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Book Review

A Review of 'Topics in Graph Theory'

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Dr. N.K. Sudev, Topics in Graph Theory – A book on Graph Theory, Owl Books, Thiruvananthapuram, Kerala India, 2018, 190 pp; ISBN 978-93-856662-1-6.

Graph Theory is a branch of discrete mathematics which interweaves the link between mathematics and almost all the 'new-gen' subjects. In truth, the subject continues to be an inevitable part in the study of Computer Science and uses mathematical logic in the field of computer sciences to bring out certain results. That's why Graph Theory is given as a core paper for computer engineering students.

There are a lot of basic or introductory books on graph theory available in literature. But "Topics in Graph Theory" by Dr. N.K. Sudev alias Sudev Naduvath seems to be a standalone reference book on Graph Theory. It is prepared as per the syllabus of the fifth semester Bachelor of Technology program in Computer Science and Technology under APJ Abdul Kalam Technological University, Thiruvananthapuram, Kerala, India. Being a text book, even though it does not go to the depth of the subject matter, it is definitely a beginner's guide or a handy dictionary to the enthusiast. The narrative is similar to that of a teacher who teaches in a class. Few topics such as join and ring sum, deletion and fusion, subdivision and smoothing, vertex-cut and edge-cut, 1-isomorphism and 2-isomorphism, path matrix and cycle matrix which usually less explained in other books on introductory studies are dealt with

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extensively in the book. Besides motivating quotes, aesthetic sense in drawings and illustrations leave a high impression by the readers towards the book. The book is rich with application level problems and algorithms.

Sudev has done justice to the syllabus as well as to the students on covering the syllabus. In a bird's eye view the book has the following features: well explained and well-structured concepts, several examples for aspirants of learning, sufficient Bibliography, solved question papers, easy to refer index which cover even minute references and so on. Since it is primarily meant as a textbook for engineering students, Sudev made an extra effort to draw considerable number of diagrams and flow charts of algorithms and also explained them step by step. Though there are few typos, mastery of the author on the subject matter is explicitly seen while we go through the chapters.

The textbook is divided into ten chapters. The first chapter starts with a meaningful quote from the "Ambassador of Graph Theory" Frank Harary. It says, "Graph Theory serves as a mathematical model for any system involving a binary relation". It introduces the basic definitions, degree sequences, fundamental classes of graphs, subgraphs, spanning subgraphs and isomorphic graphs. For the characterization of degree sequences, the theorem known as *Havel-Hakimi Theorem*[2] could be added so that the students can easily identify whether the given sequence is graphical or not. The graph operations such as union, intersection, complement, join, ringsum, deletion, fusion, subdivision, smoothing are discussed in the second chapter. The third chapter is on the connectedness of graphs. Paths, cycles, distances, edge-deleted and vertex-deleted subgraphs are deliberated here.

The discourse in the fourth chapter includes the "Konigsberg Seven Bridge Problem", which is believed to be the foundation of Graph Theory. It may be a mere coincidence that the two pillars to whom the development of 'Konig(s)-Berg Seven Bridge Problem' (Graph

[2] Both Havel and Hakimi proved it separately. A non-decreasing sequence $s : d_1, d_2, \ldots, d_n$; $n \geq 2$ of non negative integers, where $d_1 \geq 1$ is graphical if and only if the sequence $s_1 : d_2 - 1, d_3 - 1, \ldots, d_{d1}+1 - 1, d_{d1} +^2, \ldots, d_n$ is graphical[1] [2].

Theory) in the 20th century owes much are Denes Konig and Claude Berg: The former wrote the first book on Graph Theory in 1936 after exactly 200 years of the publication of first article on graph theory by Leonhard Euler whereas the latter wrote the second prominent book on Graph Theory and introduced hyper graphs and is considered as an apostle of graph theory[3]. The fundamental concepts like Eulerian and Hamiltonian graphs, Fleury's algorithm, weighted graphs and the classic problems like 'Chinese Postman problem' and 'Travelling Salesman's Problem' are also seen in this chapter.

The fifth chapter deals with the directed graphs and their equivalence relations and networks. I think it covers almost all basic definitions related to digraphs. Unfortunately, no exercise problems are seen in the end of the chapter. Trees and their properties are elaborated in the sixth chapter. Though the connectedness is introduced in the third chapter and discussed in the sixth chapter, connectedness perceived through the vertex-cut, edge-cut, cut-sets, 1-isomorphism and 2-isomorphism and plenty of theorems related to cut-vertex and cut-edge make this chapter a unique one.

The planarity of graphs is introduced through the 'Three Utility Problem' in chapter eight. Related discourses like Kuratowski graphs, geometrical dual are followed in the chapter. Even though the concept of combinatorial dual graph is given in the syllabus, it is not described in the textbook. The reason may be that Whitney recognized the equivalence of geometrical dual graph and combinatorial dual graph [4]. The ninth chapter extensively explains the matrix representation of graphs such as incidence matrix, cycle matrix, cut-set matrix, adjacency matrix and path matrix. The final chapter explains various types of algorithms including the algorithm for connectedness and components, for spanning tree, for minimal spanning tree and for shortest path. The interconnections between various types of matrices are beautifully presented. The flow charts for these algorithms are added in the appendices.

As a textbook, a list of symbols used in the textbook for easy reference, answer key or hints for answers of exercise problems and the reference list of books (with reference to the pages) given in

chapter wise are preferably added. Keeping a little bit of history of the emergence of graph theory would strengthen the interest in students. Though the general mathematical concepts like pigeon-hole principle (Ref. Theorem 4.5.3 Dirac Theorem) and use of contra positive (Ref. Theorem 4.5.4 Ore's Theorem) are familiar to the students, they could be added in the appendix as glossary.

In short, the book "Topics in Graph Theory" by Sudev goes beyond the realm of a text book and provides a background on the basics of Graph Theory. The book is a synthesis of the flexibility in the writing and expertise of the author, hence the term 'flexpertise'[5] is an apt word to describe this book.

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