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Effect of Coriolis force and Gravity Modulation on the onset of Double Diffusive Convection in a Weak Electrically Conducting Boussinesq-Stokes Suspensions with Saturated Porous Media

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Abstract

The effect of gravity modulation along with rotation is analysed in Boussinesq-Stokes suspensionin a porous media subjected to double diffusive convection. The study consists of a linear and a non-linear analysis. The thermal Rayleigh number is obtained using a regular perturbation technique. The study suggests that instead of taking electrically non-conducting fluid it is better to consider electrically conducting fluid with weak electrical conductivity as this ensures a stable environment in the presence of a magnetic field. Taylor number is found to stabilize the system. The nonlinear analysis is done using a truncate Fourier series expansion that give rise to a system of Lorenz equations. Nusselt and Sherwood numbers are used to quantify the heat and mass transfer, respectively. Frequency of modulation stabilizes the system which can be controlled to our desire.

Keywords: Boussinesq-Stokes suspension, Gravity modulation, Rayleigh-Bénard convection, Double diffusive convection

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1. Introduction

The application of fluid mechanics has been on a steep rise the last century. Aeronautical, biomedical, civil, marine, and mechanical engineers as well as astrophysicists, geophysicists, space researchers, meteorologists, physical oceanographers, physicists, and mathematicians have used this knowledge to tackle a multitude of complex flow phenomena. A class of fluid which considers couple stresses in addition to the classical Cauchy stress was developed by Stokes (1966). This fluid theory is discussed in detail by Stokes (1984) in his treatise "Theories of fluids with microstructure" wherein he also presented a list of problems discussed by researchers with reference to this theory. It is referred to as couple stress fluid or Boussinesq-Stokes fluid. Some of the problems of recent interest can also be seen in Swamy*et. al.* (2005). Gaikwad and Kouser (2014) investigated double diffusive convection in porous media. The fluid considered was a Boussinesq-Stokes suspension with internal heat source. It was found that addition of a solute delays the process of convection and internal heat source could be used as an external means to influence the stability of the system.

Another important aspect of the problem is Coriolis force. The Coriolis Effect is a deflection of moving objects when they are viewed in a rotating reference frame. One of the earlier studies on rotation modulation was done by Pearlstein (1981). He found out the behavior of a fluid subjected to two-component convection. This was followed by Sharma (2006) who did an investigation of the modulation of rotation in viscoelastic liquids. The effects of modulation of rotation on the transfer of heat was investigated by Bhaduria and Kiran(2014). The destabilizing effects of the speed of rotation on the convective system was established. Shalini and Mahantesh (2018) investigated Rayleigh-Benard convection in nanofluids with and without rotation and found the destabilizing effects of rotation. For stationary convection, the suspended particles accelerated the convection. Julien (2019) studied the scaling laws in Rayleigh-Benard convection with and without magnetic fields and rotation. Scaling laws are useful for studying the various characteristics of the flow. These were compared with the existing data.

Convection in porous media has recently been studied since it has many practical, especially biological significance. The effects of gravity/temperature modulation on magneto-convection in a weak electrically conducting fluid with internal angular momentum was investigated by linear stability analysis by Siddheshwar and Pranesh(1999). The temperature modulation gave rise to sub-critical motion. It was found that gravity modulation causes a delay in convection. Sameena and Pranesh (2016) studied the effects of g-gitter on the onset of convection in a weak electrically conducting fluid and found that time-periodic body force leads to delay in convection. Bhadauria *et. al.* (2009) investigated the effect of gravity modulation on the onset of Darcy convection in a rotating porous medium. It was concluded that gravity modulation delays the onset of convection as does rotation. Nield and Simmons (2019) briefly explained the process of convection in porous media. The interaction of the porous media with the heat transfer in a single-phase flow was the main focus. The study consisted of theoretical and experimental part. Sivakumar and Saravana (2009) investigated the effect of Gravity Modulation on the Onset of Convection in a Horizontal Anisotropic Porous Layer. A linear stability theory was used to investigate the effect of gravity modulation on the onset of convection in a homogeneous anisotropic porous layer heated from above. The Brinkman model with anisotropic permeability was considered. Free-convection flow past an infinite vertical porous plate with periodic suction and gravity modulation was investigated by Baljinder (2010). Bhadauria and Kiran (2015) examined the instability of double diffusive magneto-convection in a Newtonian fluid with gravity modulation. Using linear matrix differential operator method, the Ginzburg–Landau amplitude equation for the problem was obtained. They found the stabilizing effects of magnetic field and gravity modulation.

Naturally occurring phenomena are mostly unsteady because of the periodicity of the principal driving forces. More specifically, if heat is introduced slowly the basic temperature gradient is uniform, the instability usually appears in the form of rolls. If heat is introduced rapidly, then the basic temperature gradient is non-uniform being a function of position and time, the instability manifesting in the form of columnar instability. The modulation of such temperature, gravity or introduction of rotation to a stable system greatly affects its stability. This is observed in space experiments, oceanography, production of industrial materials and crystal growth among many others. If the introduction of such modulation can stabilize an otherwise unstable flow, then the efficiency can be increased in many processing techniques. For many alloys, although some instabilities can be neglected, convection due to earth's gravitational field remains prominent. This has led to experiments being conducted in space where the gravitational acceleration is low. However, time-dependent g-jitter from inherent mechanical vibrations may induce buoyancy-driven convection. Hence, it is important to understand how these vibrations may be controlled to achieve the best results.

The study of the present problem is taken up with these motivations. Although the studies on gravity modulations are many, this paper aims to find answers to the question of stability of double diffusive convection in Boussinesq-Stokes suspension with weak electric field in the presence of rotation. Most fluids in nature are non-Newtonian in nature and the presence of suspended particles are unavoidable. Such fluids exhibit couple stress behaviour or can be considered as Boussinesq-Stokes suspension. The scope of this papers is to interpret and understand the mechanism of enhancing or suppressing convection.

2. Mathematical Formulation

Consider a layer of a finitely electrically conducting, Boussinesq fluid with suspended particles confined between two finite horizontal walls distanced apart. The Coriolis force is induced by the rotation of the cylinder. An additional concentration gradient is introduced apart from temperature gradient. A Cartesian co-ordinate system is taken with origin in the lower boundary (fig 1).

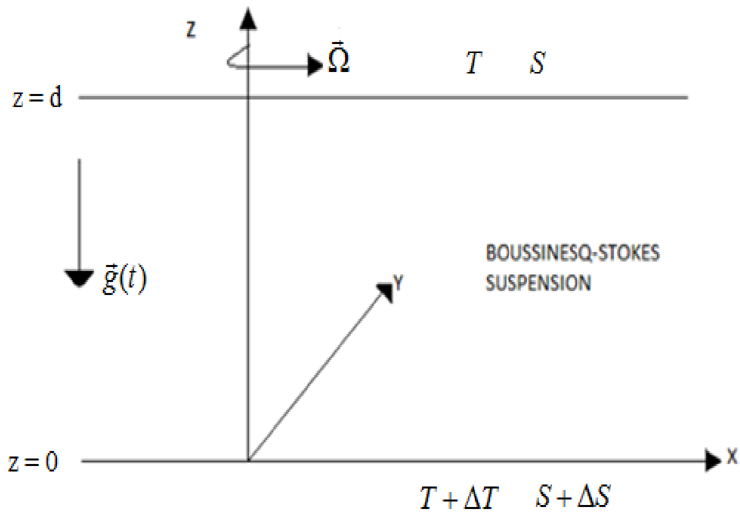

Fig 1: Physical configuration of the problem.

The governing equations are (Venezian (1969)):

Equation of continuity:

$$\nabla.\vec{q} = 0 \tag{1}$$

Equation of conservation of linear momentum:

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}+\frac{2}{\varepsilon}(\vec{\Omega}\times\vec{q})\right]=-\nabla p+\rho\vec{g}+\frac{\mu'}{\kappa}\nabla^2\vec{q}-\frac{\mu_{eff}}{\kappa}\vec{q}-\mu_m^2 H_0^2\sigma\vec{q} \tag{2}$$

Equation of conservation of energy:

$$\gamma\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T \tag{3}$$

Equation of conservation of species:

$$\gamma\frac{\partial S}{\partial t}+(\vec{q}.\nabla)S=\kappa_s\nabla^2 S \tag{4}$$

Equation of state:

$$\rho = \rho_0\left[1-\alpha(T-T_0)+\alpha_s(S-S_0)\right] \qquad (5)$$

where, $\vec{q}$ is the velocity, p pressure, $\vec{H}$ magnetic field, B magnetic induction, T temperature, ρ density , $\vec{g}$ acceleration due to gravity, χ thermal conductivity, α coefficient of thermal expansion, ρ_0 reference density, T_0 reference temperature, μ_m magnetic permeability, μ dynamic viscosity, μ' couple stress viscosity, μ_{eff} effective viscosity, σ electrical conductivity, γ porosity, S is concentration κ_S is solutal diffusivity, α_S is the coefficient of solutal expansion and t time. The above equations are solved for free-free isothermal boundary conditions. The gravity modulation is considered as $\vec{g}(t) = g_0(1+\varepsilon cos\Omega t)\hat{k}$, where ε is the small amplitude and Ω is the frequency.

3. Basic State

Initially we assume that the fluid is at rest and is described by

$$\vec{q} = \vec{q}_b = 0, p = p_b(z), \rho = \rho_b(z), T = T_b(z,t), S_b = S_b(z), \vec{\Omega} = \Omega_0\hat{k} \qquad (6)$$

When these quantities are substituted in the governing equations, we get the following set of equations:

$$\frac{dp_b}{dz} + \rho_b g(1+\varepsilon\cos\Omega t) = 0 \qquad (7)$$

$$\frac{\partial^2 T_b}{\partial z^2} = 0 \qquad (8)$$

$$T_b = T_0 - \frac{\Delta T}{d}z, \;\; S_b = S_0 - \frac{\Delta S}{d} \qquad (9)$$

4. Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation as follows

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', S = S_b + S', \vec{\Omega} = \Omega_0 \hat{k} + \vec{\Omega}' \quad (10)$$

The prime indicates that the quantities are infinitesimal perturbations.

Substituting equation (10) into governing equations (1) to (5) and using the basic state equations, the linearized equations governing the infinitesimal perturbations takes the following form:

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial \vec{q}'}{\partial t} + \frac{1}{\varepsilon^2}(\vec{q}'.\nabla)\vec{q}' + \frac{2}{\varepsilon}((\Omega_0\hat{k} + \vec{\Omega}') \times \vec{q}')\right] = -\nabla(p_b + p') - (\rho_b + \rho')g(1 + \varepsilon\cos\Omega t)\hat{k}$$
$$+\frac{\mu'}{\kappa}\nabla^2\vec{q}' - \frac{\mu_{eff}}{\kappa}\vec{q}' - \mu_m^2 H_0^2\sigma\vec{q}' \quad (11)$$

$$\gamma\frac{\partial T'}{\partial t} + (\vec{q}.\nabla)T_b + (\vec{q}.\nabla)T' = \kappa\nabla^2 T' \quad (12)$$

$$\gamma\frac{\partial S'}{\partial t} + (\vec{q}.\nabla)S_b + (\vec{q}.\nabla)S' = \kappa_s\nabla^2 S' \quad (13)$$

$$\rho' = -\alpha\rho_0 T' + \alpha_S\rho_0 S' \quad (14)$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial \vec{q}'}{\partial t} + \frac{2}{\varepsilon}(\Omega_0\hat{k} \times \vec{q})\right] = -\nabla p' + \rho_0\alpha g(1 + \varepsilon\cos\Omega t)T'\hat{k}$$
$$+\frac{\mu'}{\kappa}\nabla^2\vec{q}' - \frac{\mu_{eff}}{\kappa}\vec{q}' - \mu_m^2 H_0^2\sigma\vec{q}' \quad (15)$$

$$\gamma\frac{\partial T'}{\partial t} + (\vec{q}.\nabla)T_b = \kappa\nabla^2 T' \quad (16)$$

The perturbation equations are non-dimensionalized using the following definitions:

$$w^* = \frac{w'}{\kappa / d}, t^* = \frac{t}{d^2/\kappa}, T^* = \frac{T'}{\Delta T}, \nabla^* = d\nabla, (x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right) \quad (17)$$

Equation (17) is used to get a set of dimensionless equations given below:

$$M_1\left(\partial/\partial t - \nabla^2\right)T = w\ , \quad (18)$$

$$M_1\left(\partial/\partial t - \frac{1}{Le}\nabla^2\right)S = w\ , \quad (19)$$

$$\frac{1}{pr}\frac{\partial(\nabla^2 w)}{\partial t} + Ta^{1/2}\frac{\partial V}{\partial z} = R(1+\varepsilon\cos\Omega t)\nabla_1^2 T + \frac{C}{D_a}\nabla^4 w - \frac{1}{D_a}\nabla^2 w' - M^2\nabla^2 w' \quad (20)$$

$$\frac{1}{pr}\frac{\partial V}{\partial t} - Ta^{1/2}\frac{\partial w}{\partial z} = \frac{C}{D_a}\nabla^2 V - \frac{1}{D_a}V - M^2 V \quad (21)$$

Where $R = \frac{\alpha g \Delta T\, d^3 \rho_o}{\mu\chi}$ is thermal Rayleigh number, $Rs = \frac{\alpha_s \rho_0 g \Delta S d^3}{\mu\kappa}$ is the solutal Rayleigh number, $M = \sqrt{\frac{\mu_m^2 H_0^2 d^2 \sigma}{\mu}}$ is the Hartmann number, $\Pr = \frac{\mu}{\rho_o \chi}$ is the Prandtl number, $T_a = \left(\frac{2\Omega_0 \rho_0 d^2}{\mu}\right)^2$ is the Taylor number, $C = \frac{\mu'}{\mu d^2}$ is the couple stress parameter and $Le = \frac{\kappa}{\kappa_S}$ is the Lewis number.

Equations (18) – (21) are combined in order to eliminate S and V so as to obtain a single equation in terms of the temperature, T.

$$\left[\left(\frac{1}{pr}\frac{\partial}{\partial t}\nabla^2-\frac{C}{D_a}\nabla^4+\frac{1}{D_a}\nabla^2+M^2\nabla^2\right)\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)+TaD^2\right]\left(M_1\frac{\partial}{\partial t}-\nabla^2\right)T=$$
$$\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)R(1+\varepsilon\cos\Omega t)\nabla_1^2 T \qquad (22)$$

Each of T_n is required to satisfy the boundary conditions. The marginally stable solution of the problem is the general solution of the above equation i.e. $T_0=\sin(\pi z)e^{i(lx+my)}$, corresponding to the lowest mode of convection with the corresponding eigen value R_0.

5. Stability Analysis

The modulation of gravitation the present problem is of form $\vec{g}(t)=g_0(1+\delta cos\gamma t)\hat{k}$, where δ is the amplitude and γ is the frequency of modulation. Thus, the eigen-values of the present problem differ from those of the ordinary Benard convection by quantities of order δ.

The solution of equation (22) is in the following form:

$$(R,T)=(R_0,T_o)+\delta(R_1,T_1)+\delta^2(R_2,T_2)+\ldots \qquad (23)$$

This expansion is substituted into equation (22) and the coefficients of various powers of δ are equated on either side of the equation. The resulting system of equations is:

$$LT_0=0 \qquad (24)$$

$$LT_1=\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)(R_1+R_0\cos\Omega t)\nabla_1^2 T_0 \qquad (25)$$

$$LT_2=\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)\left[(R_1+R_0\cos\Omega t)\nabla_1^2 T_1+(R_2+R_1\cos\Omega t)\nabla_1^2 T_0\right] \qquad (26)$$

where

$$L=\left[\left(\frac{1}{pr}\frac{\partial}{\partial t}\nabla^2-\frac{C}{D_a}\nabla^4+\frac{1}{D_a}\nabla^2+M^2\nabla^2\right)\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)+TaD^2\right]\left(M_1\frac{\partial}{\partial t}-\nabla^2\right)-$$

$$\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)R_0\nabla_1^2 \qquad (27)$$

The zeroth order problem is the one with no modulation. The solution of *eq.* (39) that is derived from the coefficients of ε^0 is used in convection, with a uniform thermal modulation. The marginal stable solutions are

$$\psi_0 = Sin(\pi\alpha x)\ Sin(\pi z),$$

The eigenvalue, Ra_0 is given by

$$R(a,\delta)=R_0(a)+\delta^2R_2(a)$$

It was shown by Venezian (1969) that the critical value is determined by $O(\delta^2)$, by evaluating R_0 and R_2 at $a=a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a=a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c}) one has to substitute $a=a_0$ in R_2, where a_0 is the value at which R_0 given by equation (30) is minimum.

Equation (24) gives the expression for the Eigen value R_0:

$$R_0=\frac{(k^2X_1^2+\pi^2Ta)k^2}{a^2X_1} \qquad (28)$$

where

$$X_1 = \left(\frac{C}{D} k^2 + \frac{1}{D_a} + M^2 \right) \tag{29}$$

The consequent equations when solved gives the correction Rayleigh number, R_{2c}

$$R_{a2} = \frac{Le}{2a^2 k^2 |L(\Omega, n)|^2} \begin{bmatrix} (B_1 + B_2)(A_1 Y_1 - A_2 Y_2 + A_3 Y_2) + A_1 B_3 Y_2 \\ + A_2 B_2 Y_1 + A_3 B_2 Y_1 \end{bmatrix} \tag{30}$$

Where

$$A_1 = \frac{2\Omega^2 M_1 a^2 R_{a0}}{pr} + \frac{a^2 C k^4 R_{a0}}{DLe} - \frac{a^2 k^2 R_{a0}}{DLe} - \frac{M^2 k^2 a^2 R_{a0}}{Le}$$
$$- \frac{2a^2 M_1 \Omega^2 R_s}{pr} - \frac{a^2 C k^4 R_s}{DLe} + \frac{a^2 k^2 R_s}{DLe} + \frac{a^2 M^2 k^2 R_s}{Le} \tag{31}$$

$$A_2 = \frac{a^2 \Omega M_1 R_{a0}}{D} + a^2 M^2 M_1 \Omega R_{a0} - \frac{a^2 \Omega M_1 R_s}{D} - a^2 M^2 M_1 \Omega R_s \tag{32}$$

$$A_3 = \frac{-a^2 k^2 R_{a0}}{Lepr} - \frac{a^2 M_1 C k^2 R_{a0}}{D} - \frac{a^2 M_1 R_{a0}}{D} - a^2 M^2 M_1 R_{a0}$$
$$+ \frac{a^2 k^2 R_s}{Lepr} + \frac{a^2 M_1 C k^2 R_s}{D} + \frac{a^2 M_1 R_s Y_3}{D} + a^2 M^2 M_1 R_s \tag{33}$$

$$B_1 = \frac{M_1 a^2 \Omega^2 R_{a0}}{pr} - \frac{M_1 a^2 \Omega^2 R_s}{pr} \tag{34}$$

$$B_2 = \frac{-a^2 k^2 X_1 R_{a0}}{Le} + a^2 k^2 X_1 R_s \tag{35}$$

$$B_3 = \frac{a^2 M_1 \Omega R_{a0}}{D} + a^2 M^2 M_1 \Omega R_{a0} - \frac{a^2 M_1 \Omega R_s}{D} - a^2 M^2 M_1 \Omega R_{a0} \tag{36}$$

$$X_1 = \frac{Ck_1^2}{D} - \frac{1}{D} + M^2 \tag{37}$$

$$X_n = \frac{Ck_n^2}{D} - \frac{1}{D} + M^2 \tag{38}$$

The operator L is given by

$$L(\Omega, n) = Y_1 + iY_2 \tag{39}$$

$$Y_1 = \frac{2\Omega^2 M_1 k_n^2 X_n}{pr} - X_n^2 k_n^4 - \frac{\Omega^2 k_n^4}{pr^2} - \pi^2 Tak_n^2 + aR_0 X_n \tag{40}$$

$$Y_2 = \Omega M_1 k_n^2 X_n^2 - \frac{\Omega^3 M_1 k_n^2}{pr^2} + \frac{2\Omega k_n^4 X_n}{pr} + \pi^2 Ta\Omega M_1 - \frac{\Omega a^2 R_0}{pr} \tag{41}$$

6. Non-Linear Theory

The finite amplitude analysis wherein the following substitutions are used:

$$\psi = A(t)\sin(\pi\alpha x)\sin(\pi z) \tag{42}$$

$$V = B(t)\sin(\pi\alpha x)\cos(\pi z) + D(t)\sin(2\pi\alpha x) \tag{43}$$

$$\theta = E(t)\cos(\pi\alpha x)\sin(\pi z) + F(t)\sin(2\pi z) \tag{44}$$

$$\varphi = G(t)\cos(\pi\alpha x)\sin(\pi z) + H(t)\sin(2\pi z) \tag{45}$$

Using these, eqns. we get

$$\frac{1}{pr}\frac{\partial(\nabla^2\psi)}{\partial t} + \frac{1}{pr}J(\nabla^2\psi,\psi) - Ta^{1/2}\frac{\partial V}{\partial z} = -R_a(1+\varepsilon\cos\Omega t)\frac{\partial\theta}{\partial x} + R_s(1+\varepsilon\cos\Omega t)\frac{\partial\phi}{\partial x}$$
$$-\frac{C}{D_a}\nabla^4\psi + \frac{1}{D_a}\nabla^2\psi + M^2\nabla^2\psi \tag{47}$$

$$M_1\frac{\partial\varphi}{\partial t} - J(\psi,\phi) + \frac{\partial\psi}{\partial x} = \nabla^2\phi\,, \tag{48}$$

$$\frac{1}{pr}\frac{\partial V}{\partial t}-\frac{1}{pr}J(\psi,V)+Ta^{1/2}\frac{\partial \psi}{\partial z}=\frac{C}{D_a}\nabla^2 V-\frac{1}{D_a}V-M^2 V \tag{49}$$

Substituting equations (46) – (49) in the previous set of four equations results in the following expressions for the different modes. The over dot denotes the time derivative.

$$\dot{A}(t)=\frac{pr}{k^2}Ta^{1/2}\pi B(t)-\frac{pr}{k^2}\pi\alpha R_a(1+\varepsilon\cos\Omega t)E(t)+\frac{pr}{k^2}\pi\alpha R_s(1+\varepsilon\cos\Omega t)G(t) \tag{50}$$

$$-\frac{Cpr}{D_a}k^2 A(t)+\left(\frac{1}{D_a}+M^2\right)prA(t) \tag{51}$$

$$\dot{B}(t)=-\pi prTa^{1/2}A(t)-\frac{k^2 prC}{D_a}B(t)-\left(\frac{1}{D_a}+M^2\right)prB(t) \tag{52}$$

$$\dot{C}(t)=\frac{-\pi\alpha A(t)B(t)}{2}-\frac{4\pi^2\alpha^2 Cpr}{D_a}C(t)-\left(\frac{1}{D_a}+M^2\right)prC(t) \tag{53}$$

$$\dot{E}(t)=\frac{-\pi\alpha}{M_1}A(t)-\frac{k^2}{M_1}E(t) \tag{54}$$

$$\dot{F}(t)=\frac{\pi^2\alpha A(t)E(t)}{2M_1}-\frac{4\pi^2}{M_1}F(t) \tag{55}$$

$$\dot{G}(t)=\frac{-\pi\alpha}{M_1}A(t)-\frac{k^2}{M_1}G(t) \tag{56}$$

$$\dot{H}(t)=\frac{\pi^2\alpha A(t)G(t)}{2M_1}-\frac{4\pi^2}{M_1}H(t) \tag{57}$$

7. Results and Discussions

The problem attempts to address the effects of gravity modulation and rotation on double diffusive in a weak electrically conducting

Boussinesq-Stokes suspension. The parameters that appear in the problem are Le, Ra. Rs, Ta, δ and ω. They affect the rate of convection and the amount of heat/mass transfer. The influence of these parameters on the correction Rayleigh number are discussed.

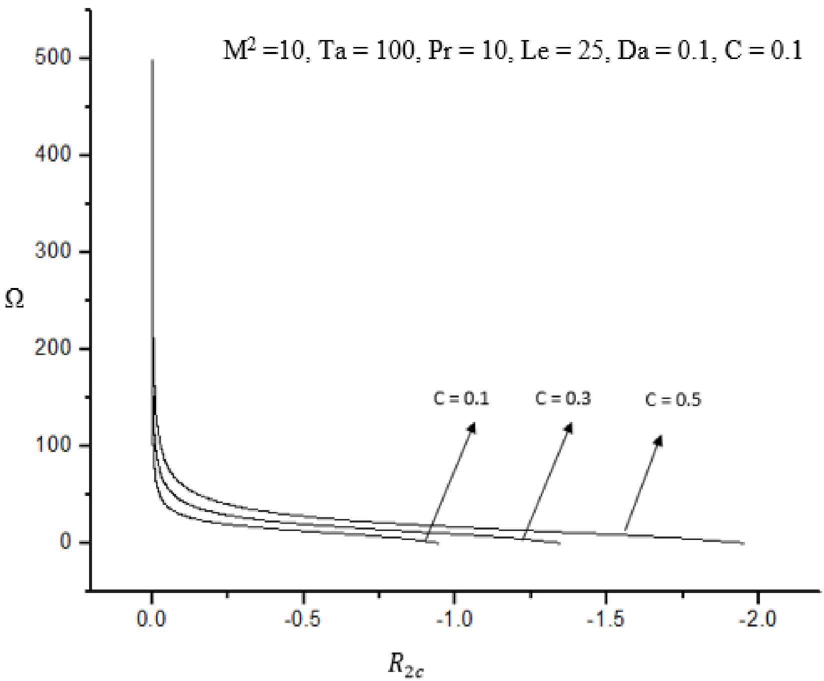

Fig 2: Plot of R_{2c} vs Ω for different values of C

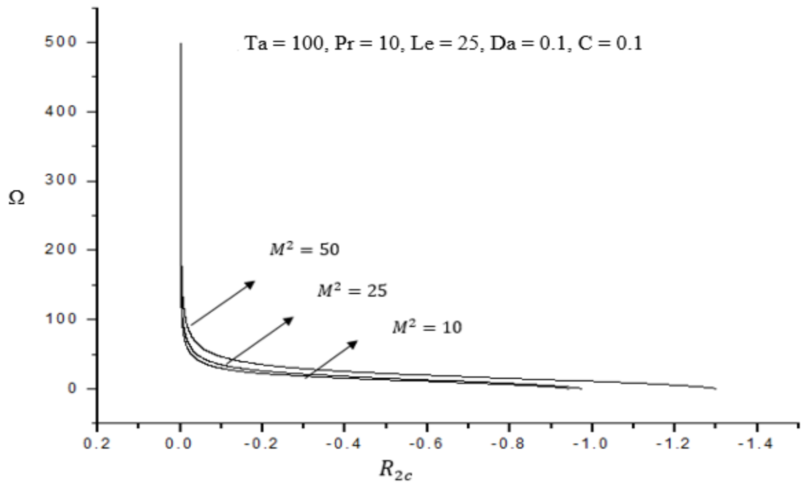

Fig 3: Plot of R_{2c} vs Ω for different values of M^2

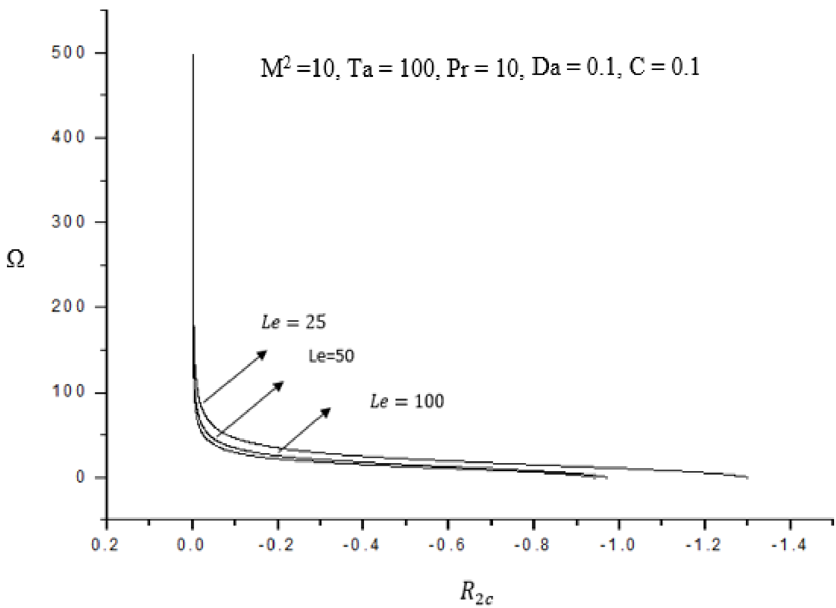

Fig 4: Plot of R_{2c} vs Ω for different values of Le

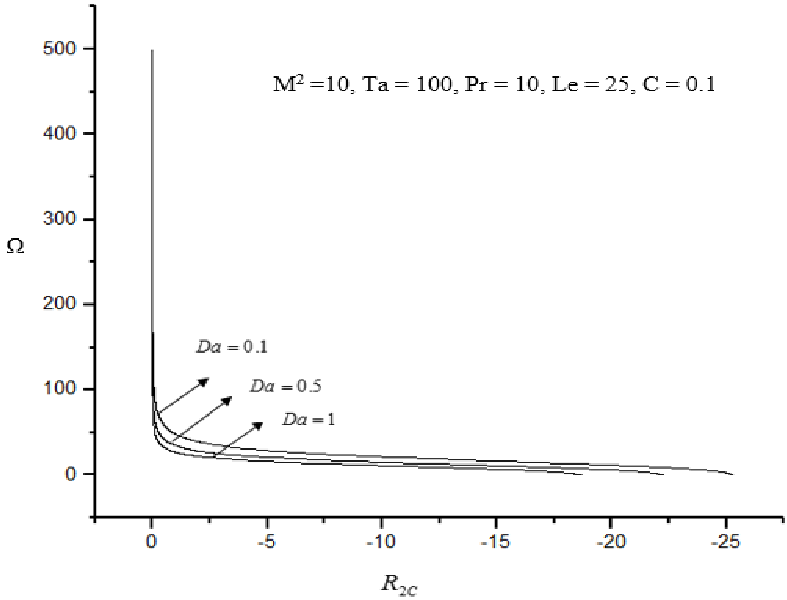

Fig 5: Plot of R_{2c} vs Ω for different values of Da

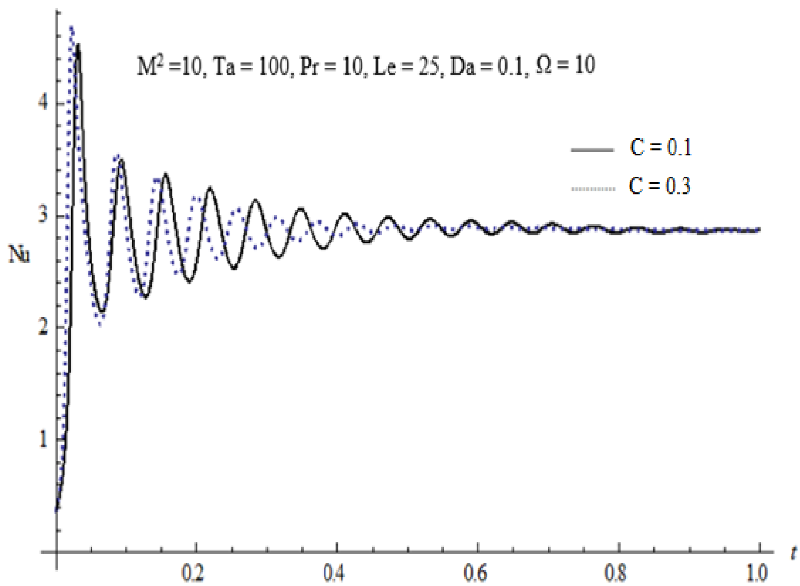

Fig 6: Graph of Nusselt number versus time for different values of C

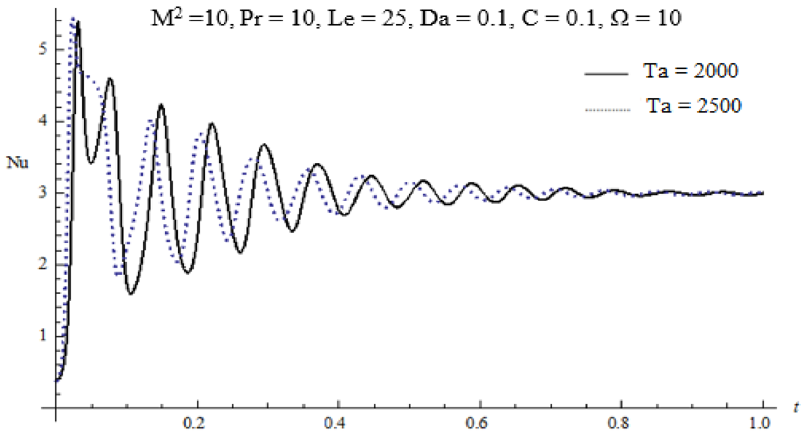

Fig 7: Graph of Nusselt number vs time for different values of Ta

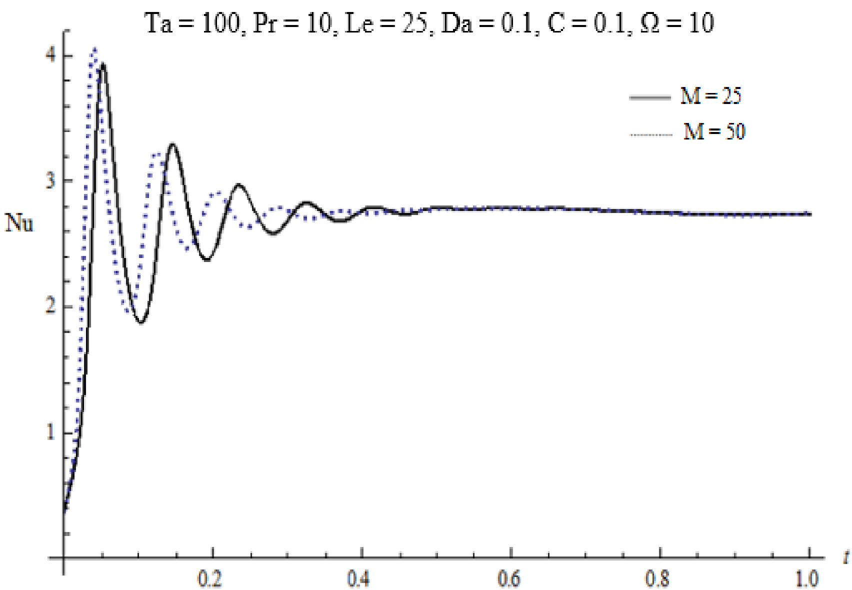

Fig 8: Graph of Nusselt number vs time for different values of M

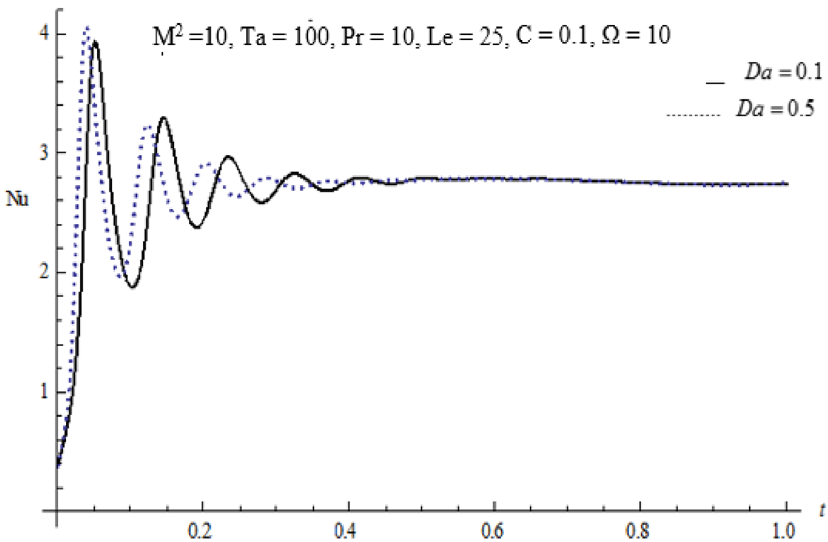

Fig 9: Graph of Nusselt number versus time for different values of Da

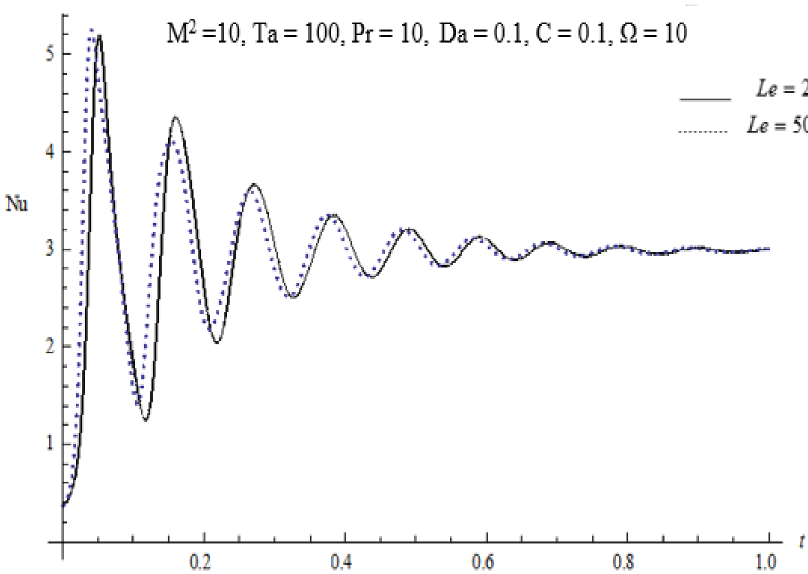

Fig 10: Graph of Nusselt number versus time for different values of Le

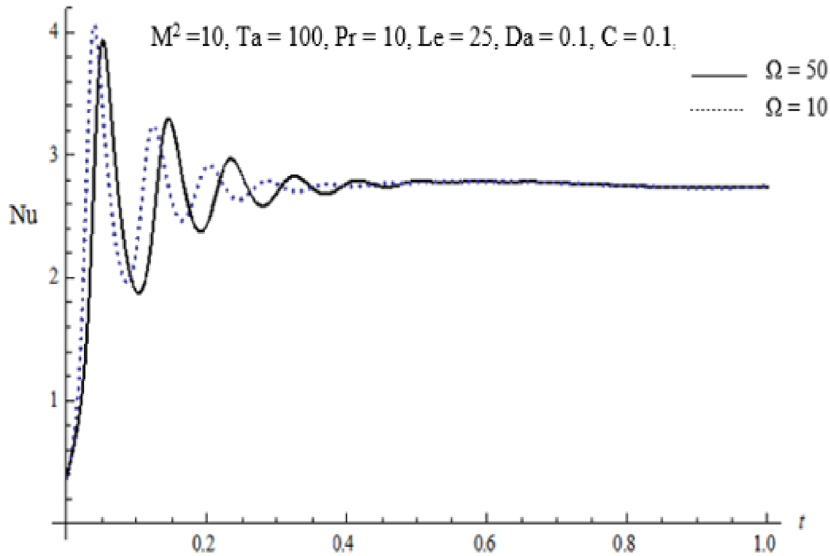

Fig 11: Graph of Nusselt number versus time for different values of Ω

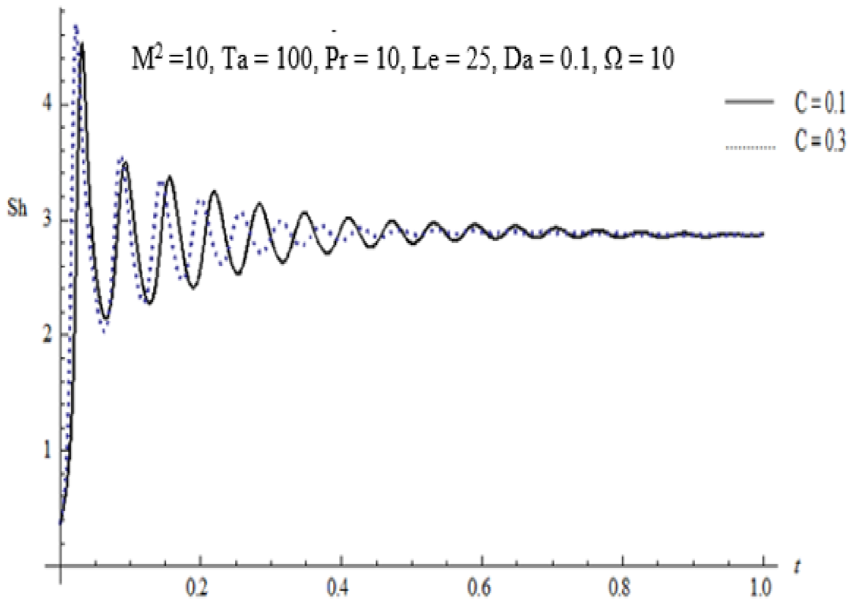

Fig 12: Graph of Sherwood number versus time for different values of C

The amplitude of modulation is assumed to be small. Hence the results depend on the frequency of modulation. The period of gravity modulation is large when the value of $\Omega < 1$. The modulation considered affects the entire bulk of the fluid and hence causes a disturbance. However, this disturbance is negligible for larger values of the frequency since buoyancy force takes a mean value and the system attains an equilibrium.

Figure 2 is the plot of correction Rayleigh number R_{2c} versus Ω for different values of C. We see that C is directly proportional to correction Rayleigh number. It is the ratio of couple stress viscosity to dynamic viscosity. As C increases so does the couple stress viscosity which in turn delays convection. Figure 3 is the plot of correction Rayleigh number R_{2c} versus Ω for different values of M^2. From the figure it is observed that with increase in M^2, R_{2c} becomes more negative. Magnetic field induces viscosity into the fluid and the magnetic lines are distorted. These magnetic lines hinder the growth of disturbances. Figure 4 is the plot of correction Rayleigh number R_{2c} versus Ω for different values of Le. We note that as Le increases, R_{2c} increases, that is it becomes less negative. Le is the ratio of thermal to solutal diffusivities. As the Lewis number increases the solutal diffusivity decreases and so does the rate of convection. Figures 5 is the plot of correction Rayleigh number R_{2c} versus Ω for different values of Da. We observe that as Da increases R_{2c} increases. Da is a result of the porous media and is indicative of the permeability of the medium. When the fluid flows through such a medium convection gets delayed.

Figures 6 – 11 depict the nonlinear analysis. They are plots of Nusselt number versus time for different values of the parameters. Nusselt number is indicative of the heat transfer. From these it is clear that couple stress parameter, Taylor number, Darcy number, electric field, Lewis number and frequency of modulation are all in favour of reducing the convective motion. As their values increase, Nusselt number decreases which implies that the heat transfer is reduced. These results reinforce the results obtained for the linear analysis. Figure 12 is a plot of Sherwood number versus time for varying values of the couple stress parameter. It is found that it supresses the mass transfer. The results for Sherwood number for all the parameters are similar to that obtained for Nusselt number.

8. Conclusions

From the study we conclude that the gravity modulation or g-jitters leads to delay in convection. It is clear that instead of taking electrically non-conducting fluid it is better to consider electrically conducting fluid with weak electrical conductivity as this ensures a stable environment in the presence of a magnetic field. The Taylor number, couple stress, Frequency of modulation, stabilizes, Lewis number the system. Hence, modulation is a powerful tool that can be used to manipulate the convective system to our desire.

This study can be further extended to modulations of rotation, temperature field and electric field.

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