f_3^2 + (1/2)\ \mathrm{Pr}\, e^{-\eta} + \varepsilon_o d^* e^{-\eta}(\sigma^* - 1) f_5}{\varepsilon_o + \sigma^*(1-\varepsilon_o) + \varepsilon_o d^* e^{-\eta}(1-\sigma^*)},$$

$$\phi = f_6,\ \frac{df_6}{d\eta} = f_7,$$

$$\frac{df_7}{d\eta} = -\frac{1}{2} Sc\ f_1\ f_7. \tag{19}$$

where $f_1 = f,\quad f_2 = f',\ f_3 = f'',\ f_4 = \theta,\ f_5 = \theta'\ ,\ f_6 = \phi,\ f_7 = \phi'$ and a prime denotes differentiation with respect to η.

The boundary conditions (15) and (16) now take the form

$$f_1(0)=0,\ f_2(0)=0,\ f_3(0)=P,\ f_4(0)=1,\ f_5(0)=Q,\ f_6(0)=1,\ f_7(0)=R \tag{20}$$

$$f_2(\infty)=1,\ f_4(\infty)=0,\ f_6(\infty)=0. \tag{21}$$

To solve the system of first-order differential equations along with boundary conditions, we need seven initial conditions, but we have only two initial conditions on f, one initial condition on θ and one initial condition on ϕ. The third condition on f $(i.e. f''(0))$, second condition on $\theta(i.e.\theta'(0))$ and second condition on ϕ (i.e., $\phi'(0)$) are not prescribed, which are determined by employing numerical shooting method and using the two ending boundary condition given in equation (15 & 16). The selection of an appropriate finite value of η_∞ is to be made. A good guess of the initial condition in the shooting technique is to be made on which the convergence depends. The accuracy of the assumed initial conditions is checked by comparing the calculated values of the dependent variable at the terminal point with its given value at that point. If any difference exists, improved values of the assumed initial conditions must be obtained and the process is repeated. The iterative process is terminated when the difference between two successive values reached 10^{-6}, then the solution is said to have converged results. The slight deviation in the values may be due to the use of Runge-Kutta-Fehlberg method which has fifth order accuracy whereas; Mohammadein and El-Shaer (2004) have used fourth-order Runge-Kutta method which has only fourth order accuracy who has analysed the influence of variable permeability with heat transfer. Thus the present results are more accurate compared to their results.

4 Results and Discussion

The flow of steady, laminar, incompressible viscous fluid in a vertical heated plate embedded in a saturated porous medium has been investigated in presence of internal heat generation in the energy equation. The Brinkman extended Darcy equation is used along with the continuity equation, energy and species concentration equation to illustrate the flow behavior in sparsely packed porous medium. The exponential form of internal heat generation is considered. The partial differential equations are converted into ordinary differential equation using the similarity solution method. The system of first-order differential equations (14)-(16) are solved numerically using shooting technique with Runge-Kutta-Fehlberg method. In order to know the accuracy of the method used, computed values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ were obtained for buoyancy ratio $N=0$ and compared with those obtained by Mohammadein and El-Shaer (2004) with only the heat transfer, for the variable permeability (d = 3.0, d* = 1.5) case and good agreement has been obtained with their results. The values are tabulated in the Table 1 for $\varepsilon_o = 0.4, Ec = 0.1, \Pr = 0.71, \mathrm{Sc} = 0.22$ with selected values of Gr/Re^2, σ^* and $\alpha^*/\sigma\mathrm{Re}$ for both uniform permeability (UP) i.e. d = d*= 0 and variable permeability (VP) i.e., d, d* ≠ 0 cases. The slight deviation in the values may be due to the use of Runge-Kutta-Fehlberg method which has fifth order accuracy. As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters.

The velocity distributions for various values of buoyancy ratio N have been depicted in fig. 1. It is observed that increase in the value of buoyancy ratio N there is a slight increase in the velocity profile and the boundary layer also varies slightly. The temperature distributions are shown for various values of buoyancy ratio N in fig. 2, it is observed that increase in value of buoyancy ratio N there is a decrease in the temperature profile. This shows that heat transfer is faster as we increase buoyancy ratio parameter due to the presence of the internal heat generation term which is an exponential term. The concentration profiles are shown for various values of buoyancy ratio N in fig. 3, it is observed that increase in value of buoyancy ratio N there is a decrease in the concentration profile similar to the temperature profile, whereas the rapid decrease starts when N = 5.

The velocity profiles for various values of mixed convection parameter Gr/Re^2 is shown in fig. 4, it is observed that increase in value of Gr/Re^2 there is an increase in the velocity boundary layer, for higher value of Gr/Re^2 that is at $Gr/\mathrm{Re}^2=2$,the velocity boundary layer increases rapidly near the plate and slowly moves down towards away from the plate. The temperature and concentration profiles for various values of Gr/Re^2 are shown in the figures 5 and 6. It is observed that increase in the value of the Gr/Re^2, the profiles decreases, and for each value of Gr/Re^2 increasing, temperature profiles decreases faster compared to the concentration profiles. Here it is observed that uniform permeability dominates more than the variable permeability in velocity, temperature and concentration profiles.

The velocity profiles increases as we increase the value of $\alpha^*/\sigma\mathrm{Re}$ as shown in the fig.7, this is due to Reynolds number leading to high viscous forces which has very high relative importance for giving the flow conditions and high porosity, with the moderate ratio of viscosities. Also it is observed that uniform permeability is more prominent than the variable permeability. The temperature and concentration profiles are shown in fig. 8 and 9, it is observed that increase in the value of $\alpha^*/\sigma\mathrm{Re}$ decreases the temperature and concentration profiles. It is also observed that uniform permeability and variable permeability flow behavior are same compared to any magnitude of $\alpha^*/\sigma\mathrm{Re}$.

Increase in Prandtl number, the velocity profiles decreases which is shown in fig.10. Temperature distributions is shown for various values of Prandtl number in fig. 11, it is observed that increase in prandtl number leads to decrease in the temperature boundary layer. It is also observed that the variable permeability is more prominent than the uniform permeability.

Variation of σ^* for temperature profiles is shown in fig. 12. It is observed that decrease in the value of σ^* leads to decrease in the temperature profile. Both uniform permeability and variable permeability decays smoothly for all the values of σ^*. An interesting factor can be observed here is when $\sigma^* = 2.0$ both uniform permeability and variable permeability behaves same, whereas for $\sigma^* = 4.0$ and $\sigma^* = 6.0$ the uniform permeability slows down and not much prominent when compared to variable permeability cases.

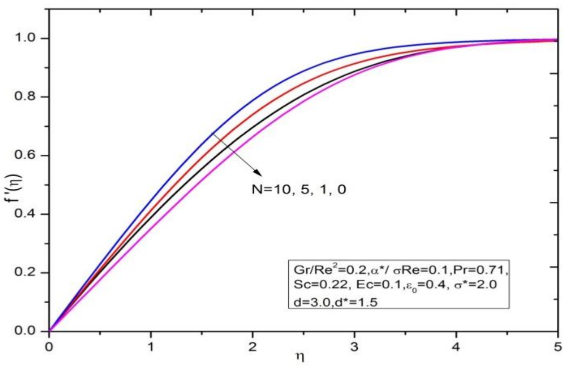

Fig. 1 Velocity distributions for various values of buoyancy ratio N for variable permeability (VP) case

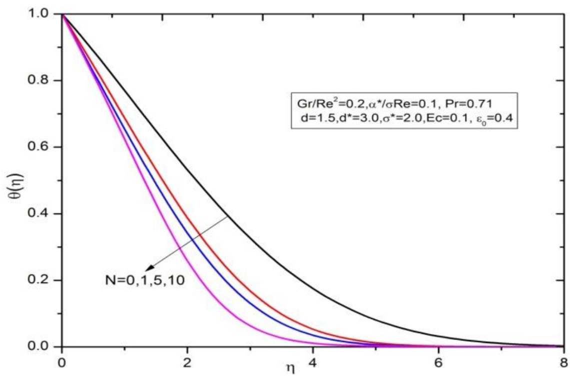

Fig. 2 Temperature distributions for various values of buoyancy ratio N for variable permeability (VP) case

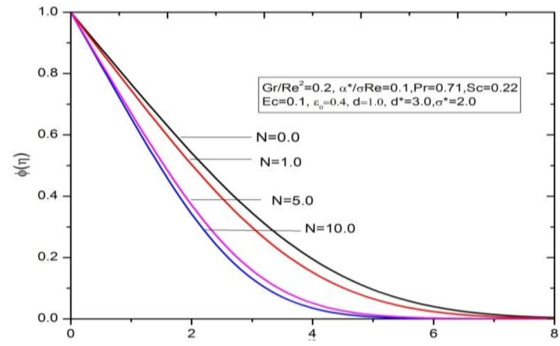

Fig. 3 Concentration distribution for various values of buoyancy ratio N for variable permeabiltiy case.

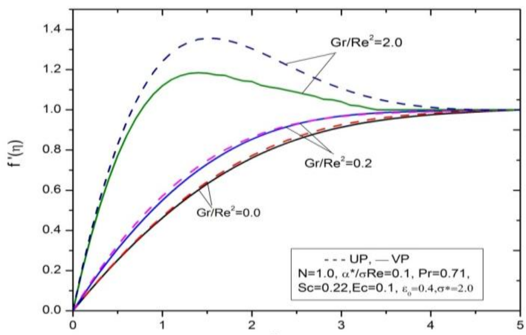

Fig. 4 Velocity distributions for various values of Gr/Re² for both variable permeability and uniform permeability cases

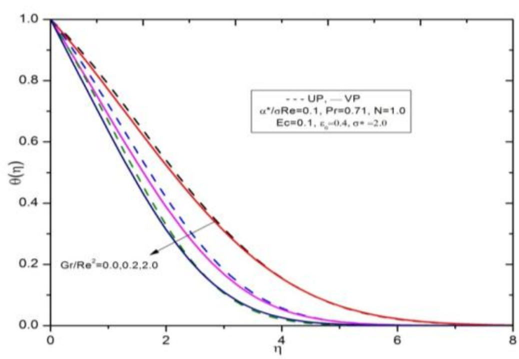

Fig 5 Temperature distributions for various values of Gr/Re^2 for both Variable permeability and uniform permeability cases

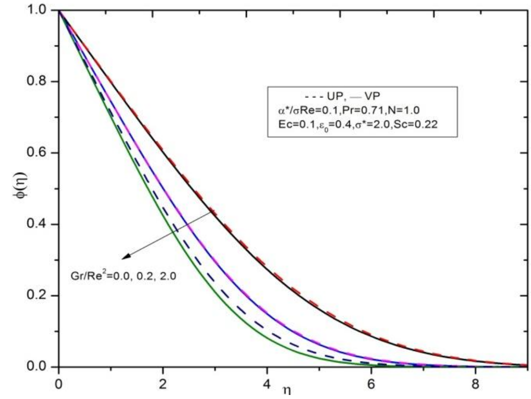

Fig. 6 Concentration profiles for different values of Gr/Re^2 for both Variable permeability and uniform permeability cases

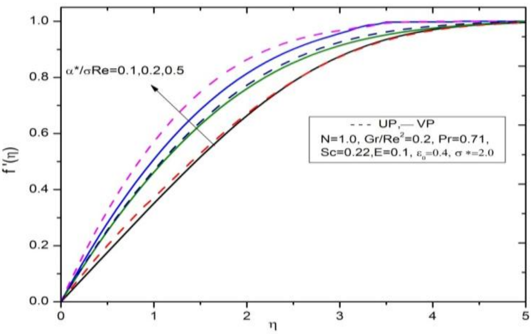

Fig. 7 Velocity profiles for various values of $\alpha^* / \sigma\,\mathrm{Re}$ for variable permeability and uniform permeability cases

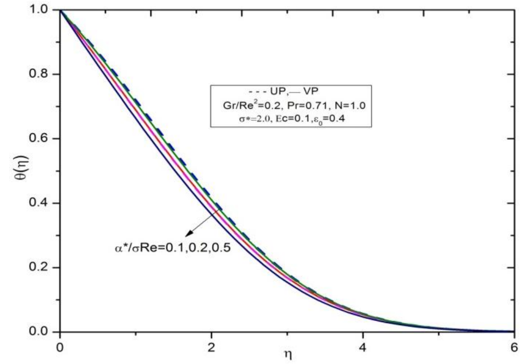

Fig. 8 Temperature profiles for various values of $\alpha^* / \sigma\,\mathrm{Re}$ for variable permeability and uniform permeability cases

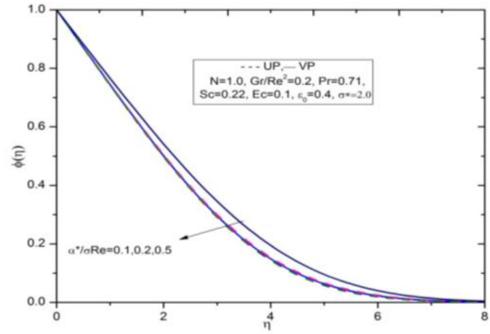

Fig. 9 Concentration profiles for various values of $\alpha^{*}/\sigma\,\mathrm{Re}$ for variable permeability and uniform permeability cases

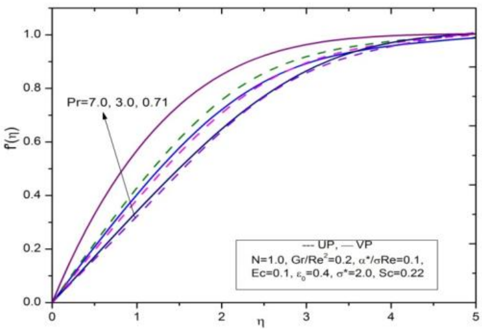

Fig. 10 Velocity profiles for various values of Prandtl number for variable permeability and uniform permeability cases

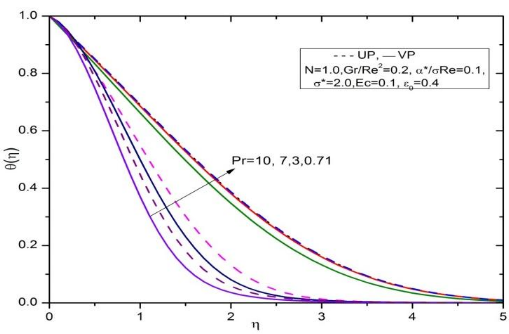

Fig. 11 Temperature profiles for various values of Prandtl numbers for variable permeability and uniform permeability cases

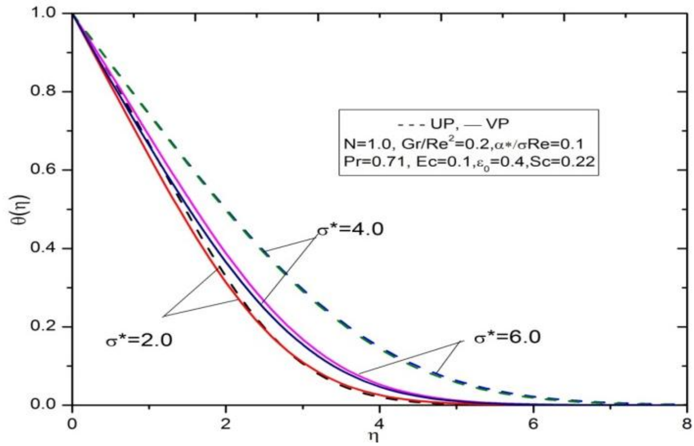

Fig. 12 Temperature profiles for various values of $\sigma*$ for variable permeability and uniform permeability cases

Acknowledgement

The authors are grateful to the Research Centre M S Ramaiah Institute of Technology, Atria Institute of Technology, Vivekananda Institute of Technology, for all the support and also the financial support from VTU research scheme project.

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