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Characteristic Study of Combined Effects of MHD and Coriolis Force on Free Convection in a Rectangular Cavity with Isotropic and Anisotropic Porous Media

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Abstract

This investigation deals with the classic Rayleigh-Bénard problem for an laminar, viscous, unsteady incompressible fluid flow heated from below is extended to three-dimensional convection in a finite geometry with isotropic and anisotropic porous media rotating with constant angular velocity and the magnetic field is applied in the vertical direction. For the given physical set-up, governing partial differential equations are transformed to a set of non-dimensional ordinary differential equations using similarity transformation. This demands to apply Fourier series method to study the characteristic of velocity, temperature and concentration for the effect of Taylors number, Rayleigh number, Hartmann's number and Prandtl number for both anisotropic and isotropic porous media. The results of steam function and isotherms on various parameters have been discussed and found to be good agreement for the physical system.

Keywords: Rayleigh-Bénard convection, Isotropic and anisotropic porous media, Free convection, Coriolis force, MHD
Mathematics Subject Classification (2010): 35Q30, 76E25, 76W05

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1. Introduction

Applications of natural convection in rectangular box with nonuniformly heated from the bottom under magnetic field in solar collectors, temperature controlling of electronic devices and in nuclear equipment's, in recent years has received considerable attention. Such flow system is also use in the modeling of environmental and geophysical phenomena. Churchill and Wilkes [1], Newell and Schmidt [2] and Patterson and Imberger [3], have studied numerically the natural convective flow and heat transfer characteristics in a rectangular enclosure with uniform heat flux from the side walls. An exhaustive review of the work on natural convection in a rectangular box has been comprehended by Ostrach [4], Catton [5], Bejan [6] and Yang [7]. These studies are on the natural convection of electrically non-conducting fluids, but very little has been done on the natural convection of electrically conducting fluids in rectangular box in the presence of a magnetic field in spite of its various applications in the areas of nuclear engineering (nuclear and fusion reactors) and crystal growth. When an electrically conducting fluid is subjected.

The flow takes place in the presence of an applied uniform magnetic field, which results in magneto hydrodynamic interaction in the dimension of the flow, due to this interaction the flow becomes highly anisotropic, in the direction of the magnetic field. This results in a strong damping of 3-D velocity disturbances, having only those which normal to the magnetic field. MHD flow in three-dimensional as the presence of electrically conducting walls, slight misalignment of the field with the parallel walls, development of the velocity profile, etc. Result in significant three-dimensional effects, especially if the so-called parallel layers are involved. Examples of such flows include buoyant convection in a cavity in a vertical and transverse magnetic fields as explained by Gelfgat [8], Hurle [9] and Aleksandrova [10], Henry [11], Ben [12] and Raju [13]. Under the influence of the magnetic field, an electric current is induced by the fluid motion. Normally, the interaction between electric current and the motion reduces the fluid velocity, Chandrasekhar [14]. Free convection of an electrically conducting fluid is studied by Raptis and Vlahos [15] and Ozoe and Okada [16]. Natural convection through vertical plates of conducting fluid under the effect of magnetic field is studied by Emery [17]. Free convection in 2D and 3D enclosures under magnetic field, when the side walls are non-uniformly heated is studied numerically by Ozoe and Okada [16] and Suresh Babu [18]. These studies investigate with isothermal walls (side).

With the application mentioned above and its importance in various fields, to our knowledge much work has not given for the study of characterstic of velocity, temperature and concentration with combined effect of MHD and corolies force in a finite geometry filled with isotropic and anisotropic porous media is the main objective of the present work. In this study we have considered 3- dimensional thermal convection of an incompressible fluid in a horizontal rectangular channel which is rotating with an angular velocity ω in a vertical Z−direction. The walls of the channel are considered to be non-porous and heat conducting. The rectangular channel is heated differentially to establish non uniform temperature gradient in the vertical Z- direction. The channel is filled with incompressible fluid saturated by anisotropic media. The channel is rectangular with height 'h' and width'a' and we choose a cartesian coordinate system with $Zaxis$ in the vertical direction and X–$axis$ in the horizontal direction perpendicular to the channel axis. The horizontal channel walls are at $z = 0$ and $z = h$. The vertical walls are at $x = \pm\frac{a}{2}$. Magnetic field is applied in the transverse direction.

Nomenclature

Latin symbols

a	width of the rectangular channel
g	Acceleration due to gravity ($g = 9.8ms^{-2}$)
H	Height of the rectangular channel
p_1	Pressure (Pa)
$\vec{q}$	Velocity components of u,v,w (ms^{-1})
$\vec{B}$	Magnetic field
R_a	Thermal Rayleigh Number
R_c	Solutal Rayleigh Number
S	Concentration
S_0	Reference Concentration
ΔS	Characteristic Concentration difference
P_1	Pressure
c	Specific hear at constant pressure
s	Deviation from the static concentration
T	Temperature(K)
T_0	Reference temperature
t	Time (s)
ΔT	Characteristic temperature difference
T_a	Taylors Number

Greek symbols

κ	Thermal diffusivity in isotropic case
k	Permeability in isotropic case
ν	Thermal viscosity
ξ	Anisotropic ratio
η	Aspect ratio
Ω	Uniform angular velocity of the system
β	Thermal expansion coefficient(K^{-1})
χ	Constant thermal diffusivity
ρ	Density (kgm^{-3})
ρ_0	Reference density
ψ	Stream function
σ	Deviation from the static concentration
θ	Deviation from static temperature

Other symbols

x, y, z	Cartesian coordinates
$\widehat{i}$	Unit vector normal in x - direction
$\widehat{k}$	Unit vector normal in z - direction
∇	Three dimensional gradient operator
∇^2	Three dimensional Laplacian operator

2. Mathematical Formulation

A three - dimensional free convection in a rectangular porous box, non-uniformly heated from down is considered. The porous media is considered to be saturated and an-isotropic by an incompressible homogeneous fluid. The rectangular box is of width a and height $'h'$, we choose vertical direction of the box as Z axis, the horizontal walls of the box are at $z = 0$ and $z = h$ and the horizontal direction along the length of the box as X axis, vertical walls are at $X = -a/2$ and $X = a/2$ in Figure 1. For the study, the Prandtl-Darcy number is assumed to be big so the inertia terms can be ignored and appeal to Boussinesq approximation, the three dimension model of the Darcy-Boussinesq equations takes the form

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} = 0, \tag{1}$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\upsilon}{k_x}u - 2\Omega v + \frac{u}{\rho_0}\sigma B_0^2 = 0, \tag{2}$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\upsilon}{k_y}v + 2\Omega u + \frac{v}{\rho_0}\sigma B_0^2 = 0, \tag{3}$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\rho}{\rho_0}g + \frac{\upsilon}{k_z}w = 0, \tag{4}$$

$$c\frac{\partial T}{\partial t} + v.\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \kappa_x\frac{\partial^2 T}{\partial x^2} + \kappa_z\frac{\partial^2 T}{\partial z^2}, \tag{5}$$

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + w\frac{\partial S}{\partial z} = \sigma_x\frac{\partial^2 S}{\partial x^2} + \sigma_z\frac{\partial^2 S}{\partial z^2}, \tag{6}$$

$$\rho = \rho_0[1 - \beta(T - T_0) + \alpha(S - S_0), \tag{7}$$

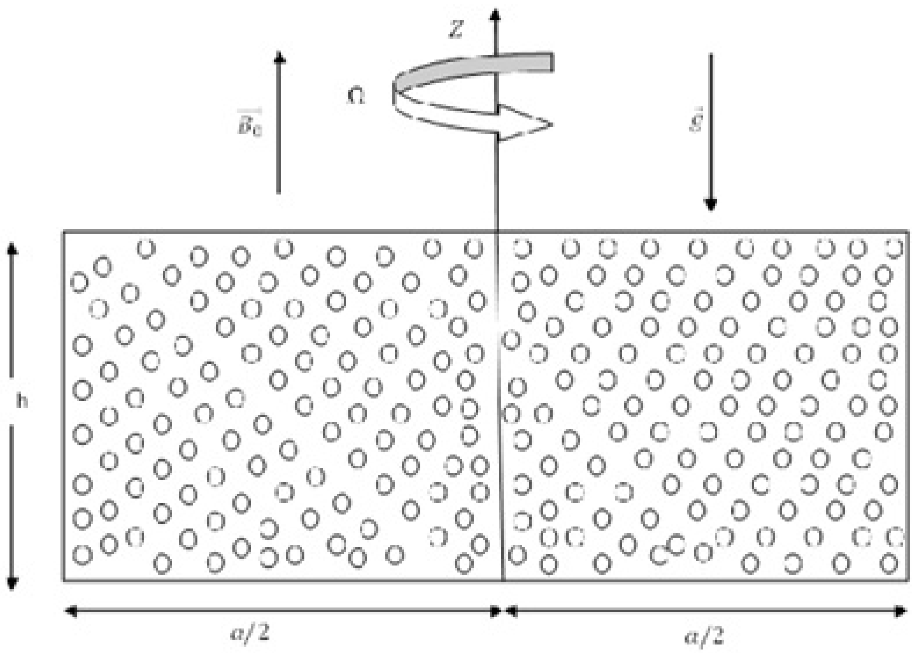

Figure 1: Physical configuration.

The lower and the upper walls of the box are at isothermal temperatures T_0 and $T_0+\Delta T$, here ΔT is the absolute temperature. All the walls of the box are considered to impermeable and heat conducting. From the governing equations it follows that a static conduction exists only if the static temperature distribution is independent of x and linearly dependent on z.

$$T = \left[T_0 + \Delta T\left(1 - \frac{z}{h}\right)\right] + \theta$$

$$S = \left[S_0 + \Delta S\left(1 - \frac{z}{h}\right)\right] + s \tag{8}$$

Where θ and s are the deviations from the static temperature and concentration respectively. Since the flow is axis symmetric, we introduce the stream function by $\psi = \psi(x, y)$

$$u = \frac{\partial\psi}{\partial z}, \qquad w = -\frac{\partial\psi}{\partial x} \tag{9}$$

Non-dimensional variables are introduced by asterisks (*)

$$u = \frac{\kappa_x a u^*}{h^2},\ v = \frac{\kappa_y a v^*}{h^2},\ w = \frac{\kappa_z w^*}{h},\ t = \frac{c h^2 t^*}{\kappa_z};$$

$$x = ax^*,\ y = ay^*, z = hz^*,$$

$$\psi = \frac{\kappa_z a\psi^*}{h},\ \ \theta = \Delta T\theta^*,\ \ T_0 = \Delta T T_0^*,\ \ p = \frac{v k_z \rho_0 p^*}{k_z},\ S = \Delta S\, S^* \tag{10}$$

On introduction of above expressions into equations (1)-(7), the governing equation takes the form:

$$\left(\xi\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\psi + \xi R_a\frac{\partial\theta}{\partial x} - \xi R_S\frac{\partial s}{\partial x} - T_a\frac{\partial v}{\partial z} = 0 \tag{11}$$

$$\chi\frac{\partial v}{\partial z} + T_a\frac{\partial^2\psi}{\partial z^2} = 0 \tag{12}$$

$$P_c\left(\zeta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)s - \frac{\partial\psi}{\partial t} = \frac{\partial s}{\partial t} \tag{13}$$

$$\left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\theta - \frac{\partial\psi}{\partial x} = \frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z} \tag{14}$$

where Ra is the Darcy-Rayleigh number and Rs is the Darcy Solutal Rayleigh number and defined by

$$R_a = \frac{\beta g \Delta T k_z h}{\kappa_z \upsilon}, \qquad R_S = \frac{\alpha g \Delta S k_z h}{\kappa_z \upsilon}. \tag{15}$$

Where ξ, η, ζ and χ are the anisotropy aspect ratio of permeability and diffusivities of temperature of the fluid.

$$\xi = \frac{k_x}{k_z}\left(\frac{h}{a}\right)^2, \ \eta = \frac{\kappa_x}{\kappa_z}\left(\frac{h}{a}\right)^2, \ \zeta = \left(\frac{h}{a}\right)^2$$

$$\chi_c = \frac{k_x}{k_y}\left(1 + \frac{k_y}{\nu}\frac{B_0}{\rho_0}\right), \ T_c = \frac{k_x}{\nu}\left(2\Omega - \frac{B_0}{\rho_0}\right) \tag{16}$$

The required boundary conditions for completely thermal conducting and impermeable boundaries give

$$S = \psi = \theta = \frac{\partial v}{\partial z} = 0 \text{ on } \begin{cases} x = -\frac{1}{2}, \ x = \frac{1}{2} & 0 < z < 1 \\ z = 0, \ \ z = 1 & -\frac{1}{2} < x < \frac{1}{2} \end{cases} \tag{17}$$

3. Linear Stability and Steady Flow Patterns

Natural convection, mentioned by linear versions of the equation (11) to (14). The solution of these equations can be expended in Fourier series as

$$\psi = e^{\sigma t}\left[\frac{C_0}{2} + \sum_{n=1}^{\infty} C_n(x)\, \cos n\pi z + D_n(x)\, \sin n\pi z\right] \tag{18}$$

$$\psi = e^{\sigma t}\left[\frac{C_0}{2} + \sum_{n=1}^{\infty} C_n(x)\, \cos n\pi z + D_n(x)\, \sin n\pi z\right] \tag{19}$$

$$v = e^{\sigma t}\left[\frac{A_0}{2} + \sum_{n=1}^{\infty} A_n(x)\, \cos n\pi z + B_n(x)\, \sin n\pi z\right] \tag{20}$$

$$S = e^{\sigma t}\left[\frac{S_0}{2} + \sum_{n=1}^{\infty} S_n(x)\, \cos n\pi z + H_n(x)\, \sin n\pi z\right] \tag{21}$$

where C_n, D_n, F_n, G_n, A_n, B_n, S_n and H_n are the function of only and is the growth rate. The boundary conditions (17) are satisfied if for all.

On substituting the equation (18) - (21) to the linearized governing equations, we get the subsequent set of differential equations:

$$\left(\xi\frac{d^2}{dx^2} - n^2\pi^2\right)D_n + \xi R_a\frac{dG_n}{dx} - \xi R_S\frac{dH_n}{dx} + T_c n\pi A_n = 0 \quad (22)$$

$$\chi A_n + T_a n\pi D_n = 0 \quad (23)$$

$$P_c\left(\zeta\frac{d^2}{dx^2} - n^2\pi^2\right)H_n - \frac{dD_n}{dx} = \sigma H_n \quad (24)$$

$$\left(\eta\frac{d^2}{dx^2} - n^2\pi^2\right)G_n - \frac{dD_n}{dx} = \sigma G_n \quad (25)$$

and the boundary conditions for D_n, G_n, A_n and Hn as below

$$D_n\left(\frac{1}{2}\right) = D_n\left(-\frac{1}{2}\right) = 0, \qquad A_n\left(\frac{1}{2}\right) = A_n\left(-\frac{1}{2}\right) = 0,$$

$$G_n\left(\frac{1}{2}\right) = G_n\left(-\frac{1}{2}\right) = 0, \qquad H_n\left(\frac{1}{2}\right) = H_n\left(-\frac{1}{2}\right) = 0. \quad (26)$$

We can conclude from equation (11) - (14) and from boundary condition (2.17) that σ to be real. Thus, to find critical Rayleigh number R_{a_c} which is a function of (ξ, η, ζ and χ), for the marginal stability we can substitute in the equations (24) and (25). The set of equations (22) to (25) together with the boundary conditions (26) gives a self-adjoint eigenvalue problem with R_a as the eigenvalue, R_{a_c} is the smallest eigenvalue. The general solution is given by

$$D_n(x, R_a) = [C_1\cos px + C_2\sin px + C_3\cos qx + C_4\sin qx] \quad (27)$$

$$G_n(x, R_a) = s\,[rC_1\sin px - rC_2\cos px + C_3\sin qx - C_4\cos qx] \quad (28)$$

$$H_n(x, R_a) = t\,[rC_1\sin px - rC_2\cos px + C_3\sin qx - C_4\cos qx] \quad (29)$$

$$A_n = \frac{-n\pi T_a}{\xi}\left[C_1 \cos px + C_2 \sin px + C_3 \cos qx + C_4 \sin qx\right] \tag{30}$$

where C_1, C_2, C_3 and C_4 are arbitrary constants and

$$p_q = \frac{1}{2\sqrt{\xi}}\left\{\left[\sqrt{\left(R_a - \frac{R_s}{P_c}\right) - n^2\pi^2\left(2 + \frac{T_aT_c}{\xi}\right) + 2n^2\pi^2\sqrt{1 + \frac{T_aT_c}{\xi}}}\right] \pm\left[\sqrt{\left(R_a - \frac{R_s}{P_c}\right) - n^2\pi^2\left(2 + \frac{T_aT_c}{\xi}\right) - 2n^2\pi^2\sqrt{1 + \frac{T_aT_c}{\xi}}}\right]\right\} \tag{31}$$

where,

$$r = \frac{q\left(\xi p^2 + n^2\pi^2\left(1 + \frac{T_aT_c}{\xi}\right)\right)}{p\left(\xi q^2 + n^2\pi^2\left(1 + \frac{T_aT_c}{\xi}\right)\right)},$$

$$s = \frac{\xi q^2 + n^2\pi^2\left(1 + \frac{T_aT_c}{\xi}\right)}{\xi\, q\left(R_a - \frac{R_s}{P_c}\right)},$$

$$t = \frac{\xi q^2 + n^2\pi^2\left(1 + \frac{T_aT_c}{\xi}\right)}{\xi\, q\,(R_aP_c - R_s)}. \tag{32}$$

Here, $p \neq q$ is assured by the boundary conditions at $R_a \neq R_{a_c}$ from (26) boundary condition we get the non-trivial solution of the given problem when

$$I.(1-r)\sin\left(\frac{p+q}{2}\right) - (1+r)\sin\left(\frac{p-q}{2}\right) = 0,\ and\ C_2 = C_4 = 0 \tag{33}$$

$$II.(1-r)\sin\left(\frac{p+q}{2}\right) + (1+r)\sin\left(\frac{p-q}{2}\right) = 0\ and\ C_2 = C_4 = 0 \tag{34}$$

In the case of $\xi = \eta = \zeta = \chi$, which is fulfilled for an isotropic medium, it is solved analytically and is found. When $\xi \neq \eta \neq \zeta \neq \chi$, that is, anisotropic case is evaluated numerically.

3.1 ***The isotropic case:***

In this case the condition $\xi = \eta = \zeta = \chi$ is satisfied if

$$\frac{k_x}{k_z} = \frac{\kappa_x}{\kappa_z}$$

i. e., the proportion of the parallel and perpendicular component of the permeability and thermal diffusivity are equal. The following condition for case I and II are obtained at $r = 1$

$$p - q = 2m\pi, \ for\, m = 1, 2, 3, \ldots \tag{35}$$

This gives

$$R_a = 4\pi^2 \left(\xi m^2 + \tfrac{n^2}{2}\left(1 + \tfrac{T_a T_c}{2\xi} + \sqrt{1 + \tfrac{T_a T_c}{\xi}}\right)\right) + \tfrac{R_s}{P_c} \tag{36}$$

where$n = 1, 2, 3, ...and m = 1, 2, 3, ...$

Equation (36) gives the eigenvalues and the smallest of it is the critical Rayleigh number

$$Ra_c = 4\pi^2 \left(\xi + \frac{1}{2}\left(1 + \frac{T_a T_c}{2\xi} + \sqrt{1 + \frac{T_a T_c}{\xi}}\right)\right) + \frac{R_s}{P_c} \tag{37}$$

The smallest eigenvalue corresponds to and for an isotropic medium it can be written as

$$Ra_c = 4\pi^2 \left[\left(\frac{h}{a}\right)^2 + \frac{1}{2}\left(1 + \left(\frac{a}{h}\right)^2 \frac{T_a T_c}{2} + \sqrt{1 + \left(\frac{a}{h}\right)^2 T_a T_c}\right)\right] + \frac{R_S}{P_c} \tag{38}$$

As the limit $(\frac{h}{a}) \rightarrow 0$ and the channel tends to infinitesimal horizontal porous layer. In such case critical Rayleigh number $R_{a_c} = 4\pi^2$ it is in line with a well-known conclusion for the permeable layers Boris [19]. The critical value from the equation (38) is not same as the conclusion found for a channel with absolutely insulating walls done by Sutton [20]. In case $h = a$, i. e., we get a square box, equation (38) gives $R_{a_c} = 8\pi^2$ whereas the result corresponding to perfectly insulating lateral walls gives. Since, in this case the heat transfer over the walls. A greater critical value is expected with conducting lateral wall box.

The flow for moderately super Critical Rayleigh number is defined by the flow at the onset of natural convection. Since the equations (33) and (34) coincides when$\xi = \eta = \zeta = \chi$, i. e., when $r = 1$, the boundary value problem gives two linearly independent solutions. It can also seen from the stationary linearized form of the equation (11) and (14).

If ψ_0 , θ_0 , S_0 *and* v_0 are the solutions at $R_a = Ra_c$, then $\psi_1 = -\xi R_a\theta_0,\ \theta_1 = \psi_0$ and $v_1 = v_0$ are linearly independent solutions. The two set of solutions are given by,

$$\psi^{(1)} = Q\cos Kx\ \sin\pi x\ \sin\pi z$$

$$\theta^{(1)} = Q\, s\sin Kx\ \sin\pi x\ \sin\pi z$$

$$v^{(1)} = \frac{-n\pi T_a}{\xi} Q\cos Kx\ \sin\pi x\cos\pi z$$

$$s^{(1)} = -Q\sin Kx\ \sin\pi x\ \sin\pi z \tag{39}$$

$$\psi^{(2)} = \frac{S}{s}\cos Kx\ \sin\pi x\ \sin\pi z$$

$$\theta^{(2)} = S\ \sin Kx\ \cos\pi x\ \sin\pi z$$

$$v^{(2)} = -\frac{St}{s}\sin Kx\ \cos\pi x\ \cos\pi z$$

$$s^{(2)} = -\frac{\pi S T_a}{s\xi}\cos Kx\ \cos\pi x\sin\pi z \tag{40}$$

Where, Q and S and are amplitude constants. The solutions (40) yields a symmetric flow pattern consisting of 2n cells, where the number of cells depends on ξ. The solution (41) yields a symmetric flow pattern consisting of $2n \pm 1$ cells.

Table 1 computed values for R_{a_c} for various values of ξ and η. The main diagonal corresponds to the isotropic case.

Table 1: Computed values for R_{a_c} for various values of ξ and η.

$\frac{\xi}{\eta}$	0.125	0.25	0.5	1	2	4	8
0.125	15656	8019	4154	2189	1183	663	392
0.25	30805	15661	8024	4159	2194	1188	668
0.5	60915	30814	15671	8034	4169	2204	1198
1	120874	60935	30834	15691	8054	4189	2223
2	240420	120913	60974	30874	15730	8093	4228
4	478986	240499	120992	61053	30953	15809	8172
8	955376	479144	240657	121150	61212	31111	15968

ξ/η	0.125	0.25	0.5	1	2	4	8
0.125	15656	8019	4154	2189	1183	663	392
0.25	30805	15661	8024	4159	2194	1188	668
0.5	60915	30814	15671	8034	4169	2204	1198
1	120874	60935	30834	15691	8054	4189	2223
2	240420	120913	60974	30874	15730	8093	4228
4	478986	240499	120992	61053	30953	15809	8172
8	955376	479144	240657	121150	61212	31111	15968

Table(1)

3.2 *The Anisotropic case*

This case deals with the condition $\xi \neq \eta \neq \zeta \neq \chi$ the non - trivial solutions for D_n, A_n, H_n and G_n when the equations (3.16) and (3.17) are fulfilled. Case I gives the solution in the form of

$$D_n(x) = \left[\sin px - \frac{\sin\frac{p}{2}}{\sin\frac{q}{2}}\sin qx\right],$$

$$G_n(x) = -s\left[r\cos px - \frac{\sin\frac{p}{2}}{\sin\frac{q}{2}}\cos qx\right],$$

$$H_n(x) = -t\left[r\cos px - \frac{\sin\frac{p}{2}}{\sin\frac{q}{2}}\cos qx\right],$$

$$A_n(x) = \frac{-n\pi T_a}{\chi}\left[\sin px - \frac{\sin\frac{p}{2}}{\sin\frac{q}{2}}\sin qx\right].$$

and for case II

$$D_n(x) = \left[\cos px - \frac{\cos\frac{p}{2}}{\cos\frac{q}{2}}\cos qx\right],$$

$$G_n(x) = -s\left[r\sin px - \frac{\cos\frac{p}{2}}{\cos\frac{q}{2}}\sin qx\right],$$

$$H_n(x) = t\left[r\sin px - \frac{\cos\frac{p}{2}}{\cos\frac{q}{2}}\sin qx\right],$$

$$A_n = \frac{-n\pi T_a}{\chi}\left[\cos px - \frac{\cos\frac{p}{2}}{\cos\frac{q}{2}}\cos qx\right].$$

Solutions (a) and (b) are defined for an infinite numbers of eigenvalues. Let R_{a_1} and R_{a_2} be the two smallest values in each of the above case. These values will exist at $n = 1$. Form equation (33) and (34) R_{a_1} and R_{a_2} are calculated for a given value of ξ, η, ζ and χ. R_{a_c}= min.$\{R_{a_1}$ and $R_{a_2}\}$. Normally $R_{a_1} and\ R_{a_2}$ are not equal, it means there exist an exclusive solution i. e., there exists a different laminar flow pattern at the convection.

4. Summary

In this study, the effect of uneven temperature gradient on the free convection in a horizontal rectangular box in three dimensions is investigated. The three dimensional problem is transformed to a two dimensional double diffusive convection problem, in which diffusing components are temperature and solute in a isotropic and anisotropic rectangular channels. The channel is considered to impermeable and heat conducting. The channel is heated non- uniformly from below and added solutes to build a linear concentration and temperature distributions in the perpendicular directions. Apart from Boussinesq approximation, which states density remains constant throughout the momentum equation except for the body force and also the following assumptions have been considered.

- Large heating at the walls implies the non-dimensional parameters Darcy-prandtl numbers are large and hence the inertial and viscous terms are neglected in the momentum equation.

- Flow is symmetric with respect to $Y - axis$ and thereby, introduced the stream function which enables to determine the critical Rayleigh number and solutal Rayleigh number based on the linear stability theory.

The critical Rayleigh number R_{a_c} obtained by solving the resulting eigenvalue problem for($\xi \neq \eta \neq \zeta \neq \chi$) in the anisotropy case, whose eigenvalue is found to be

$$Ra_c = \pi^2\left[4\eta + \left(1 + \sqrt{\frac{\eta}{\xi}\left(1 + \frac{T_a T_c}{\chi}\right)}\right)^2\right] + R_S \tag{41}$$

The critical Rayleigh number R_{a_c} for the corresponding isotropic case ($\xi = \eta = \zeta = \chi$) as a particular case of the above equation whose eigenvalue is found to be

$$Ra_c = 4\pi^2 \left[\xi + \frac{1}{2} \left(1 + \frac{T_a T_c}{2\xi} + \sqrt{\left(1 + \frac{T_a T_c}{\xi} \right)} \right) \right] + R_s \tag{42}$$

The result is in accordance with the previous result, when $T_a = 0$ it reduces to Rayleigh number found in the non-rotating case, when $T_a = 0$ and $\xi = \eta = \zeta = \chi$ (in the isotropic case), as the limit $(\frac{h}{a}) \to 0$ it reduces to the standard results $R_{a_c} = R_s + 4\pi^2$ and $R_{a_c} = 4\pi^2$ when $R_s = 0$ in the absence of the second diffusing components which is in line with the acclaimed result for the porous layers Bories [19]. Two sets of solution which are linearly independent are derived, presents a different nice steady flow patterns at moderately super critical Rayleigh number.

5. Conclusion

Governing partial differential equations are transformed to ordinary differential equations using similarity transformation. Solutions of the ordinary differential equations are found by using Fourier series method. From these solutions critical Rayleigh numbers, stream function and isotherms of the physical configuration are calculated to understand the combined effects of Magneto hydrodynamics and coriolis force on free convection in a rectangular cavity with isotropic and anisotropic porous media rotating with angular velocity. The following observations have been main in flow pattern of the streamlines and isothermal lines.

Figure (8) Represents the plotted graph of critical Rayleigh number R_{a_c} versus $\frac{\xi}{\eta}$ ratio of permeability to thermal diffusivity. The observation shows that the critical Rayleigh number Rac varies inversely with ratio $\frac{\xi}{\eta}$. The critical Rayleigh number are further increases with increasing Taylors number, Solutal Rayleigh number and the effects of rotation therefore, is to destabilize the system more significantly. Observation from Steady flow Patterns. The number of cells found to be increased with the increase in the Taylor's number for both isotropic and anisotropic cases. Figure (2) and (4). Increase in Taylor's number increases the coriolis force, which intern increases the number of rotation. Increase in rotation causes the increases the streamlines and isothermal lines. A similar pattern is also seen as T_c (modified Taylors number) is increased Figure (7).

The anisotropic ratios also greatly influence the flow patterns. The isothermal line shows the increasing oscillatory flow behaviour with rotation in anisotropy. The isothermal line becomes more flattened with the anisotropy. The present analytical study of combined effects of MHD and coriolis force present a beautiful flow patterns of stream lines and isothermal lines as shown in Figures (2) to (9).

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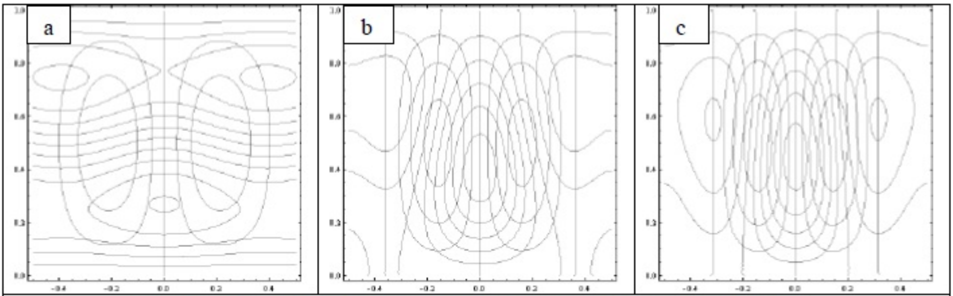

Fig. 2. Flow pattern Isothermal lines and Stream lines in isotropic case (T_a-varying)

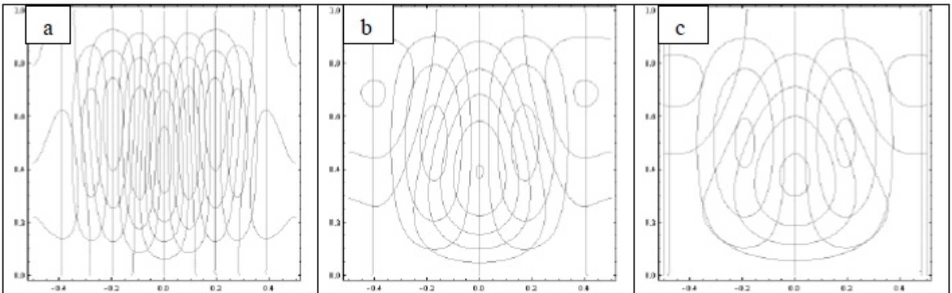

Fig. 3. Flow pattern Isothermal lines and Stream lines in isotropic case (ξ-varying)

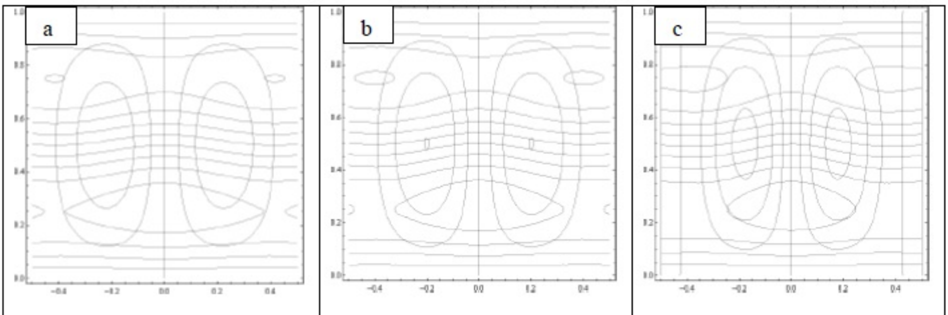

Fig. 4. Flow pattern Isothermal lines and Stream lines in anisotropic case (T_a-varying)

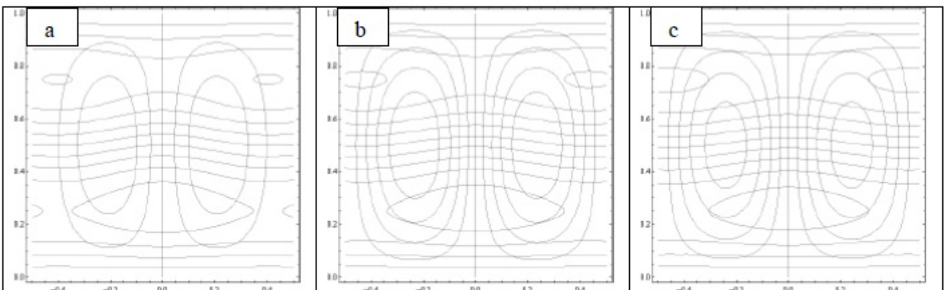

Fig. 5. Flow pattern Isothermal lines and Stream lines in anisotropic case (ξ-varying)

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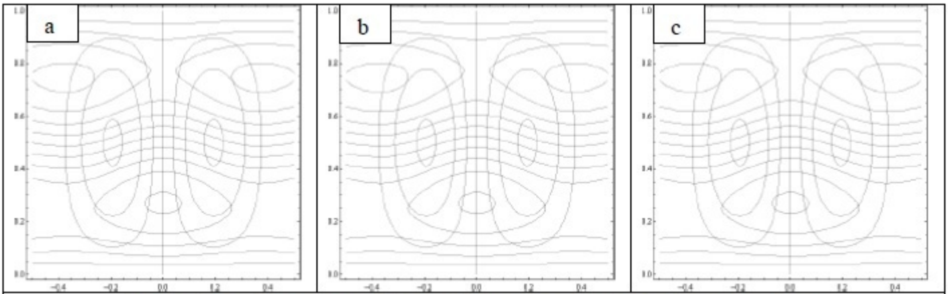

Fig. 6. Flow pattern Isothermal lines and Stream lines in anisotropic case (χ-varying)

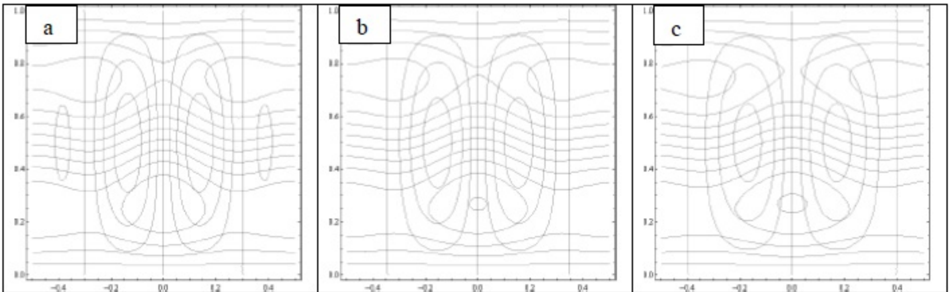

Fig. 7. Flow pattern Isothermal lines and Stream lines in anisotropic case (χ_c-varying)

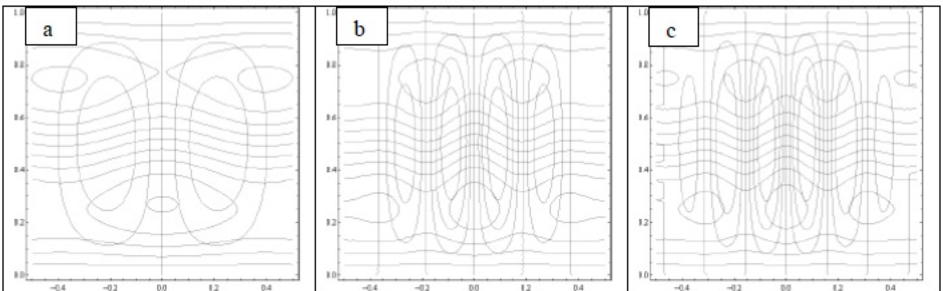

Fig. 8. Flow pattern Isothermal lines and Stream lines in anisotropic case (T_c-varying)

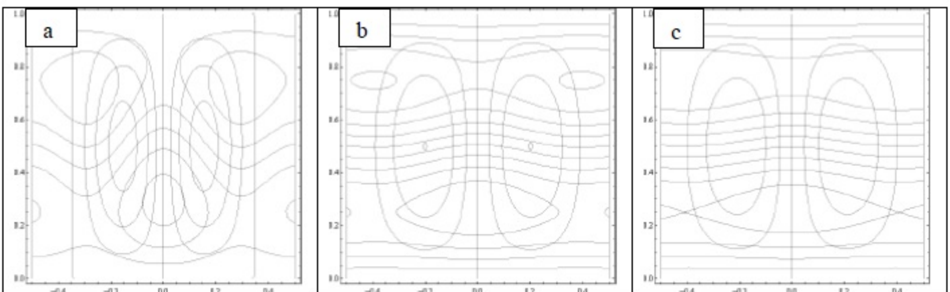

Fig. 9. Flow pattern Isothermal lines and Stream lines in anisotropic case (η-varying)

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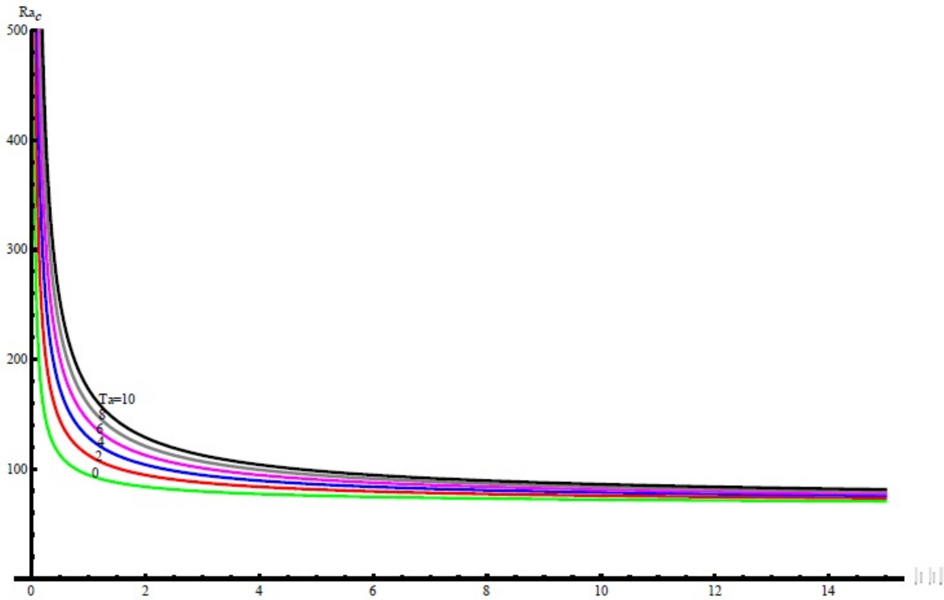

Fig. 10. Plot Ra_c vs ξ / η (Rs=50, ξ=0.5, η=0.125)

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