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L(t, 1)-Colouring of Cycles

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Abstract

For a given finite set T including zero, an L(t, 1)-colouring of a graph G is an assignment of non-negative integers to the vertices of G such that the difference between the colours of adjacent vertices must not belong to the set T and the colours of vertices that are at distance two must be distinct. For a graph G, the L(t, 1)-span of G is the minimum of the highest colour used to colour the vertices of a graph out of all the possible L(t, 1)-colourings. We study the L(t, 1)-span of cycles with respect to specific sets.

Keywords: L(t, 1)-colouring, Communication networks, Channel assignment, Radio frequency, Colour span, Cycles.

Mathematics Subject Classification (2010): 05C38, 05C78

1. Introduction

The concept of graph colouring finds its use in the optimal assignment of radio frequencies to radio stations in a specific region. The first kind of channel assignment problem was brought into picture by Metzger[4]. The T-colouring problem introduced by Hale[3] is one of the first types of graph colouring used in radio channel assignment. In this colouring, the vertices must be assigned colours in such a way that the difference of colours of any two neighbouring vertices should not belong to the given set T. Later, Roberts[8] in his private communication with Griggs proposed that the disturbance in transmission of signals is not only due to neighbouring transmitters but also due to the transmitters at distance 2. This led to the study of L(2, 1)-colouring. Labelling vertices of graphs at distance 2 was studied extensively by Griggs and Yeh[10].

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We introduced another type of colouring called L(t, 1)-colouring which finds its foundation in T-colouring and L(2, 1)-colouring[6] and studied the L(t, 1)-colouring of Wheel Graphs in [7]. One of the major difficulties when it comes to colour any graph when the set T comes in picture is the random gaps in the set T. The randomness of the set T makes it tough to work on bound related problems for graph of larger size.

In this work we study the bounds for L(t, 1)-span of cycles [5]. All standard definitions and notations related to graphs are according to [9].

2. L(t, 1)-span of Paths and Cycles

L(t, 1)-colouring is defined as follows.

Definition 2.1. *Let G=(V, E) be a graph and let $d(u, v)$ be the distance between the vertices u and v of G. Let T be a finite set of non-negative integers containing 0. An L(t, 1)-colouring of a graph G is an assignment c of non-negative integers to the vertices of G such that $|c(u) - c(v)| \notin T$ if $d(u, v) = 1$ and $c(u) \neq c(v)$ if $d(u, v) = 2$[6].*

Definition 2.2. *[6] For a graph G with a given set T and all the L(t, 1)-colourings c of G, L(t, 1)-span of G denoted by the symbol $\lambda_{t,1}(G)$ is*

$$\lambda_{t,1}(G) = min\,\{\max_{u,v\in V(G)} \{|c(u) - c(v)|\}\} \tag{1}$$

Next, we find the bounds of L(t, 1)-span of cycles for some specific sets T.

The following lemma is very trivial.

Lemma 2.3. *A minimum of three colours are required to colour any connected graph G with $n \geq 3$ in L(t, 1)-colouring.*

Theorem 2.4. *For a finite set T of even numbers containing 0 and 2 the L(t, 1)-span of the cycles of length $n \geq 3$ is given by*

$$\lambda_{t,1}(C_n) = \begin{cases} 3 & if\;\; n \equiv 0(mod\; 4) \\ c_\sigma & if\;\; n \equiv 1(mod\; 4) \\ 5 & if\;\; n \equiv 2(mod\; 4) \\ c_\sigma & if\;\; n \equiv 3(mod\; 4) \end{cases}$$

where c_σ = least even integer not occurring in set T.
Furthermore, if $2 \notin T$ then,

$$\lambda_{t,1}(C_n) = \begin{cases} 2 & if\;\; n \equiv 0(mod\; 3) \\ 3 & if\;\; n \equiv 1(mod\; 3) \\ a & if\;\; n \equiv 2(mod\; 3) \begin{cases} a = 5\; if\; 4 \in T \\ a = 4\; if\; 4 \notin T \end{cases} \end{cases}$$

Proof. We prove the theorem in two parts. In the first part, let us assume that T contains 0 and 2. Let the vertices of the cycle be labelled as $v_1, v_2, \ldots, v_{n-1}, v_n, v_1$. Let f be the colouring function defined from $V(G) \to \mathbb{N} \cup \{0\}$.

Case 1: $n \equiv 0 \pmod 4$

The following colouring gives the least possible L(t, 1)-colouring.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 4) \\ 1 & \text{if } i \equiv 2 (mod\ 4) \\ 2 & \text{if } i \equiv 3 (mod\ 4) \\ 3 & \text{if } i \equiv 0 (mod\ 4) \end{cases}$$

By this colouring, the vertices that are adjacent will have the colour difference 1 or 3 which is not a part of set T and the vertices which are at distance 2 will get distinct colours. Figure 1 shows the colouring for such a graph.

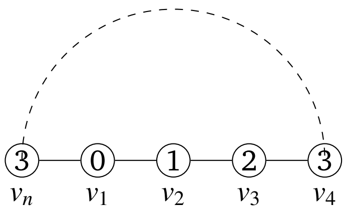

Figure 1: L(t, 1)-colouring of cycle C_n for $n \equiv 0(mod\ 4)$

Case 2: $n \equiv 1 \pmod 4$

The colouring given below gives the least possible L(t, 1)-colouring for all vertices of the cycle for $1 \leq i \leq n-1$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 4) \\ 1 & \text{if } i \equiv 2 (mod\ 4) \\ 2 & \text{if } i \equiv 3 (mod\ 4) \\ 3 & \text{if } i \equiv 0 (mod\ 4) \end{cases}$$

and

$$f(v_n) = c_\sigma$$

where c_σ is the least even integer not occurring in the set T. Here, we see that $|f(v_i) - f(v_{i+1})| = 1$ for $1 \leq i \leq n-2$. Therefore, none of the adjacent vertices $\{v_i, v_{i+1}\}$ for $1 \leq i \leq n-2$ would have colour difference an even number.

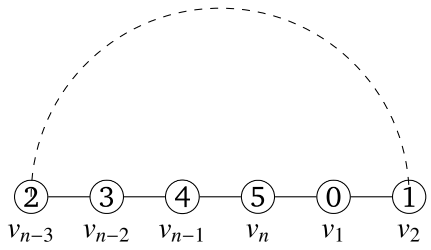

Figure 2: L(t, 1)-colouring of cycle C_n for $n \equiv 2(mod\ 4)$

Moreover, $|f(v_{n-1}) - f(v_1)| = |c_\sigma - 3|$, which is an odd number and $|f(v_n) - f(v_1)| = |c_\sigma - 0| = c_\sigma$. Hence for adjacent vertices f gives a colouring such that colour difference does not belong to T.

For vertices at distance 2, $|f(v_i) - f(v_{i+2})| = 2$ for $1 \leq i \leq n-3$, $|f(v_{n-2}) - f(v_n)| = |2 - c_\sigma|$ which is at least 2. Also, $|f(v_{n-1}) - f(v_1)| = |3 - 0| \geq 1$ and $|f(v_n) - f(v_2)| = |c_\sigma - 1| \geq 3 \geq 1$.

Case 3: $n \equiv 2\ \ (mod\ 4)$

The colouring given below gives the least possible L(t, 1)-colouring for all vertices of the cycle for $1 \leq i \leq n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1(mod\ 4) \\ 1 & \text{if } i \equiv 2(mod\ 4) \\ 2 & \text{if } i \equiv 3(mod\ 4) \\ 3 & \text{if } i \equiv 0(mod\ 4) \end{cases}$$

and

$$f(v_i) = \begin{cases} 4 & \text{if } i = n-1 \\ 5 & \text{if } i = n \end{cases}$$

We can give the same argument as in case 1 for vertices $v_i, 1 \leq i \leq n-2$. From the Figure 2, $f(v_{n-3}) = 2, f(v_{n-2}) = 3, f(v_{n-1}) = 4, f(v_n) = 5, f(v_1) = 0, f(v_2) = 1$. Hence the above colouring f justifies the condition for L(t, 1)-colouring.

Case 4: $n \equiv 3\ \ (mod\ 4)$

The colouring given below gives the least possible L(t, 1)-colouring for all vertices of the cycle for $1 \leq i \leq n-1$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1(mod\ 4) \\ 1 & \text{if } i \equiv 2(mod\ 4) \\ 2 & \text{if } i \equiv 3(mod\ 4) \\ 3 & \text{if } i \equiv 0(mod\ 4) \end{cases}$$

and

$f(v_n) = c_\sigma$

In this colouring adjacent vertices have colour difference an odd number and non-adjacent vertices have distinct colours. This concludes the proof of the first part.

In the second part of the proof, let us consider the situation that $2 \notin T$. The proof is obtained by considering three cases.

Case 1: $n \equiv 0 \pmod 3$

Let the colouring be given as follows

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

As $2 \notin T$, the above colouring implies that the colour difference of adjacent vertices and vertices at distance 2 are either 1 or 2. Hence it is an optimal L(t, 1)-colouring since we are using the colours 0, 1 and 2.

Case 2: $n \equiv 1 \pmod 3$

For $1 \leq i \leq n-1$, the colouring is given below.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$f(v_n) = 3$

The vertices which are adjacent will have the colour difference 1 or 3 which are not a part of set T and the vertices which are at distance 2 will get distinct colours. For v_n neither 0 nor 2 would fit in as they are the colours of the adjacent vertices. For the two vertices which are at the distance 2 from v_n f assigns the colour 1. Hence v_n is forced to have the colour 3, which is the least number satisfying the condition for L(t, 1)-colouring. Hence f gives the least possible L(t, 1)-colouring.

Case 3: $n \equiv 2 \pmod 3$

The colouring given below gives the least possible L(t, 1)-colouring for all vertices of the cycle for $1 \leq i \leq n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

Here, $(n-1)^{th}$ and n^{th} vertices can be coloured depending on the presence of element 4 in the set T.
If $4 \in T$ then,

$$f(v_i) = \begin{cases} 3 & \text{if } i = n-1 \\ 5 & \text{if } i = n \end{cases}$$

Since, $|f(v_i) - f(v_{i+1})| \in \{1,2,5\} \notin T$ for $1 \leq i \leq n-1$, $f(v_i) \neq f(v_{i+2})$ for $1 \leq i \leq n-2$, $f(v_{n-1}) \neq f(v_1)$and $f(v_n) \neq f(v_2)$. Thus we get a least possible L(t, 1)-colouring.
If $4 \notin T$ then,

$$f(v_i) = \begin{cases} 3 & \text{if } i = n-1 \\ 4 & \text{if } i = n \end{cases}$$

Here, the vertices which are adjacent will have the colour difference 1 or 4 which are not part of the set T and the vertices which are at distance 2 will get distinct colours. Hence, the conditions for L(t, 1)-colouring are satisfied. □

Remark 2.5. *When $2 \notin T$, span is always less than 5 i.e., the value of $\lambda_{t,1}(C_n)$ does not depend on the highest value of the set T.*

Corollary 2.6. *For a finite set T containing 0 and all consecutive even integers from 2 such that max{T}=r, the L(t,1)-span of the cycles is given by:*

$$\lambda_{t,1}(C_n) = \begin{cases} 3 & \text{if } n \equiv 0 (mod\ 4) \\ r+2 & \text{if } n \equiv 1 (mod\ 4) \\ 5 & \text{if } n \equiv 2 (mod\ 4) \\ r+2 & \text{if } n \equiv 3 (mod\ 4) \end{cases}$$

We consider the case of 0 and odd integers alone.

Theorem 2.7. *For a finite set T of odd numbers containing 0 and 1, the L(t, 1)-span of the cycles for length $n \geq 3$ is given by:*

$$\lambda_{t,1}(C_n) = \begin{cases} 4 & \text{if } n \equiv 0 (mod\ 3) \\ 6 & \text{if } n \equiv 1 (mod\ 3) \\ a & \text{if } n \equiv 2 (mod\ 3) \begin{cases} a = 8 & \text{if } 3 \in T \\ a = 4 & \text{if } 3 \notin T \end{cases} \end{cases}$$

If $1 \notin T$, then

$$\lambda_{t,1}(C_n) = \begin{cases} 2 & \text{if } n \equiv 0 (mod\ 3) \\ a & \text{if } n \equiv 1 (mod\ 3) \begin{cases} a = 4 & \text{if } 3 \in T \\ a = 3 & \text{if } 3 \notin T \end{cases} \\ 4 & \text{if } n \equiv 2 (mod\ 3) \end{cases}$$

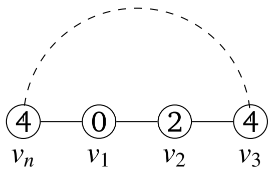

Figure 3: L(t, 1)-colouring of cycle C_n for $n \equiv 0 (mod\ 4)$

Proof. Let the vertices of the cycle be labelled as $v_1, v_2, \ldots, v_{n-1}, v_n, v_1$. Let us assume that T contains 0 and 1. Let f be a function defined from $V(G) \rightarrow \mathbb{N} \cup \{0\}$.

Case 1: $n \equiv 0 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 2 & \text{if } i \equiv 2 (mod\ 3) \\ 4 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

Figure 3 shows the colouring for such graph.

Case 2: $n \equiv 1 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \leq i \leq n-1$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 2 & \text{if } i \equiv 2 (mod\ 3) \\ 4 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_n) = 6$$

The vertices which are adjacent will get the colours in such a way that the difference between the colours will be an even number which does not belong to set T and the vertices which are at distance 2 will get distinct colours. Any number less than 6 will violate the condition for L(t, 1)-colouring, so this is the least possible L(t, 1)-colouring.

Case 3: $n \equiv 2 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \leq i \leq n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 2 & \text{if } i \equiv 2 (mod\ 3) \\ 4 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

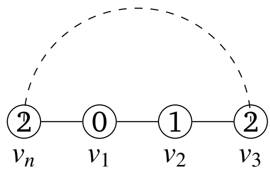

Figure 4: L(t, 1)-colouring of cycle C_n for $n \equiv 0(mod\ 4)$

Here, $(n-1)^{th}$ and n^{th} vertices can be coloured depending on the presence of element 3 in set T.
If $3 \in T$ then

$$f(v_i) = \begin{cases} 6 & \text{if } i = n-1 \\ 8 & \text{if } i = n \end{cases}$$

The vertices which are adjacent will get the colours in such a way that the difference between the colours will be an even number $\in \{2, 8\}$ which does not belong to set T and the vertices which are at distance 2 will get distinct colours.

If $3 \notin T$ then

$$f(v_i) = \begin{cases} 1 & \text{if } i = n-1 \\ 3 & \text{if } i = n \end{cases}$$

In this case, the vertices which are adjacent will get the colours in such a way that the difference between the colours will be an even number $\in \{2, 3\}$ which does not belong to set T and the vertices which are at distance 2 will get distinct colours.

Now let us consider the situation that $1 \notin T$. The proof for this is obtained by considering three cases.

Case 1: $n \equiv 0 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1(mod\ 3) \\ 1 & \text{if } i \equiv 2(mod\ 3) \\ 2 & \text{if } i \equiv 0(mod\ 3) \end{cases}$$

Figure 4 gives an L(t, 1)-colouring for such graph.

Case 2: $n \equiv 1 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n-1$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1(mod\ 3) \\ 1 & \text{if } i \equiv 2(mod\ 3) \\ 2 & \text{if } i \equiv 0(mod\ 3) \end{cases}$$

Here, n^{th} vertex can be coloured depending on the presence of element 3 in set T.
If $3 \in T$ then

$$f(v_n) = 4$$

The vertices which are adjacent will get the colours in such a way that the difference between the colours will belong to set $\{1, 4\}$ which does not belong to set T and the vertices which are at distance 2 will get distinct colours.

If $3 \notin T$ then

$$f(v_n) = 3$$

Similarly, here the vertices which are adjacent will get the colours in such a way that the difference between the colours will belong to set $\{1, 3\}$ which does not belong to set T and the vertices which are at distance 2 will get distinct colours.

Case 3: $n \equiv 2 \pmod 3$
The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_i) = \begin{cases} 3 & \text{if } i = n-1 \\ 4 & \text{if } i = n \end{cases}$$

It is easy to check that the vertices which are adjacent will get the colours in such a way that the difference between the colours will be an even number $\in \{1, 4\}$ which does not belong to set T and the vertices which are at distance 2 will get distinct colours. □

Remark 2.8. *Here, $\lambda_{t,1}(C_n)$ does not depend on the highest value r of set T.*

Theorem 2.9. *For a finite set T containing 0 and multiples of m where $m \ge 3$, the L(t, 1)-span of the cycles*

$$\lambda_{t,1}(C_n) \le 4.$$

Proof. Consider a finite set $T = \{0, m, 2m, 3m, \dots, r\}$, where r is the $\max\{T\}$.

For m=1, L(t, 1)-colouring becomes L(p, q)-colouring, where p=r+1, and q=1. The bound for which was studied by J. P. Georges and D. W. Mauro in [2].

For m=2, bound is given in Corollary 1. For m=3, we will show that the bound is 4 for following three cases.

Let the vertices of cycle be labelled as $v_1, v_2, \dots, v_{n-1}, v_n, v_1$. Let us assume that T contains elements $\{0, 3, 6, \dots, r\}$. Let f be a function defined from $V(G) \to \mathbb{N} \cup \{0\}$.

Case 1: $n \equiv 0 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1 or 2 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Since 0, 1, 2 are the least possible colours therefore it gives an optimal L(t, 1)-colouring.

Case 2: $n \equiv 1 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n-1$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_n) = 4.$$

The vertices which are adjacent will get the colour with colour difference 1, 2 or 4 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case.

Case 3: $n \equiv 2 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_i) = \begin{cases} 3 & \text{if } i = n-1 \\ 4 & \text{if } i = n \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1 or 4 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case. Therefore, for m=3, $\lambda_{t,1}(C_n) \le 4$.

For m=4; we will show that the bound is 4 for following three cases.

Case 1: $n \equiv 0 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1 or 2 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case.

Case 2: $n \equiv 1 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_n) = 3$$

The vertices which are adjacent will get the colour with colour difference 1, 2 or 3 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case.

Case 3: $n \equiv 2 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \le i \le n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

$$f(v_i) = \begin{cases} 3 & \text{if } i = n-1 \\ 4 & \text{if } i = n \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1 or 4 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case. Therefore, for m=4, $\lambda_{t,1}(C_n) \leq 4$.

For $m \geq 5$, We can colour any cycle using the following sequence:

Case 1: $n \equiv 0 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \leq i \leq n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1 or 2 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case.

Case 2: $n \equiv 1 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \leq i \leq n$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_n) = 3$$

The vertices which are adjacent will get the colour with colour difference 1, 2 or 3 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case.

Case 3: $n \equiv 2 \pmod 3$

The following colouring gives the least possible L(t, 1)-colouring for all vertices of cycle for $1 \leq i \leq n-2$.

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 (mod\ 3) \\ 1 & \text{if } i \equiv 2 (mod\ 3) \\ 2 & \text{if } i \equiv 0 (mod\ 3) \end{cases}$$

and

$$f(v_i) = \begin{cases} 4 & \text{if } i = n-1 \\ 3 & \text{if } i = n \end{cases}$$

The vertices which are adjacent will get the colour with colour difference 1, 2 or 3 which are not an element of the set T and the vertices which are at distance 2 will get distinct colours. Thus it gives an optimal L(t, 1)-colouring in this case. Therefore, $\lambda_{t,1}(C_n) \leq 4$. Hence the result. □

Remark 2.10. *$\lambda_{t,1}(C_n)$ does not depend on the highest value r of set T when the gap between the terms of T starts increasing.*

3. Conclusion

In this paper, we found the bounds for L(t, 1)-span of cycles for some specific set T. The work done in this paper can be extended to various other classes of graphs, for various set T.

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