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Study of Two-Dimensional, all –Time Dispersion of a Solute in a Fluid –Saturated Porous Medium

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Abstract

The paper presents the mathematical formulation which describes the dispersion of solute in a laminar flow in a sparsely packed porous medium. The effect of interphase mass transfer on dispersion in a unidirectional flow through a horizontally extent of infinite porous channel is examined using the generalized dispersion model of Sankarasubramanian and Gill. The model brings into focus three important coefficients namely the exchange coefficient, the convection coefficient and the dispersion coefficient. The time-dependent dispersion coefficient and mean concentration distribution are computed and results are represented graphically. The problem finds many applications in waste water management, in chromatography and in biomechanical problems.

Keywords: Dispersion; Porous medium

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1. Introduction

Dispersion of solute in porous media is seen in many real life problems such as ground water pollution and chromatography. Many of the problems associated with the transport of pollutants involve interphase mass transfer. Taylor [8] was the first to study the dispersion of passive solute in a Hagen-Poiseuille flow and the limitations of the classical paper by Taylor [8] was overcome Aris [1] and Aris [1] also studied the dispersion of solute using a statistical approach. The study of dispersion by Taylor [8] and Aris [1] was applied to only long times and this limitation was overcome by Gill and Sankarasubramanian [4] and Barton [2]. Gill [3] gave a note on dispersion of transient dispersion problems. Later Gill and Sankarasubramanian [4] presented the elegant all-time approaches to study dispersion of passive solute in Newtonian fluid flows. Exact solutions are obtained for the effect of interphase mass transfer on dispersion in unidirectional flow through a horizontally extended porous channel using generalized model of Sankarasubramanian and Gill [5]. Siddeshwar and Manjunath ([6], [7]) have studied the convective diffusion process in a Non–Isothermal Plane-Poiseuille flow. The model brings into focus three coefficients namely exchange coefficient, convective coefficient and dispersion coefficient. The effect of wall reaction rate parameter and its effect on these three coefficients are also studied.

2. Mathematical Formulation

The physical configuration considered in this problem is as shown in figure. Consider an infinite horizontally extended sparsely packed porous medium bounded by solvent-impermeable walls of width 2h. The solute undergoes first order heterogeneous chemical reaction with the bounding walls of the channel. The flow is assumed to be steady, unidirectional and fully developed and the Newtonian fluid is considered to be incompressible.

The Basic equations for the flow are:

$$\mu' \frac{d^2u}{dy^2} - \frac{\partial p}{\partial x} - \frac{\mu}{\kappa} u = 0 \tag{1}$$

$$\frac{\partial p}{\partial y} = 0 \tag{2}$$

where p, μ and κ are respectively the pressure, dynamic viscosity and permeability of the porous medium. Elimination the pressure p between the Equation (1) and (2) and using the following boundary condition for the flow we get:

$$u=0 \quad at \quad y=\pm h. \tag{3}$$

$$\mu' \frac{d^2u}{dy^2} - \frac{\mu}{\kappa} u = \frac{d\psi}{dx} \tag{4}$$

We consider dispersion of passive solute in this fully developed flow through a parallel plate and into this flow a slug of concentration is introduced $C = C_0 f(x, y)$. The mass balance equation for the solute C undergoing heterogeneous chemical reaction is given by:

$$\frac{\partial C}{\partial t} + u(y)\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right). \tag{5}$$

The initial and boundary conditions for the Equation (5) are:

$$C(0, x, y) = C_o f(x, y) , \tag{5.a}$$

$$-D\frac{\partial C}{\partial y}(t, x, h) = \kappa_s C(t, x, h) , \tag{5.b}$$

$$\frac{\partial C}{\partial y}(t, x, 0) = 0 , \tag{5.c}$$

$$C(t, \infty, y) = \frac{\partial C}{\partial y}(t, \infty, y) = 0 , \tag{5.d}$$

$$C(t, x, 0) = \textit{finite} \tag{5.e}$$

Where, C_o is the reference concentration and κ_s is the rate constant of first order catalysed by the wall. Equation (5.a) is the general form of initial concentration distribution. Equation (5.b) is a balance of concentration flux with first order chemical reaction catalysed by the wall. Equation (5.c) is the concentration C which is symmetric about the centre line in the system. Equation (5.d) implies the solute concentration is zero at the distance far removed from the source. Equation (5.e) is based on the assumption that concentration about the centre line is always finite.

Now the Equation (4) can be solved for the velocity profile by introducing the following non-dimensional parameters

$$\tau = \frac{tD}{h^2}, \quad X = \frac{x}{hPe}, \quad Y = \frac{y}{h}, \quad \theta = \frac{C}{C_o}, \quad P_x = -\frac{h^3}{\rho_o \gamma^2}\frac{d\psi}{dx}, \quad U = \frac{uh}{\gamma P_x} \qquad (6)$$

Where,

$$Pe = \frac{u_o h}{D}, \quad \text{(Peclet Number)}$$

$$\beta = \frac{\kappa_s h}{D}, \quad \text{(Reaction rate parameter).}$$

Using Equation (6) in Equations (3) and (4), we get their non-dimensional form as:

$$\frac{d^2U}{dY^2} - \Lambda\sigma^2 U = -\Lambda \qquad (7)$$

The solution for the Equation (7) can be obtained with the boundary condition as:

$$U = 0 \quad at \quad Y = \pm 1 \qquad (8)$$

$$U(Y) = \frac{1}{\sigma^2}\left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right] \qquad (9)$$

Where,

$\sigma^2 = \frac{h^2}{\kappa}$, (Porous Parameter)

$\Lambda = \frac{\mu}{\mu'}$, (Brinkman Number)

Substituting Equation (6) in Equations (5 - 5.e), we get the following non-dimensional form as:

$$\frac{\partial\theta}{\partial\tau} + U(Y)\frac{\partial\theta}{\partial X} = \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}, \tag{10}$$

$$\theta(0, X, Y) = \phi(X)\psi(Y), \tag{11}$$

$$\frac{\partial\theta}{\partial Y}(\tau, X, 1) = -\beta\theta(\tau, X, 1), \tag{12}$$

$$\frac{\partial\theta}{\partial Y}(\tau, X, 0) = 0, \tag{13}$$

$$\theta(\tau, \infty, Y) = \frac{\partial\theta}{\partial Y}(\tau, \infty, Y) = 0, \tag{14}$$

$$\theta(\tau, X, 0) = \mathit{finite}. \tag{15}$$

The Solution of the Equation (10), subjected to the conditions ((11)-(15)) is now assumed in the form (Gill and Sankarasubramanian ([4], [5]))

$$\theta(\tau, X, Y) = f_o(\tau, Y)\theta_m(\tau, \mathrm{X}) + \sum_{\mathrm{k}=1}^{\infty} f_k(\tau, Y)\frac{\partial^k\theta_m}{\partial X^k}, \tag{16}$$

Where,

$$\theta_m = \int_0^1 \theta\, dy \tag{17}$$

Using the definition of the θ_m and integrating the Equation (10) we get:

$$\frac{\partial\theta_m}{\partial\tau} = \frac{1}{P_e^2}\frac{\partial^2\theta_m}{\partial X^2} - \beta\theta(\tau, X, 1) - \frac{1}{\sigma^2}\frac{\partial}{\partial X}\int_0^1\left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right]\theta\, dY \tag{18}$$

Substituting Equation (16) in Equation (18) we get the dispersion model of in the form

$$\frac{\partial\theta_m}{\partial\tau} = K_o\theta_m + K_1\frac{\partial\theta_m}{\partial X} + K_2\frac{\partial^2\theta_m}{\partial X^2} + K_3\frac{\partial^3\theta_m}{\partial X^3} + \ldots\ldots\ldots\ldots, \tag{19}$$

Where $K_i's$ are given by

$$K_i(\tau) = \frac{\delta_{i2}}{Pe^2} - \beta f_i(\tau,1) - \frac{1}{\sigma^2}\int_0^1\left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right]f_{i-1}\,dY\ ,\ i = 0,1,2,\ldots.. \tag{20}$$

Here $f_{-1} = 0$

Substituting Equation (16) in Equation (10) and using the generalized dispersion model given by Equation (19) in the resulting equation, we get the equations for $f_0, f_{1,}\ f_2, \ldots\ldots\ldots\ldots\ldots$ in the form:

$$\frac{\partial f_k}{\partial\tau} = \frac{\partial^2 f_k}{\partial Y^2} - \frac{1}{\sigma^2}\left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right]f_{k-1} + \frac{1}{Pe^2}f_{k-2} - \sum_{i=0}^{k}K_i f_{k-i}\ ,\ \ k = 0,1,2,\ldots \quad 21)$$

Where $f_{-2} = f_{-1} = 0.$

The corresponding boundary conditions for the solution of Equation (21) are:

$$f_k(\tau,0) = \mathit{finite}\,, \tag{22}$$

$$\frac{\partial f_k}{\partial Y}(\tau,1) = -\beta f_k(\tau,1)\ , \tag{23}$$

$$\frac{\partial f_k}{\partial Y}(\tau,1) = 0\ , \tag{24}$$

$$\int_0^1 f_k(\tau,Y) = \delta_{k0}\,, \qquad k = 0,1,2,3,\ldots\ldots\ldots\ldots \tag{25}$$

Substituting $k = 0$ in Equation (22) we get the differential equation for f_0 as:

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial Y^2} - f_0 K_0 \tag{26}$$

The initial condition for the Equation (26) (i.e., f_0) by taking $\tau = 0$, we get:

$$\theta_m(\tau, 0) = \int_0^1 \theta(0, X, Y)\, dY. \tag{27}$$

Taking $\tau = 0$ and substituting Equation (17) in (27) and setting $f_k(Y) = 0$ $(k = 1, 2, 3, ...)$ gives the initial condition for f_0 as

$$f_0(0, Y) = \frac{\theta(0, X, Y)}{\theta_m(0, Y)} \tag{28}$$

We can clearly see that from the above Equation is function of Y only on the left hand side and on the right hand side is function of both X and Y. This is clear justification of initial concentration distribution must be of separable function of X and Y.

Substituting Equation (11) into Equation (28) we get:

$$f_0(0, Y) = \frac{\psi(Y)}{\int_0^1 \psi(Y) dY}. \tag{29}$$

From Equation (20) we get K_0 as

$$K_0(\tau) = -\beta f_0(\tau, 1) \tag{30}$$

Substituting Equation (30) in Equation (26) given an exclusive Equation for f_0 and the solution of f_0 subject to the conditions ((22) – (25)) and (28) we get the form

$$f_0(\tau, Y) = \frac{\sum_{n=0}^{9} A_n \exp[-\mu_n{}^2 \tau] \cos \mu_n Y}{\sum_{n=0}^{9} \frac{A_n}{\mu_n} \exp[-\mu_n{}^2 \tau] \sin \mu_n} \tag{31}$$

Where μ_n's are the roots of:

$$\mu_n \tan \mu_n = \beta, \qquad n = 0(1)9, \tag{32}$$

and $A_n's$ are given by:

$$A_n = \frac{4\int_0^1 \psi(Y)\cos \mu_n Y dY}{\left[1 + \frac{\sin 2\mu_n}{2\mu_n}\right]\int_0^1 \psi(Y) dY}, \qquad n = 0(1)9. \tag{33}$$

Having obtained f_0 now we get K_0 from Equation (30) as:

$$K_0(\tau) = \frac{\sum_{n=0}^{9} A_n \mu_n \exp[-\mu_n^{\ 2}\tau]\sin \mu_n}{\sum_{n=0}^{9} \frac{A_n}{\mu_n} \exp[-\mu_n^{\ 2}\tau]\sin \mu_n}, \qquad n = 0(1)9. \tag{34}$$

Now we consider the case of initial concentration occupying the entire cross section of the parallel plate channel, we take $\psi(Y) = 1$ and $K_0(\tau)$ for this is:

$$K_0(\tau) = \frac{\sum_{n=0}^{9} \frac{1}{(\mu_n^{\ 2} + \beta^2 + \beta)} \exp[-\mu_n^{\ 2}\tau]}{\sum_{n=0}^{9} \frac{1}{\mu_n^{\ 2}(\mu_n^{\ 2} + \beta^2 + \beta)} \exp[-\mu_n^{\ 2}\tau]}, \quad n = 0(1)9. \tag{35}$$

Now let we proceed for long time analysis i.e., as $\tau \to \infty$ we get the asymptotic solution for $K_0(\tau)$ from Equation (35) as:

$$K_0(\infty) = -\mu_0^{\ 2} \tag{36}$$

Where μ_0 is the first root of the Equation (32)

Now we look for $K_1(\infty)$ from Equation (20) if we know $f_0(\infty, Y)$ and $f_1(\infty, Y)$. Likewise $K_2(\infty)$ requires the knowledge of K_0, K_1, f_0, f_1 and f_2. Now as $\tau \to \infty$ in Equation (31) reduces to:

$$f_0(\infty, Y) = \frac{\mu_0 \cos \mu_0 Y}{\sin \mu_0} \tag{37}$$

We now go ahead and find K_1, f_1, f_2 and K_2. For asymptotically long time i.e. $\tau \to \infty$, Equations (20) and (21) gives us $K_k's$ and $f_k's$ in the form:

$$K_k(\infty) = \frac{\delta_{i2}}{Pe^2} - \beta f_i(\infty,1) - \frac{1}{\sigma^2}\int_0^1 \left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right] f_{i-1}(\infty, Y)\, dY, k = 0, 1, 2,..... \tag{38}$$

$$\frac{d^2 f_k}{dY^2} + \mu_0{}^2 f_k = \frac{1}{\sigma^2}\left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right] f_{k-1} - \frac{1}{Pe^2} f_{k-2} + \sum_{i=1}^{k} K_i f_{k-i}, k = 0, 1, 2,.. \tag{39}$$

The $f_k's$ must satisfy the conditions ((22) – (25)) and this permits the Eigen function expansion in the form:

$$f_k(\infty, Y) = \sum_{j=0}^{\infty} B_{j,k} \cos(\mu_j Y), \qquad k = 0,1,2,........... \tag{40}$$

Substitution Equation (40) in Equation (39) and multiplying the resulting equations by $\cos(\mu_j Y)$ and integrating between the limits 0 and 1, we get after simplification:

$$B_{j,k} = \frac{1}{\mu_j{}^2 - \mu_0{}^2}\left[\frac{1}{P_e^2} B_{j,k-2} - \sum_{i=1}^{k} K_i B_{j,k-i} + \frac{1}{\left(1 + \frac{\sin 2\mu_j}{2\mu_j}\right)} \sum_{l=0}^{9} C_{j,l} B_{l,k-l}\right], k = 1, 2,.. \tag{41}$$

Where,

$$C_{j,l} = \frac{2}{\sigma^2}\int_0^1 \left[1 - \frac{\cosh(\sqrt{\Lambda}\sigma)Y}{\cosh(\sqrt{\Lambda}\sigma)}\right] \cos(\mu_j Y)\cos(\mu_l Y), \tag{42}$$

$$B_{j,-1} = 0, \quad B_{j,0} = 0 \qquad for \quad j = 1(1)9 \tag{43}$$

The first expansion coefficient $B_{0,k}$ in the Equation (41) can be expressed in terms of $B_{j,k}$ by using the conditions ((22) – (25)) as:

$$B_{0,k} = -\frac{\mu_0}{\sin\mu_0}\left[\sum_{j=1}^{\infty} B_{l,k}\frac{\sin\mu_j}{\mu_j}\right], \qquad k = 1,\ 2, \ldots\ldots\ldots \tag{44}$$

Further from (37) and (40), it can be seen that

$$B_{0,0} = \frac{\mu_0}{\sin\mu_0} \tag{45}$$

Now substituting $k=1$ in Equation (38) and using Equations (42), (43) and (45) in the resulting equation we get:

$$K_1(\infty) = -\frac{2}{\sigma^2\left(1+\frac{\sin 2\mu_0}{2\mu_0}\right)}\int_0^1\left[1-\frac{\cosh(\sqrt{\Lambda\sigma})Y}{\cosh(\sqrt{\Lambda\sigma})}\right]\cos(\mu_j Y)\cos(\mu_l Y)\,dY. \tag{46}$$

Now substituting $k=1$ in Equation (38) and using Equations (41), (43) and (45) in the resulting equation we get:

$$K_2(\infty) = \frac{1}{Pe^2} - \frac{\sin\mu_0}{\mu_0\left(1+\frac{\sin 2\mu_0}{2\mu_0}\right)}\sum_{j=l}^{9} C_{j,0}B_{j,l} \tag{47}$$

Using the asymptotic coefficients $K_0(\infty)$, $K_1(\infty)$ and $K_2(\infty)$ in Equation (19) one can determine the mean concentration distribution as a function of X, τ and the parameters of the problem Λ, σ, Pe and β. This distribution is valid only for long times and is a gross approximation at short and moderate times.

The initial condition for solving Equation (19) can be obtained from Equation (11) by taking the cross-sectional average. Since we are making long time evaluations of the coefficients an unfortunate side effect is the non-dependence of θ_m on the initial concentration distribution.

In view of this we just note that the solution of Equation (19) with asymptotic coefficients can be obtained by Fourier transforms as:

$$\theta_m(\tau, X) = \frac{\exp\left\{K_0(\infty)\tau - \frac{(X + K_1(\infty))^2}{4K_2(\infty)\tau}\right\}}{2Pe\sqrt{\pi K_2(\infty)\tau}}, \tag{48}$$

Where $\theta_m(\tau,\infty) = 0$, $\frac{\partial \theta_m}{\partial X}(\tau,\infty) = 0$ as required by Equation (14) and $K_0(\infty)$, $K_1(\infty)$ and $K_2(\infty)$ are given by the Equations (36), (46) and (47).

Results and discussion

In this paper, the solution of the Darcy Brinkman momentum equation is used in the study of dispersion. We assumed that the bounding walls of the channel undergo first order chemical reaction with the solute and hence causes interphase mass transfer. It is also assumed that the chemical reaction is weak and hence β is used as the perturbation parameter to solve the equations. Three coefficients, viz., the exchange coefficient K_0 the convective coefficient K_1 and the diffusion coefficient K_2 arise in the model. The main aim of this work is to study the effect of β on K_0, K_1 and K_2.With this objective, we have plotted $-K_0$, $-K_1$ and $K_2 - Pe^{-2}$ as a function of β.

Figure 1 is a plot of the velocity distribution as a function of the non-dimensional transverse coordinate Y and Brinkman number Λ, which shows that velocity increases as Λ increases. Figure 2 is a plot of the velocity distribution as a function of the non-dimensional transverse coordinate Y and porous parameter σ, which shows that velocity decreases as σ decreases. Figure 3 is a plot of exchange coefficient $-K_0$ versus β. It may be observed from Figure 3 that as β increases, exchange coefficient $-K_0$ increases steadily. It can readily seen that as $\beta \to 0$, $-K_0$ vanishes. This means that due to interphase mass transfer, the term exchange coefficient exists in the model and clearly the exchange coefficient $-K_0$ is independent of the flow.

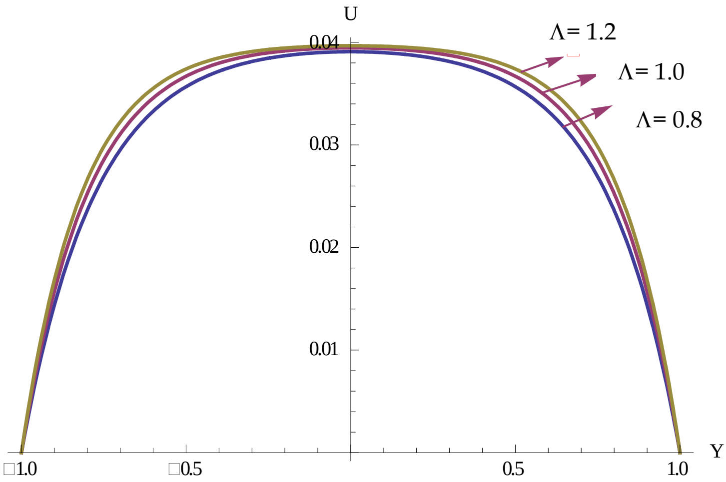

Fig 1: Plots of filter velocity distribution for various values of Λ, for $\sigma = 5$

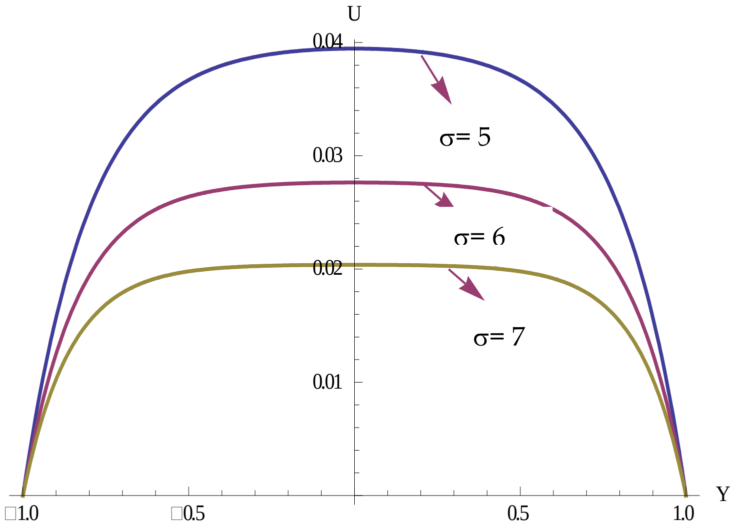

Fig 2: Plots of filter velocity distribution for various values of σ, for $\Lambda = 1$

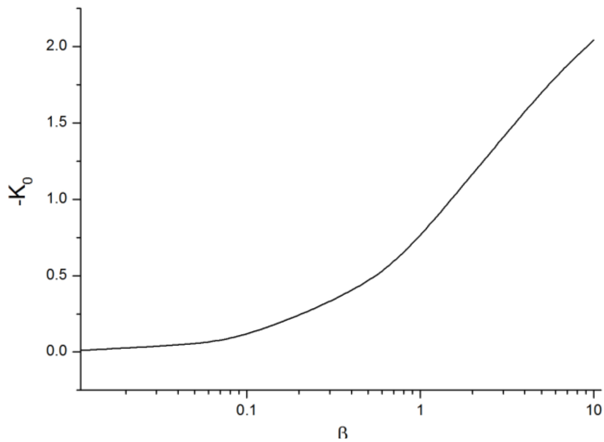

Fig 3: Plots of dimensionless convective coefficient $-K_0$ against dimensionless reaction rate parameter β .

Figures (4) - (5) are plots of convective coefficient $-K_1$ versus β for various values of Λ and σ. As β increases, the convective coefficient increases. A physical explanation for this is, the effect of the wall reaction is to deplete the solute in the slower moving wall region and therefore the solute distribution is weighed in favour of the fast moving central region. It can be found that as the porous parameter σ for a given non-zero value of β , the convective coefficient $-K_1$ decreases and for the increasing values of Brinkman number Λ the convective coefficient $-K_1$ increases. This is because an increase in σ will reduce the filter velocity and hence the convention while an increase in the value of Λ will increase the filter velocity and the advection.

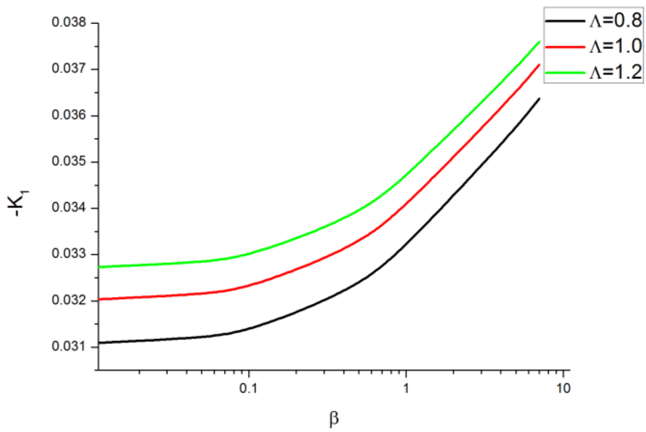

Fig 4: Plots of dimensionless convective coefficient $-K_1$ against dimensionless reaction rate parameter β for different values of Λ.

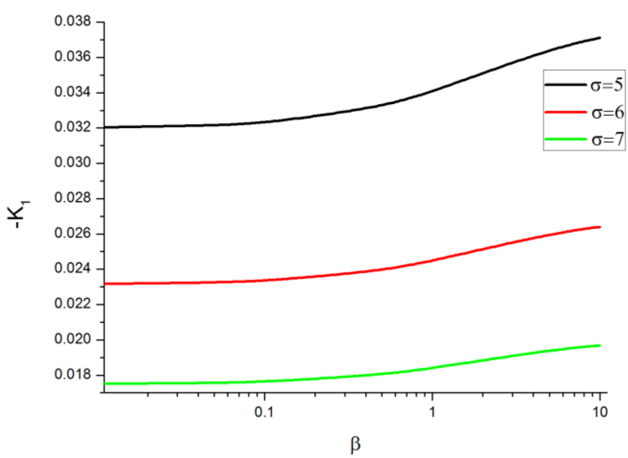

Fig 5: Plots of dimensionless convective coefficient $-K_1$ against dimensionless reaction rate parameter β for different values of σ.

Figures (6) - (7) are plots of dispersion coefficient $K_2 - Pe^{-2}$ versus β for various values of Λ and σ. From figures (6) - (7), it may be observed that as β increases, the dispersion coefficient $K_2 - Pe^{-2}$ decreases. This means due to interphase mass transfer the dispersion in axial direction is decreased effectively. This is only to be anticipated because as the wall reaction parameter β increases there is increase in predominance of transverse transport over molecular diffusion. We also found from figures (6) - (7) that as the porous parameter σ and Brinkman number Λ increases, the dispersion coefficient decreases.

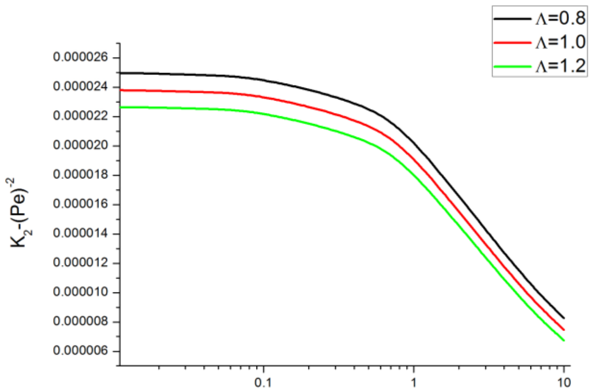

Fig 6: Plots of dimensionless dispersion coefficient $K_2 - Pe^{-2}$ against dimensionless reaction rate parameter β for different values of Λ.

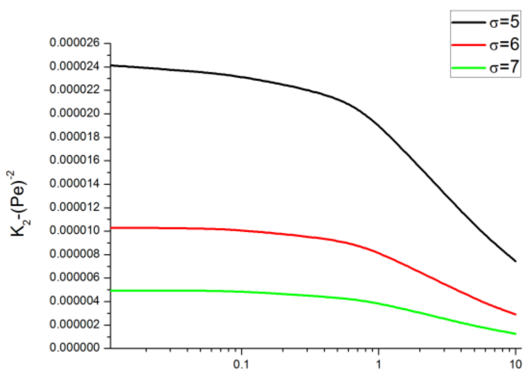

Fig 7: Plots of dimensionless dispersion coefficient $K_2 - Pe^{-2}$ against dimensionless reaction rate parameter β for different values of σ.

We now proceed to discuss the results obtained for mean concentration in the case of wall reaction (i.e. $\beta \neq 0$). In the case of wall reaction it is to be noted that the distribution of the mean concentration at small and moderate times is approximate and we find the same to be accurate for long times. This is because the values of $-K_1$ and K_2 have been evaluated asymptotically.

Figure (8) is a plot of θ_m versus X for various values of Λ and for a fixed value of σ, for a given time. It can be seen readily from figure (8) that for a given Λ and β, at a given time, the mean concentration θ_m starts increasing initially with X and we see that further increase in X the mean concentration reaches a maximum and then decreases to zero. Because of decrease in the value of K_2 we see that the peak of the men concentration at a given time increases as Λ increases. For higher values of Λ, the distribution of concentration starts at an earlier position and ends early.

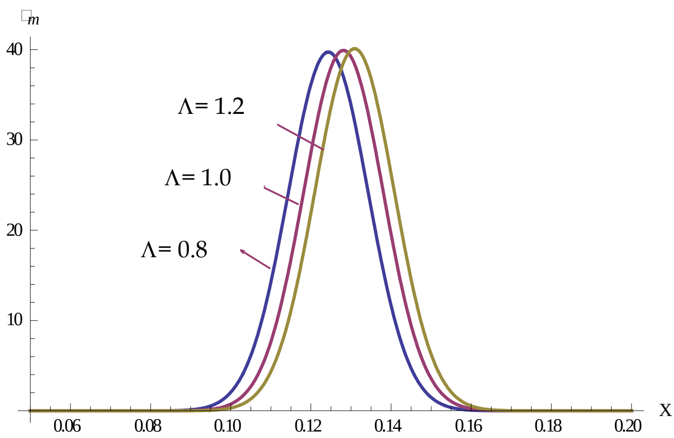

Fig 8: Plots of dimensionless mean concentration θ_m against axial distance X for different values of Λ, $\sigma = 5, \ \beta = 0.01$.

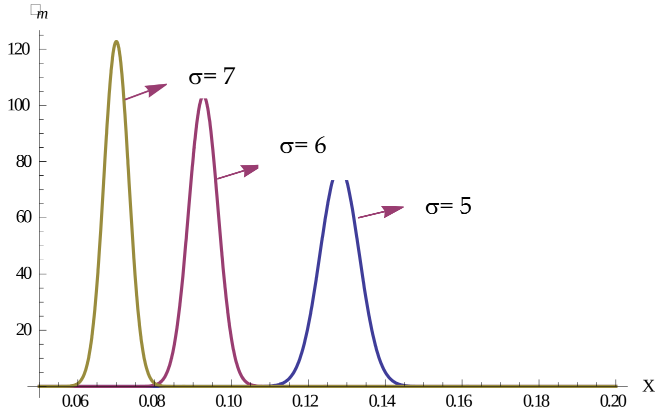

Fig 9: Plots of dimensionless mean concentration θ_m against axial distance X for different values of σ, $\Lambda = 1,\ \beta = 0.01$.

Figure (9) is a plot of θ_m versus X for various values of σ and for a fixed value of Λ. It can be seen readily from figure (9) that for a given σ and β, at the given time, the mean concentration θ_m starts increasing initially with X and we see that further increase in X the mean concentration reaches a maximum and then decreases to zero. Because of decrease in the value of K_2 we see that the peak of the men concentration at a given time increases as σ increases.

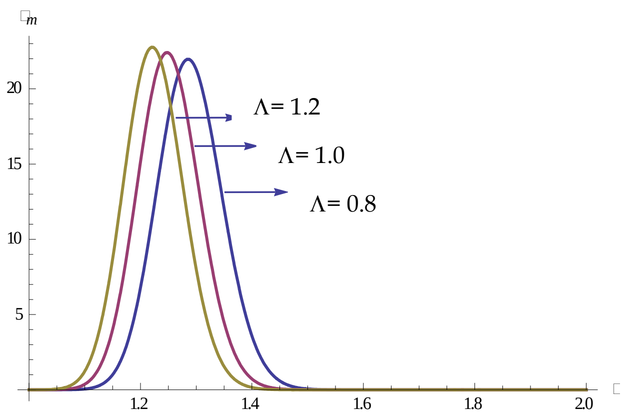

Fig 10: Plots of dimensionless mean concentration θ_m against time τ for different values of Λ, $\sigma = 5,\ \beta = 0.01$.

Figure (10) is a plot of θ_m versus τ for various values of Λ and for a fixed value of σ. It can be found that for a given Λ and for a fixed position, and for a given wall reaction parameter β, the mean concentration at a particular time starts increasing and later the mean concentration reaches maximum and then decreases to zero which signifies the dispersion of the solute is complete. It can be seen readily that as Λ increases, the dispersion starts early. This is because an increase in Λ will cause the solute to arrive early at the fixed position. As Λ increases, the peak of the mean concentration also increases.

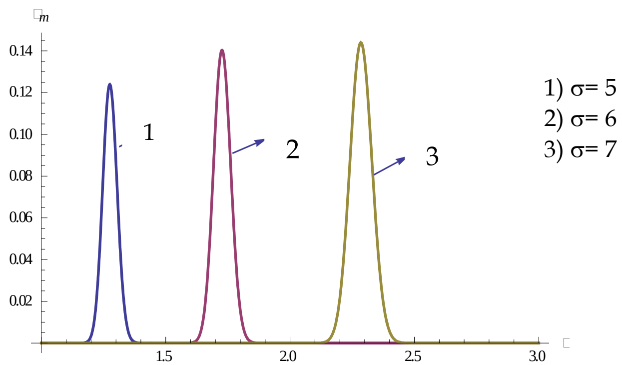

Fig 11: Plots of dimensionless mean concentration θ_m against time τ X for different values of σ, $\Lambda = 1,\ \beta = 0.01$.

Figure (11) is a plot of θ_m versus τ and for various values of σ and for a fixed value of Λ. It can be found that for a given σ, for a fixed position, and for a given wall reaction parameter β, the mean concentration at a particular time starts increasing and later the mean concentration reaches maximum and then decreases to zero which signifies the dispersion of the solute is complete. It can be seen readily that for higher values of σ the dispersion of solute starts slowly and as a result of it, the solute takes longer time to disperse completely.

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