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Effect of Rotational Modulation on Rayleigh–Bénard Convection in a Couple Stress Liquid

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Abstract

The Rayleigh-Bénard convection in a couple stress liquid with rotational modulation is studied using the linear analysis based on normal mode technique. The stability of a horizontal layer of fluid heated from below is examined when, in addition to a steady rotation, a time-periodic sinusoidal perturbation is applied. The expression for Rayleigh number and correction Rayleigh number are obtained using regular perturbation method. The expression for correction Rayleigh number is obtained as a function of frequency of modulation, Taylor number, Couple Stress parameter and Prandtl number. It is observed that rotational modulation leads to delay in onset of convection. Rotation modulation is an example of external control of internal convection.

Keywords: Rotational Modulation, couple stress fluid, Rayleigh-Bénard convection, correction Rayleigh number

Mathematics Subject Classification (2010): 76R50

1. Introduction

The study of free convection induced by centrifugal force in a rotating layer is of prime interest due to its importance from both theoretical as well as practical point of views. The classical problem initiated

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by Bénard[1] and Rayleigh[13] gives the condition for onset of convection. Chandrashekar[5] extended the Rayleigh[13] analysis to the rotating case. He found that the rotation viscosity plays inverted role, when compared with viscosity of the fluid, i. e., the fluid layer will be stable when the viscosity decreases, which is due to Taylor-Proudman effect. Two German researchers Küppers and Lortz [9] studied the convection pattern in an infinite horizontal fluid layer with rotation and found that for large rotation rate, the instability of a pattern are parallel but rotated through a certain angle with respect to original one. They conjectured that for the above critical rotation rate, the onset of convection must be time-dependent. A detailed study of effect of gravity and centrifugal acceleration on the stationary convection in a rotating vertical porous layer placed far away from axis of rotation is presented by Vadaz and Govender [20]. The effect of coriolis force on the stability of free convection due to centrifugal acceleration has been investigated by Govender [7].

In many practically important problems, the control of convection is important and one of the ways to achieve this is by modulating one of the parameter of the problem. The modulating centrifugal force has applications in many fields of engineering like chemical, industry, food industry, solidification of alloys and rotating turbo machinery. Bhattacharjee [2, 3] was the first to study the effect of rotational modulation in a Rayleigh-Bénard convection and found that it stabilizes the system. Later Om *et al.* [11, 12] investigated the rotational modulation in porous medium. The literature pertaining to rotational modulation is scarce.

Onset of Rayleigh-Bénard convection in liquids with suspended particles (like couple stress(Boussinesq-Stokes suspensions)/ micropolar liquids) is now well studied in the absence/presence of internal/external constraints like porous media, rotation, magnetic field, modulations of temperature/ gravity/rotation/ heat source [4, 8, 10, 14, 18, 19]. Of relevance to the present work the result that unmodulated rotation delays the onset of convection culled out from the linear stability analysis reported in the aforementioned works. To the best of authors' knowledge, there is no study available on the rotational modulation in couple stress liquid. The theoretical study of linear analysis of rotational modulation on Rayleigh-Bénard convection in couple stress liquid is the main objective of the paper. We intend to provide a fundamental understanding of how modulation of rotation would influence the thermal convection. With these objectives, we now move on to the formulation of the problem.

2. Mathematical Formulations

Consider a layer of Boussinesq, couple stress liquid confined between two parallel plates separated by a distance d apart and rotating with time dependent angular velocity. Cartesian system is taken with origin in the lower boundary and z-axis vertically upward and the rotation axis coincides with the vertical coordinate axis (Figure 1).

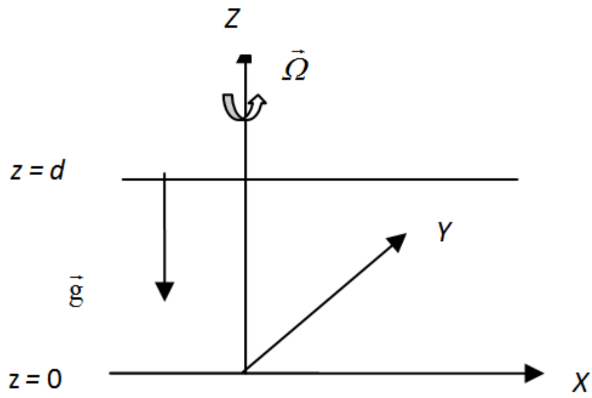

Figure 1: Physical configuration

The basic equations governing the fluid motion are:

Continuity Equation: $\nabla.\vec{q} = 0$ (1)

Conservation of Linear Momentum:

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q} + 2\vec{\Omega}\times\vec{q}\right] = -\nabla P_{ro} + \rho\vec{g} + \mu\nabla^2\vec{q} - \mu^{'}\nabla^4\vec{q} \tag{2}$$

Conservation of Energy: $\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi\nabla^2 T,$ (3)

Equation of State: $\rho = \rho_0(1 - \alpha_t(T - T_0)),$ (4)

We consider a sinusoidal time-dependent rotational speed modulation, so that

$$\vec{\Omega}(t) = \Omega_0(1 + \epsilon\cos\gamma t)\hat{k}. \tag{5}$$

where, $\vec{q}$ is the velocity, ρ_0 is the density at reference temperature, t is the time, $P_{ro} = p - \frac{1}{2|\vec{\Omega}\times r|^2}$, ρ is the density of the fluid, g is the gravitational force, μ is the viscosity of the fluid, μ' is the couple stress viscosity, T is the temperature, χ is the thermal conductivity, α_t is the thermal expansion, T_0 is the reference temperature, $\vec{\Omega}(t)$ is the angular velocity of rotation, ϵ is the amplitude of modulation and γ is the frequency of modulation. In quiescent basic state, system has a solution in the form:

$$\vec{q}_b = \vec{0}, \rho = \rho_b(z), P_{ro} = P_{ro_b}(z), T = T_b(z), \vec{\Omega} = \Omega_0\hat{k}. \tag{6}$$

Substituting equation (6) into basic governing equations (1)-(4), we obtain the quiescent state solutions as:

$$-\frac{dP_{ro_b}}{dz} - \rho_b g = 0, \frac{d^2T_b}{dz^2} = 0, \rho_b = \rho_0(1 - \alpha_t(T_b - T_0)). \tag{7}$$

Basic state is perturbed by introducing the following perturbations

$$\vec{q} = \vec{q}_b + \vec{q}', \rho = \rho_b + \rho', P_{ro} = P_{ro_b} + P'_{ro}, T = T_b + T' \tag{8}$$

where the prime indicates the finite amplitude perturbations. Substituting equation (8) into equations (1) - (4), using the basic state equations, neglecting non-linear terms, eliminating pressure, introducing the stream functions in the form $u' = \frac{\partial\psi}{\partial z}$ and $w' = -\frac{\partial\psi}{\partial x}$ and non-dimensionalizing by taking characteristic length as d, characteristic velocity as $\frac{d}{\chi}$, characteristic time as $\frac{d^2}{\chi}$, characteristic temperature as ΔT and characteristic angular velocity as Ω_0, we get following linear dimensionless equations:

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C\nabla^4\right)\nabla^2\psi = T_a^{1/2}(1 + \epsilon\cos\omega t)\frac{\partial v}{\partial z} - R\frac{\partial T}{\partial x}, \tag{9}$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C\nabla^4\right)v = -T_a^{1/2}(1 + \epsilon\cos\omega t)\frac{\partial\psi}{\partial z}, \tag{10}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = -\frac{\partial\psi}{\partial x}, \tag{11}$$

where, $Pr = \frac{\mu}{\rho_0\chi}$ (Prandtl number),$C = \frac{\mu'}{d^2\mu}$ (Couple stress parameter), $T_a = \left(\frac{2\rho_0\Omega_0 d^2}{\mu}\right)^2$ (Taylor number), $R = \frac{\rho_0\alpha_t g\Delta T d^3}{\mu\chi}$ (Rayleigh number) and

$\omega = \frac{\gamma d^2}{\chi}$ (Dimensionless frequency of modulation).

Equations (9)-(11) are solved for stress free isothermal, vanishing couple stress boundaries and hence we have at z = 0,1

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^4 \psi}{\partial z^4} = T = 0 \quad at \quad z = 0, 1 \tag{12}$$

Eliminating ψ and v from equations (9) - (11) we get,

(13)

where, $L_1 = \frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C\nabla^4$, $L_2 = \frac{\partial}{\partial t} - \nabla^2$, $D_x^2 = \frac{\partial^2}{\partial x^2}$, $D_z^2 = \frac{\partial^2}{\partial z^2}$.

In dimensionless form, the temperature boundary conditions for solving equation (13) are obtained from equations (9) - (11) in the form:

(14)

3. Perturbation Procedure

The eigen-function T and eigen-value R of the equation (13) are expanded in powers of ϵ. Thus, the eigenvalues of the present problem differ from those of the ordinary Bénard convection by quantities of order ϵ. We seek the solution of equation (13) in the form:

$$T = T_0 + \epsilon T_1 + \epsilon^2 T_2 + ..., R = R_0 + \epsilon^2 R_2 + ... \tag{15}$$

where R_0 is the Rayleigh number in unmodulated case while $R_2 \ldots R_4$ are the corrections in critical Rayleigh number due to modulation. In equation (15) the odd powers of R are not considered, because changing the sign of ϵ shifts the time origin only, which does not affect the problem of stability and thus R is independent of sign of ϵ therefore all the odd powers of R must be zero [4]. The expansion (15) is substituted into equation (13) and the coefficients of various powers of ϵ are equated on either side of the equation to obtain the following system of equations

$$LT_0 = 0, \tag{16}$$

$$LT_1 = \left(fL_1^2\nabla^2 - \frac{f'}{Pr}L_1L_2\nabla^2 + 3T_a fL_2D_z^2 - R_0 fL_1D_x^2 + \frac{R_0 f'}{Pr}D_x^2 \right) T_0, \tag{17}$$

$$LT_2 = \left(3T_a f^2 L_2 D_z^2 - R_0 f L_1 D_x^2\right) T_0 +$$

$$\left(f L_1^2 \nabla^2 - \frac{f'}{Pr} L_1 L_2 \nabla^2 + 3T a f L_2 D_z^2 - R_0 f L_1 D_x^2 + \frac{R_0 f'}{Pr} D_x^2\right) T_1, \tag{18}$$

where

$$L = R_0 L_1 D_x^2 - L_1^2 L_2 \nabla^2 - T_a L_2 D_z^2 \tag{19}$$

3.1 *Solution to the Zeroth Order Problem*

The zeroth order problem is equivalent to the Rayleigh-Bénard problem of couple stress fluid with rotation in the absence of rotational modulation is investigated by introducing vertical temperature perturbation T_0 corresponding to lowest mode (n=1) of convection as

$$T_0 = \cos(\pi \alpha x) \sin(\pi z) \tag{20}$$

Substituting equation (20) into equation (16) we obtain the expression for Rayleigh number R_0 as,

$$R_0 = \frac{\left(\kappa_\alpha^2\right)^3 \eta_\alpha}{\pi^2 \alpha^2} + \frac{T_a}{\alpha^2 \eta_\alpha} \tag{21}$$

In the absence of rotation i. e., $T_a = 0$ and setting $\pi^2\alpha^2$ as a^2 we get $R_0 = \frac{(\kappa_1^2)^3 \eta}{a^2}$ where, $K_1^2 = \pi^2 + a^2$, $\eta = 1 + Ck_1^2$, which is the expression for Rayleigh number discussed by Siddheshwar and Pranesh [15].

3.2 *Solution to the First Order Problem*

Using equation (20) in equation (17), we get

$$LT_1 = Re\left\{\left[\left(\kappa_\alpha^2 \eta_\alpha + \frac{\iota\omega}{Pr}\right) P_1 - P_2\right] e^{-\iota\omega t}\right\} T_0, \tag{22}$$

where, $P_1 = R_0 \pi^2 \alpha^2 - (\kappa_\alpha^2)^3 \eta_\alpha$ and $P_2 = 3T_a \pi^2 \kappa_\alpha^2$.

To solve equation (22), we expand the right-hand side using Fourier series expansion and obtain T_1 by inverting the operator L term by term as:

$$LT_1 = Re\left\{\frac{1}{L(\omega)}\left[\left(\kappa_\alpha^2 \eta_\alpha + \frac{\iota\omega}{Pr}\right) P_1 - P_2\right] e^{-\iota\omega t} \sin(\pi z)\right\}, \tag{23}$$

where, $L(\omega) = B_1 - \iota\omega B_2$,
$B_1 = -\omega^2\left(\frac{2(\kappa_\alpha^2)^2\eta_\alpha}{Pr} + \left(\frac{\kappa_\alpha^2}{Pr}\right)\right)$ and $B_2 = \frac{(\kappa_\alpha^2)^3\eta_\alpha}{Pr} + (\kappa_\alpha^2)^3(\eta_\alpha)^2 - \frac{\omega^2\kappa_\alpha^2}{Pr^2} + T_a\pi^2 - \frac{T_a\pi^2}{\eta_\alpha Pr}$.

We shall not solve equation (18), but will use this to determine R_2. The solvability condition requires that the time-independent part of the right hand side of equation (18) must be orthogonal to $\sin(\pi z)$, and this results in the following equation,

$$R_2 = \frac{R_0}{\kappa_\alpha^2\eta_\alpha}\left(\frac{I_1}{Pr} - I_2\right) + \frac{1}{\eta_\alpha a^2}\left(\frac{I_3 - I_4}{Pr}\right) + \frac{3T_a\pi^2}{\kappa_\alpha^2\eta_\alpha a^2}\left(\kappa_\alpha^2 I_5 + I_6\right) \tag{24}$$

where, $I_1 = \frac{-\omega^2}{2|L(\omega)|^2}\left[\frac{B_1P_1}{Pr} + B_2\left(\kappa_\alpha^2\eta_\alpha P_1 - P_2\right)\right]\sin(\pi z)$, $I_2 = \frac{1}{2|L(\omega)|^2}\left[B_1P_1\left((\kappa_\alpha^2\eta_\alpha) + (\frac{\omega}{Pr})^2\right) - \frac{\omega^2B_2P_2}{Pr} - \kappa_\alpha^2\eta_\alpha B_1P_2\right]\sin(\pi z)$,

$$I_3 = \frac{1}{2|L(\omega)|^2}\left[B_1(X_1X_3 + \omega^2X_2X_4) - \omega^2B_2(X_1X_4 - X_2X_3)\right]\sin(\pi z),$$

$$I_4 = \frac{\omega^2}{2|L(\omega)|^2}\left[B_1(X_3 - \kappa_\alpha^2X_4) - B_2(\omega^2X_4 + \kappa_\alpha^2X_3)\right]\sin(\pi z),$$

$$I_5 = \frac{1}{2},\ I_6 = \frac{1}{2|L(\omega)|^2}\left[B_1X_5 - \omega^2X_6B_2\right]\sin(\pi z),$$

$X_1 = (\kappa_\alpha^2)^2\eta_\alpha - \frac{\omega^2}{Pr}$, $X_2 = \kappa_\alpha^2\eta_\alpha + \frac{\kappa_\alpha^2}{Pr}$, $X_3 = \left[(\kappa_\alpha^2\eta_\alpha)^2 + \frac{\omega^2}{Pr^2}\right]P_1 - \kappa_\alpha^2\eta_\alpha P_2$, $X_4 = \frac{P_2}{Pr}$, $X_5 = \left[(\kappa_\alpha^2)^2\eta_\alpha + \frac{\omega^2}{Pr}\right]P_1 - \kappa_\alpha^2P_2$, $X_6 = \left[\frac{\kappa_\alpha^2}{Pr} - \kappa_\alpha^2\eta_\alpha\right]P_1 + P_2$.

4. Minimum Rayleigh Number for Convection

The value of Rayleigh number R obtained by this procedure is the eigenvalue corresponding to the eigen function T, which, though oscillating, remains bounded in time. Since R is a function of the horizontal wave number $a^2 = \pi^2\alpha^2$ and the amplitude of modulation ε, we have

$$R(a,\epsilon) = R_0(a) + \epsilon^2R_2(a) + \cdots, \tag{25}$$

It was shown by Venezian that the critical value of thermal Rayleigh number is computed upto ϵ^2, by evaluating R_0 and R_2 at $a = a_0$ [21]. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c}) one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by equation (25) is minimum.

5. Results and Discussions

The objective of this paper is to study the effects of the time periodic rotational field in couple stress fluid using linear analysis. The regular perturbation is used to compute the critical Rayleigh number R_{0c}, critical wave number a_c^2 and correction Rayleigh number R_2c. The expression for correction Rayleigh number is obtained as a function of modulating frequency and Taylor number. The linear analysis presented is based on the assumption that the amplitude of rotational modulation is small. The validity of the results obtained here depends on the value of the modulating frequency ω. Moderate values of the modulating frequency ω is considered because when ω is small, the modulation is large. As $\omega \to \infty$, the modulation effect becomes zero.

Figure 2 is the plot of R_{2c} versus ω for different values of couple stress parameter C. From the figure we observe that as C increases R_{2c} also increases and remains positive for all values of ω. The presence of suspended particles increases the viscosity of the fluid and hence makes the system stable even in the presence of modulation, but only when ω is greater than a certain value say, ω_c . The system becomes unstable for large values of ω. However, for $\omega > \omega_c$ the effect of modulation vanishes as noted by Venezian [21]. At this point we note that the plot of R_c reveals more on the stability or instability of the system.

Figure 3 is the plot of R_{2c} versus ω for different values of Taylor number T_a. In the figure we observe that as T_a increases R_{2c} also increases and remains positive for all values of ω, indicating that the effect of rotational modulation is to stabilize the flow. The stabilizing effect of rotation is due to the fact that rotation introduces an additional velocity component in fluid motion. Thus the onset of convection is delayed. It is observed that in the case when modulation is present, the critical Rayleigh number for onset of convection is larger than the one predicted by the onset of convection without modulation, i. e., the critical Rayleigh number in the case of rotational modulation is greater than that in the case of uniform rotation. It is also observed from the figure that the effect of modulation persists for larger values of modulation frequency only, when the value of Taylor number is large.

Figure 4 is the plot of R_{2c} versus ω for different values of Prandtl number Pr. In the figure we observe that as Pr increases R_{2c} also increases indicating that Prandtl number has a stabilizing effect on the system. It is appropriate to note that Pr does not affect R_0.

From the analysis and by taking $R_c = R_{0c} + \epsilon^2 R_{2c}$ the following results are obtained:

$(i) R_c(C = 0) < R_c(C \neq 0)$,

$(ii) R_c(Ta = 0) < R_c(Ta \neq 0)$,

$(iii) R_c(Pr = 10) < R_c(Pr = 15)$, $(iv) R_c(\epsilon = 0) > R_c(\epsilon \neq 0)$.

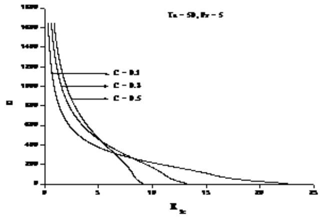

Figure 2: Plot of R_{2c} versus ω for different values of Couple Stress Parameter C

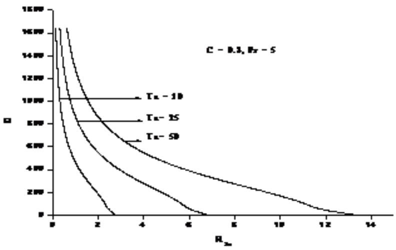

Figure 3: Plot of R_{2c} versus ω for different values of Taylor Number Ta

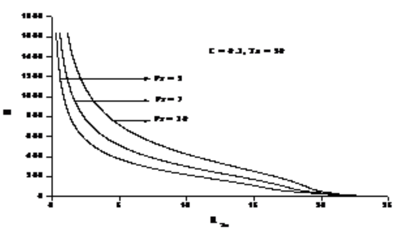

Figure 4: Plot of R_{2c} versus ω for different values of Prandtl number Pr

6. Conclusion

The following conclusions are drawn from the present study:

1. By modulating the rotation it is possible to advance or delay the onset of convection.
2. The couple stress parameter represents the presence of suspended particle in the fluid and it stabilizes the system.
3. Rotational modulation leads to stabilizing the system.

4. The couple stress parameter, Taylor number and amplitude of modulation in the case of rotational modulation ensure that the system is stable leading to a situation wherein rheometric measurements are possible.

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