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# SOME ASPECTS OF BINARY COMPACT ASTROPHYSICAL OBJECTS

Kenath Arun*

ABSTRACT

*This paper shall look into systems of compact astrophysical objects (black holes, neutron stars and white dwarfs) concerning mainly with binary systems. Determination of the orbital periods, velocities, etc, for such systems and also, the energy lost from the system through gravitational waves has been made. From the concept of energy loss, the merger of the two black holes in the system has been considered and the typical time required for the merger is calculated.*

## 1. Binary black hole

About half of all stars in the universe come in binary systems. If they are sufficiently massive to start with, then once their nuclear burning stops they end up as binary neutron stars or black holes. A binary system emits gravitational radiation [1, 2], which carries away energy. As a consequence, the orbit shrinks, slowly at first but faster and faster, until the two objects merge to a single, rapidly spinning black hole.

* Christ Junior College, Bangalore, e-mail: kenath.arun@cjc.christcollege.edu

These black holes are orbiting each other and will merge several hundred million years from now, to create an even larger black hole resulting in a catastrophic event that will unleash intense radiation and gravitational waves. The detection of a binary black hole supports the idea that black holes grow to enormous masses in the centres of galaxies by merging with other black holes. Toward the end of this process an enormous burst of gravitational waves will be produced. These gravitational waves will spread through the universe and produce ripples in the fabric of space. These ripples would appear as minute changes in the distance between any two points and could be detected [3, 4]. Gravitational waves, predicted by the theory of general relativity, are ripples in space-time. By analogy to electromagnetic waves, which are produced when a charged particle is accelerated, gravitational waves should occur when mass is accelerated. The predicted effect is quite small.

Moving masses like stars or black holes produce gravitational waves in the fabric of space-time. A more massive moving object will produce more powerful waves, and objects that move very quickly will produce more waves over a certain time period. Gravitational waves are usually produced in an interaction between two or more compact masses.

Such interactions include the binary orbit of two black holes, a merger of two galaxies, or two neutron stars orbiting each other. As the black holes, stars, or galaxies orbit each other, they send out waves of "gravitational radiation" that reach the Earth. However, once the waves do get to the Earth, they are extremely weak. This is because gravitational waves decrease in strength (red-shifts) as they move away from the source. Even though they are weak, the waves can travel unobstructed within the fabric of space-time.

We can analyse the system using classical laws like the Kepler's equations [5]. The equation of force for the stabilized system is given by equating the centrifugal force to the gravitational force. From Kepler's third law, the period is related to the separation by,

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} R^3 \qquad \ldots (1)$$

For a system of binary black holes of mass $10^8 M_\odot$ each, separated by one parsec, the orbital period is then given by $P \approx 10^3$ years. Knowing the period, we can determine the orbital velocity from, $vP = 2\pi R$. For this system, this works out to about, $v \approx 10^5$ m/s. The energy lost by the system due to the emission of gravitational waves, for a circular orbit, is given by, [6]

$$\dot{E}_{GW} = \frac{128v^{10}}{5Gc^5} \qquad \ldots (2)$$

For the above system, this works out to about $\dot{E}_{GW} \approx 10^{25}$ J/s

The system emitting gravitational waves at this rate will merge together in a time scale given by:

$$\tau_{MER} = \frac{GM^2}{R} \Big/ \dot{E}_{GW} \qquad \ldots (3)$$

In the case of the above-mentioned system, the merger time is of the order of $\tau_{MER} \approx 10^6 T_H$, where $T_H = 10^{10}$ years is the Hubble time. The total energy emitted by the system during the merger is given by $E = \dot{E}_{GW} \times \tau_{MER} \Rightarrow E = 10^{50}$ J.

This is the amount of energy released during the process of merger of two black holes. Most of the energy liberated is in the last stage of the merger when the black holes are almost in contact. At this stage, the distance of separation between the two black holes corresponds to twice the Schwarzschild radius of the black hole [7], that is,

$$R = 2\left(\frac{2GM}{c^2}\right) \qquad \ldots (4)$$

The energy emitted during this stage is given by:

$$E = \frac{GM^2}{R} = \frac{1}{4}Mc^2 \qquad \ldots (5)$$

And this corresponds to about $10^{50}$J.

Since the merger time for the system of black holes is about a million times the age of the Universe we cannot observe this process. For the merger time to be of the order of the Hubble time, we can calculate the separation between the black holes.

The merger time is given by equation (3), and the gravitational energy lose is given by equation (2). Making use of all the results obtained above for period and orbital velocity, we can easily show that for a binary black hole system with $10^8 M_\odot$ black holes, the merger time will be of the order of the Hubble time if the distance of separation is $R \approx 10^{12} m$.

## 2. Binary Systems of White Dwarfs and Neutron Stars

The expressions we have arrived in the last section are applicable to other binary systems as well, like those of white dwarfs and neutron stars. In the case of a binary system consisting of two solar mass white dwarfs, having orbital period of 5 minutes, the distance of separation is given by the Kepler's law, which comes to about $10^7$ m.

The corresponding lose of energy due to the emission of gravitational waves is given by,

$$\dot{E}_{GW} = \frac{128 v^{10}}{5 G c^5} \approx 10^{25} J/s \qquad \dots (6)$$

And the merger time is given by,

$$\tau_{MER} = \frac{GM^2}{R} \Big/ \dot{E}_{GW} \sim 10^{10} \text{ years} \qquad \dots (7)$$

In a similar way we can estimate the merger time for a binary system consisting of two neutron stars of mass $1.5 M_\odot$ each, which comes to be of the order of $10^8$ years.

## 3. Energy Emitted in the Merger Time for Equal Mass Binary System

The gravitational energy is given by, $E = \frac{GM^2}{R} \Rightarrow \frac{dE}{dR} = \frac{GM^2}{R^2}$

The rate of change of the energy with time is given by,

$$\dot{E} = \frac{dE}{dt} = \frac{dE}{dR}\frac{dR}{dt} \quad \text{... (8)}$$

That is $\dot{E}_{GW} = \frac{128v^{10}}{5Gc^5} = \frac{GM^2}{R^2}\frac{dR}{dt}$ ... (9)

Also the orbital velocity is given by the Kepler's law. From this we have:

$$\frac{dR}{dt} \approx \frac{G^3M^3}{c^5R^3} \quad \text{... (10)}$$

On integrating the above expression, we get R as a function of time.

$$R \approx \left( \frac{3(GM)^3}{c^5} t \right)^{1/4} \quad \text{... (11)}$$

The total energy is given by

$$E = \int_0^{\tau} \frac{dE}{dt} dt = \int \frac{dE}{dR}\frac{dR}{dt} dt \quad \text{... (12)}$$

On substituting the known quantities, we get the energy as a function of the merger time as,

$$E \approx \frac{4\times10^{55}}{\tau_{mer}^{1/4}} \quad \dots (13)$$

For a merger time as calculated in the previous section, this energy corresponds to, $E \approx 10^{50}$ J.

This result is in agreement with that calculated in the last section through independent reasoning. This is the amount of energy liberated during the entire merger time associated with the coalesce of two black holes by the lose of energy through the gravitational wave emission. As we have already seen, most of the energy is liberated during the last stage of the merger.

The same process in the case of a binary system of neutron stars can also be explained in the same way. A similar type of expression gives the energy associated over the entire merger time (see equation (13)),

$$E \approx \frac{4\times10^{45}}{\tau_{mer}^{1/4}} \quad \dots (14)$$

We have already determined the merger time for two neutron stars to be, $10^8$ years. Using this in the above relation, the energy is of the order of $\sim 2\times10^{41}$ J. We can see that the energy associated with the neutron star is a billion times lesser than that of the supermassive black hole system.

# 4. Conclusion

In this paper the dynamics of binary systems consisting of different compact objects have been discussed. The results obtained are consistent with other independent results and observations and these results are strong confirmation for the existence of gravitational waves and for general theory of relativity. The mergers of the binary black holes can lead to the formation of the supermassive black holes residing at the centres of galaxies. The mechanisms for the formation of these as well as the intermediate mass black hole (mass ranging form 500 to $10^4$ solar mass) are not well under stood. Some of the possible mechanisms are discussed in reference 8.

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