Mapana Journal of Sciences
2018, Vol. 17, No. 1, 25–32
ISSN 0975-3303 | https://doi.org/10.12723/mjs.44.3

Dark Energy and Cosmological Constant

Louise Rebecca,* C. Sivaram† and Arun Kenath‡

Abstract

One of the unresolved problems in cosmology is that the measured mass density of the universe has revealed a value that was about 30% of the critical density. Since the universe is very nearly spatially flat, as is indicated by measurements of the cosmic microwave background, about 70% of the energy density of the universe was left unaccounted for.Another observation seems to be connected to this mystery. Generally one would expect the rate of expansion to slow downonce the universe started expanding. The measurements of Type Ia supernovae have revealed that the expansion of the universe is actually accelerating. This accelerated expansion is attributed to the so-called dark energy (DE). Here we give a brief overview on the observational basis for DE hypothesis and how cosmological constant, initially proposed by Einstein to obtain astatic universe,can play the role of dark energy.

Keywords: Dark Energy, Cosmological Constant

1. Introduction

The detailed measurement of the mass density of the universe has revealed that ~70% of the energy density of the universe is unaccounted for. This appears to be connected to the independent observation of the non-linear accelerated expansion of the universe deduced from measurements of Type Ia supernovae [7, 6]. Generally one would expect the rate of expansion to slow down, as once the universe started expanding, the combined gravity of all its constituents should pull it back, i.e., decelerate it. So the deceleration parameter, q_0, was

*Christ Junior College, Bengaluru, India; louise.rheanna@cjc.christcollege.edu
†Indian Institute of Astrophysics, Bengaluru, India; sivaram@iiap.res.in
‡Christ Junior College, Bengaluru, India; kenath.arun@cjc.christcollege.edu

Received: December 2017. Reviewed: December 2017

expected to be a positive value. A negative q_0 would imply an accelerating universe, with repulsive gravity and negative pressure. The measurements of Type Ia supernovae have revealed just that. By measuring their flux with redshift, q_0 is determined to be –0.55. This together with the fact that the universe is flat (from CMBR) and the total matter content, $\Omega_M\tilde{0}.3$, the rest of the matter in the universe, i.e., $\Omega_{DE}\tilde{0}.3$, must be in some exotic form which is dubbed Dark Energy (DE). All postulated forms of matter yield a positive deceleration parameter, except in the case of DE, hence this accelerated expansion is attributed to this dark energy [5, 14]. This has led to the introduction of a repulsive gravity source to make the deceleration parameter negative [4]. The dimensionless quantity, deceleration parameter q measures the cosmic acceleration of the universe's expansion:

$$q = \frac{-\ddot{a}.a}{\dot{a}^2} \tag{1}$$

where a is the scale factor of the universe. The gravitational acceleration is given by: $\ddot{a} = -\frac{GM}{a^2}$. For ordinary baryonic matter, the pressure exerted is positive, i.e., P=ρv^2. The pressure contribution by radiation is given by: P= $\frac{1}{3}\rho c^2$, which is also positive. In GR pressure also contributes to gravity. Therefore the acceleration is given by:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) = -\frac{4\pi G}{3}(1 + 3\omega)\rho \tag{2}$$

Where, ρ is the energy density of the universe, P is its pressure, and ω is the equation of state of the universe. Equation 2 can be rewritten as:

$$q = \frac{1}{2}(1 + 3\omega)[1 + \frac{k}{(aH)^2}] \tag{3}$$

where, H is the Hubble parameter and $k > 0$, $k = 0$, $k < 0$ depending on whether the universe is open, flat or closed.The derivative of the Hubble parameter can be written in terms of the deceleration parameter as: $\frac{\dot{H}}{H^2} = (1 + q)$.

Except in the case of dark energy, all postulated forms of matter yield a deceleration parameter $q \geq 0$. Thus, any expanding universe should have a non-increasing Hubble parameter and the local expansion of space is always slowing.

Observations of the cosmic microwave background demonstrate that the universe is very nearly flat. Therefore, from equation 3, we have $q=\frac{1}{2}(1 + 3\omega)$. This implies that the universe is decelerating for any cosmic fluid with equation of state ω greater than $-\frac{1}{3}$. Hence a negative q, that implies an accelerated expansion of the universe, is an indication for the existence of dark energy with a negative pressure given by $-\rho c^2$.

The cosmological constant is the energy density of vacuum, originally introduced by Einstein [3] as an addition to his theory of general relativity (GR) to make the universe static. If the universe is filled with just ordinary matter and radiation, GR predicts that the gravitational attraction of all matter in the universe would pull it back and slow down the rate of expansion, i.e., decelerate it. Einstein's field equation included a cosmological constant term to counteract the gravitational attraction of all the matter in the universe, and hence make the universe static. Einstein abandoned the concept after Hubble's discovery, in 1929, that all galaxies outside the Local Group are moving away from each other. This led Einstein to call his cosmological constant "his greatest blunder".

2. Observational Evidence

Supernovae are used as standard candles for cosmological measurements. At small redshifts they could be used to measure the Hubble constant and at higher redshifts they can be used to determine the deceleration parameter. In 1998, two independent measurements from Type Ia supernovae [7, 6] show that the expansion of the universe is accelerating. The luminosity distance of an object at a red shift of z is given by:

$$D_L = \frac{cz}{H_o}[1 + \frac{(1 - q_0)}{2}z] \tag{4}$$

The luminosity distance, D_L, of an object of known intrinsic luminosity L and observed flux density, f, can be derived from the square law:

$$D_L = \sqrt{\frac{L}{4\pi f}} \tag{5}$$

For a matter dominated flat universe, Ω_m=1, i.e., a decelerating universe, the light emitted at a redshift of z=1 by a type IaSNe would travel a less distance than in a universe expanding at a constant rate. The flux from a distant supernova for matter dominant universe would be $\tilde{2}5\%$ brighter. But observations show that the supernovae are not as bright as expected for such a universe; they appear fainter, hence indicating an accelerating universe and a negative deceleration parameter. The Supernova Cosmology Project also reported the mass density and cosmological constant energy density of the universe based on the analysis of 42 type IaSNe. The magnitude

– redshift data for these SNe at redshifts between 0.18 and 0.83, indicate that the cosmological constant is non-zero and positive, with a confidence of P=99%.

3. Cosmological Constant

The Einstein's originalfield equation was of the form [2]: (6)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{6}$$

where, $R_{\mu\nu}$ is the Ricci curvature tensor,R is the curvature scalar, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the stress-energy momentum tensor. The left hand side of the equation stands for the curvature of space-time and the right hand side stands for mass and energy of the universe. In accelerated frames described by curved space-time, the curvature is caused by the distribution of matter and energy. [10, 11, 8]

Einstein wanted to apply his field equations to model the whole universe. But he wrongly assumed that the universe should be static. During that time, there was hardly any observational evidence for an expanding universe. However it is known that the gravitational attractive force of matter would cause the universe to collapse and not remain static. In about 15 months after his publication of the field equations of GR, Einstein proposed a modification to his equation to [3]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{7}$$

The Λ in equation 7 is a negative pressure term called(cosmological constant) that givesa repulsive gravitation.On very large scales the universe is spatially homogenous and isotropic. The metric takes the Robertson – Walker (RW) form:

$$ds^2 = dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{8}$$

We can abbreviate the terms within the brackets as, $d\omega^2 = d\theta^2 + sin^2\theta d\phi^2$. An alternate metric that obeys both, isotropy and homogeneity as proved by Robertson and Walker is:

$$ds^2 = dr^2 + f(k)r^2d\omega^2 \tag{9}$$

where, f(k)is a curvature function for which the universe is open, flat or closed for $k > 0, k = 0, k < 0$, respectively. In a real universe, one would expect the scale factor and the RW metric to vary with time. This gives:

$$ds^2 = -dt^2 + a(t)^2[dr^2 + f(k)r^2 d\omega^2] \quad (10)$$

The function a(t) is the scale factor that will describe the expansion of the universe. The scale factor is normalised such that at the present time we have a=1. This metric gives two solutions to the Einstein's equation:

$$(\frac{\dot{a}}{a}^2) = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \quad (11)$$

$$\frac{\dot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3} \quad (12)$$

These equations are called the Friedmann Equations. The cosmological constant was discarded when Hubble made his discovery of the expansion of the universe. There was no need for a static universe and Einstein himself thought that the cosmological constant was a blunder and the cosmological constant was abandoned completely. The observations from the type Ia supernova show that the universe is accelerating. This has led many to believe that Λ is non-zero, and Einstein may not have blundered after all.The Friedmann equation can also be written as:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{(kc^2)}{a^2} \quad (13)$$

WhereH is the Hubble's constant, Λ is replaced by $\rho_\Lambda = \frac{\Lambda}{8\pi G}$, ρ_m is the matter density and ρ_r is the radiation density. For the defined critical density, $\rho_c = \frac{(3H^2)}{8\pi G}$, we see that $(\rho_m + \rho_r + \rho_A)$ becomes equal to the critical density for a flat universe, i.e., $k = 0$. From the equality of curvature and expansion rate, this would mean that the universe expands critically. Therefore this density is the one at which the universe expands critically. The densities can be further expressed by a dimensionless parameter $\Omega = \frac{\rho}{\rho_c}$. The first Friedmann equation can be expressed in terms of Ω values and Hubble constant as they are at the present time. This is given by:

$$\frac{H^2}{(H_0^2)} = \frac{\Omega_{(r,0)}}{a^4} + \frac{\Omega_{(m,0)}}{a^3} + \frac{\Omega_{(k,0)}}{a^2} + \Omega_{(\Lambda,0)} \quad (14)$$

For a matter dominated flat universe, $\Omega_{(m,0)} = 1, \Omega_{(\Lambda,0)} = \Omega_{(r,0)} = \Omega_{(k,0)} = 0$, the above equation becomes:

$$\frac{\dot{a}}{a} = \frac{H_0}{a^{3/2}} \tag{15}$$

which has a solution $a(t) = ((\frac{3}{2})H_0 t)^{(2/3)}$. This is the Einstein–de Sitter model for the Universe. It is an expanding universe with the expansion rate inversely proportional to time. According to this model the age of the universe is around $9x10^9$ years. For a radiation dominated universe we get $a(t) = (2H_0 t)^{(1/2)}$. This is similar to the matter dominated universe that expands but decelerates eventually. This is because pressure alone does not help to balance the attractive gravitational force. It follows from the Friedmann equation that a pressure gradient alone can induce force that counteracts the gravitational force. In a homogenous universe there is no pressure gradient, therefore pressure cannot help expand the universe.

If $\Omega_{(\Lambda,0)} = 1, \Omega_{(m,0)} = \Omega_{(r,0)} = \Omega_{(k,0)} = 0$, the solution becomes $a(t) = e^{(H_0 t)}$. In this universe, the age of the universe is infinite. This is the de Sitter model of the universe, which models the universe as spatially flat and neglects ordinary matter, so the dynamics of the universe is dominated by the cosmological constant. This is the solution of the GR field equations with a background curved space of uniform curvature Λ, i.e., space-time is now not asymptotically flat (Minkowskian), i.e., at $r \to \infty$, we now have a de-Sitter space, a space of uniform positive curvature Λ. The de Sitter metric is given by:

$$ds^2 = (1 - (\Lambda r^2)/3)c^2 dt^2 - (1 - (\Lambda r^2)/3)^(- 1)dr^2 - r^2 d\omega^2 (16) \tag{16}$$

The current model of the universe suggests that the universe is flat $\Omega_{(k,0)} = 0$, but contains matter, radiation and a non-zero cosmological constant. The pressure due to a cosmological constant term, given by $\frac{(\Lambda c^4)}{8\pi G}$, might just be the missing pressure which is presumably DE.[12, 13]

4. Conclusion

The question of DE is also far from resolved. The possible models is not restricted to cosmological constant, other models include quintessence, phantom energy, etc. The current observations is consistent with a cosmological constant playing the role of repulsive gravity [9, 1]. But the models do not predict why DE constitutes close to 70% of the universe's energy density. And present observations show that even growth rate of structures cannot differentiate between the alternatives.

It is argued that dark energy appears to be an ad hoc postulate that is added to a theory in response to observations. Some alternate models to DE aim to explain the observations as merely a measurement artefact. For instance, if we are located in a region of the universe that is less dense than average, the observed cosmic expansion rate could be interpreted as an acceleration.Whether the existence of dark energy is confirmed or an alternate model turns out to be correct, the next few years promises to be exciting.

References

[1] K. Arun, S. B. Gudennavar, C. Sivaram, "Dark matter, dark energy, and alternate models: A review," *Adv. Space Res.*, vol. 60, pp. 166, 2017.

[2] A. Einstein, "The Foundation of the General Theory of Relativity," *Annalen der Physik*, vol. 49, pp. 769, 1916.

[3] A. Einstein, "Cosmological Considerations in the General Theory of Relativity," *Sitz. König. Preuss. Akad.*, part 1, vol. 142, 1917.

[4] M. Jones, and R. Lambourne, *An Introduction to Galaxies and Cosmology*. Cambridge University Press, Cambridge, 2004.

[5] P. J. E. Peebles and B. Ratra, "The cosmological constant and dark energy," *Rev. Mod. Phys.*, vol. 75, pp. 559, 2003.

[6] S. Perlmutter et al., "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," *Astrophys. J.*, vol. 517, pp. 565, 1999.

[7] A. G. Riess et al., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *Astron. J.*, vol. 116, pp. 1009, 1998.

[8] C. Sivaram, K. Arun, and O. V. Kiren, *100 years of Einstein's Theory of Relativity: An Introduction to Gravity and Cosmology*. Ane Books, New Delhi, 2016.

[9] C. Sivaram and K. P.Sinha, S"trong spin-two interaction and general relativity," *Phy. Reports*, vol. 51, pp. 111, 1979.

[10] C. Sivaram, "Mond, dark matter and the cosmological constant," *Astrophys. Space Sci.*, vol. 219, pp. 135, 1994.

[11] C. Sivaram, "A non-anthropic origin for a small cosmological constant," *Bull. Astron. Soc. India*, vol. 27, pp. 377, 1999.

[12] C. Sivaram, "On Zero-Point Fluctuations, the Cosmological Constant, and the Graviton Mass," *Astrophys. J.*, vol. 520, pp. 454, 1999.

[13] C. Sivaram, "A non-anthropic origin for a small cosmological constant, *Mod. Phys. Lett. A*," vol. 14, pp. 2363, 1999.

[14] C. Sivaram, "A Brief History of Dark Energy," *Astrophy. Space Sci.*, vol. 319, pp. 3, 2009.