ISSN 0975-3303
*Mapana J Sci*, **11**, 3(2012), 105-112
https://doi.org/10.12725/mjs.22.6

# Mixed Convection Flow over a Vertical Cone with an Applied Magnetic Field

K R Jayakumar*, A H Srinivasa† and A T Eswara‡

## Abstract

An analysis is performed to investigate the mixed convection flow over a vertical cone with an applied magnetic field when the axis of the cone is in line with the flow. The results have been obtained for assisting and opposing flows. The partial differential equations governing the non-similar flow have been solved by an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results are reported here to account the effects of magnetic field in presence of buoyancy parameter at different stream wise locations on skin friction and heat transfer coefficients.

## 1. Introduction

Convective heat transfer in steady flows over a stationary cone is important for thermal design of various types of industrial equipment's such as heat exchanger, conisters for nuclear waste disposal, nuclear reactors cooling system and geothermal reservoirs etc. In early study, Hering and Grosh [1] investigated the practical case of steady mixed convection from a vertical cone for Pr = 0.7.

* Department of Mathematics, K S School of Engineering and Management, Bangalore – 560062, India, krjk_maths@yahoo.com

† Department of Mathematics, Maharaja Institute of Technology , Mysore, Belawadi – 571438, India, ahsydv@yahoo.com

‡ Department of Mathematics, PES College of Engineering, Mandya – 571401, India, eswaram@ymail.com

Received: July 2012, Reviewed: Aug. 2012

In a further study, Himasekhar et al. [2] found the similarity solution of the mixed convection flow over a vertical cone in an ambient fluid for a wide range of Prandtl numbers. Kumari et al. [3] has presented non-similar solution for mixed convection along a vertical cone. The objective of this paper is to analyse the steady, nonsimilar, mixed convection, laminar incompressible boundary layer flow of an electrically conducting fluid over a vertical cone along with an applied magnetic field.

## 2. Analysis

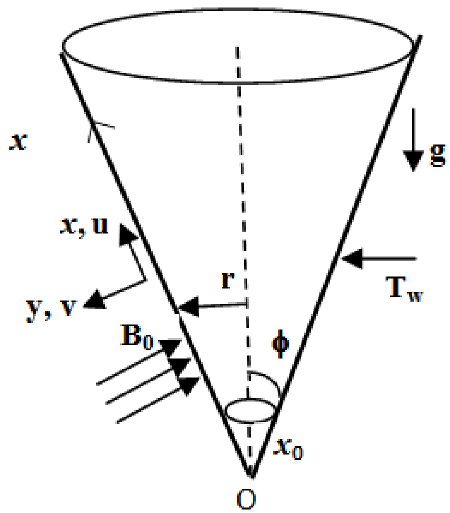


Figure 1. Flow model and co-ordinate system

Consider a vertical circular cone with a half angle $\phi$ along which a forced flow moves parallel to the axis of the cone with the free stream velocity $u_\infty$ and temperature $T_\infty$. The surface of the cone is at a uniform higher temperature $T_w$ i.e., $T_w > T_\infty$ and the forced flow is in upward direction. The physical model and co-ordinate system is presented in Fig.1. The stream wise coordinate x is measured from the apex of the cone along its generator and the transverse coordinate y is measured normal to it into the fluid, respectively. Thermo–physical properties of the fluid in the flow model are assumed to be constant. A magnetic field $B_o$ fixed relative to the fluid is applied in y–direction. It is assumed that magnetic

Reynolds number is small so that the induced magnetic field can be neglected. Under the above assumptions, the continuity, momentum and energy equations governing mixed convection flow along a vertical cone can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2}+U\frac{\partial U}{\partial x} \\ &\quad +g\beta\cos\phi\left(T-T_\infty\right)-\frac{\sigma B_0^2}{\rho}\left(u-U\right) \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are

$$\left.\begin{aligned} &u(x,0)=0,\ v(x,0)=0,\ T(x,0)=T_w \\ &u(x,\infty)=U(x),\ T\left(x,\infty\right)=T_\infty \end{aligned}\right\} \tag{4}$$

Applying the following transformations

$$\left.\begin{aligned} &\eta=y\left(\frac{m+3}{6}\frac{U}{\nu x}\right)^{1/2},\ \xi=\left(\frac{6}{m+3}\frac{\nu x}{U}\right)^{1/2},\ u=\frac{\partial\psi}{\partial y} \\ &v=-\frac{\partial\psi}{\partial y},\ \psi(x,y)=\left(\frac{6}{m+3}\nu xU\right)^{1/2}f\left(\xi,\eta\right) \\ &G\left(\xi,\eta\right)=\frac{T-T_\infty}{T_w-T_\infty},\ U=u_\infty x^{m/3} \end{aligned}\right\} \tag{5}$$

to Eqs. (1) – (3), it is found that Eq. (1) is satisfied identically, and Eqs. (2) and (3) reduce to

$$\begin{gathered} F'' + f\,F' + \frac{2m}{m+3}\left(1 - F^2\right) + \frac{6}{m+3}\lambda\,G \\ - \frac{6}{m+3} M\left(F - 1\right) = \frac{3}{m+3}\xi\left(FF_\xi - F' f_\xi\right) \end{gathered} \tag{6}$$

$$\Pr^{-1} G'' + f\,G' = \frac{3}{m+3}\,\xi\left(F\,G_\xi - G' f_\xi\right) \tag{7}$$

where

$$\left.\begin{aligned} &\frac{u}{U} = f' = F,\ \Pr = \frac{\mu C_p}{k},\ \mathrm{Re}_x = \frac{u_\infty x}{\nu},\ \lambda = \frac{Gr_x}{\mathrm{Re}_x^2} \\ &v = -\frac{1}{2}\left(\frac{6}{m+3}\,\frac{\nu U}{x}\right)^{1/2}\left[\left(\frac{m+3}{3}\right) f + \xi f_\xi - \eta f'\right] \\ &Gr_x = \frac{g\beta x^3\left(T_w - T_\infty\right)\cos\phi}{\nu^3},\ M = \frac{\sigma B_0^2}{\rho U x} \end{aligned}\right\} \tag{8}$$

The transformed boundary conditions are

$$F = 0;\ G = 1 \ \text{ at } \ \eta = 0 \quad \text{and} \quad F = 1;\ G = 0 \ \text{ as } \ \eta \to \infty \tag{9}$$

for $\xi \geq 0$.

The quantities of physical interest are as follows [4]: The skin friction coefficient and heat transfer coefficient in the form of Nusselt number, can be expressed as

$$C_f = \frac{2\left[\mu \frac{\partial u}{\partial y}\right]_w}{\rho U^2} = 2\left(\mathrm{Re}_x\right)^{-1/2}\left(\frac{m+3}{6}\right)^{1/2}\left(F'\right)_w \tag{10}$$

$$Nu = -\frac{\left[\mu \frac{\partial T}{\partial y}\right]_w}{T_w - T_\infty} = \left(\frac{m+3}{6}\right)^{1/2}\left(\mathrm{Re}_L\right)^{1/2}\left(-G_w'\right) \tag{11}$$

Here F is the dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; $\xi$ and $\eta$ are transformed coordinates; $\psi$ and f are the dimensional and dimensionless stream functions, respectively; Pr is the Prandtl number; $\rho$, $\nu$, $\alpha$ are respectively density, kinetic viscosity and thermal diffusivity; $\mathrm{Re}_x$ is the Reynold's number; $u_\infty$ is the free stream velocity; M is the non-dimensional magnetic parameter. The subscripts e, w and $\infty$ denote conditions at the edge of the boundary layer, on the wall and in the free stream, respectively; the subscripts x, y, $\xi$ denote partial derivatives with respect to x, y, $\xi$ respectively and prime $(')$ denotes derivatives with respect to $\eta$.

### 3. Results and Discussion

The set of coupled partial differential equations (6) and (7) along with the boundary conditions (9) have been solved by an implicit finite difference scheme along with quasilinearization technique [5]. Computations have been carried out for various values of $M\left(0.0 \le M \le 1.0\right)$ for assisting $\left(\lambda = 1.0\right)$and opposing $\left(\lambda = -0.5\right)$ flows. In all numerical computations *m* is taken as 1.0, *Pr* is taken as 0.7 (air) and the edge of the boundary layer $\eta_\infty$ is taken as 5.0. In order to verify the correctness of the procedure, solutions have been obtained for *m* = 0.0 and *M* = 0.0 compared for various Prandtl numbers with those of Hering and Grosh [1], and Himasekhar et al. [2]. The results are found in an excellent agreement and only some of the comparisons are shown in Table 1.

Table 1. Comparison of skin friction and heat transfer parameter results $[f''(0), -G'(0)]$ with those of Hering and Grosh [1], and Himasekhar et al. [2] for $m$ = 0.0 & M = 0.0.

| Pr | λ | Hering and Grosh [1] | | Himasekhar et al. [2] | | Present Results | |
|---|---|---|---|---|---|---|---|
| | | $f''(0)$ | $-G'(0)$ | $f''(0)$ | $-G'(0)$ | $f''(0)$ | $-G'(0)$ |
| 0.7 | 1.0 | 2.2078 | 0.6120 | 2.2012 | 0.6120 | 2.2019 | 0.6125 |
| | 10.0 | 8.5246 | 1.0173 | 8.5041 | 1.0097 | 8.5049 | 1.0105 |
| 1.0 | 1.0 | 2.0881 | 0.6998 | 2.0886 | 0.7010 | 2.0901 | 0.7015 |
| | 10.0 | 7.9420 | 1.1233 | 7.9425 | 1.1230 | 7.9445 | 1.1294 |
| 10.0 | 1.0 | 1.5630 | 1.5657 | 1.5636 | 1.5662 | 1.5658 | 1.5785 |
| | 10.0 | 5.0818 | 2.3574 | 5.0821 | 2.3580 | 5.0861 | 2.3587 |

Fig. 2 shows the effect of magnetic field *(M)* on skin friction $\left[C_f(\mathrm{Re}_x)^{1/2}\right]$ and heat transfer $\left[Nu(\mathrm{Re}_x)^{-1/2}\right]$ coefficients for assisting $(\lambda>0)$ and opposing $(\lambda<0)$ flows. As magnetic parameter increases, it is found that, both $\mathrm{C_f}(\mathrm{Re_x})^{1/2}$ and $\mathrm{Nu}(\mathrm{Re_x})^{-1/2}$ increases in case of assisting as well as opposing flows. However, initially the skin friction and heat transfer coefficients decreases for small values of ξ and then increases with the increasing values of ξ, before attaining a constant value in case of assisting flow [See Fig.2 (a) and 2(b)], whereas it is just opposite in case of opposing flow [See Fig. 2(c) and 2(d)]. This trend is same for all the values of magnetic parameter *M*. The percentage of decrease of $\mathrm{C_f}(\mathrm{Re_x})^{1/2}$ is 74.71% and $\mathrm{Nu}(\mathrm{Re_x})^{-1/2}$ is 1.33% in case of assisting flow and the corresponding percentage of skin friction coefficient is 76.62% and heat transfer coefficient is 1.25% in case of opposing flow at $\xi$ = 2.5 in the range of $M(0.0 \leq M \leq 1.0)$. Easily we can observe that the percentage of increase is more in skin friction coefficient as compared to heat transfer coefficient, since the magnetic parameter is present only in momentum equation.

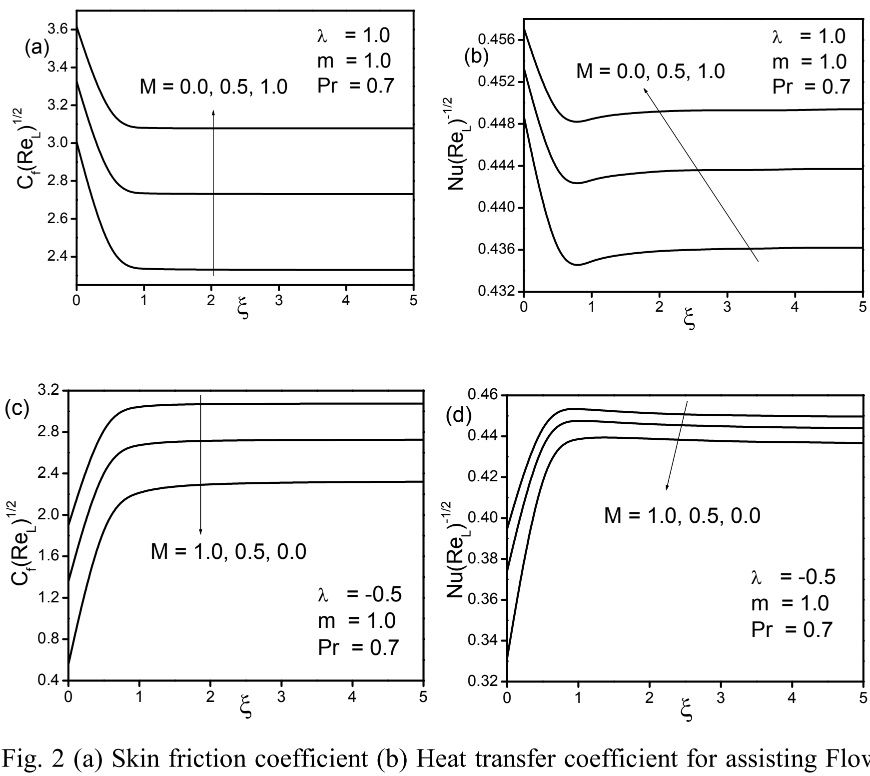


Fig. 2 (a) Skin friction coefficient (b) Heat transfer coefficient for assisting Flow and (c) Skin friction coefficient (d) Heat transfer coefficient for opposing Flow for different values of magnetic parameter (M).

## 4. Conclusions

Non-similar solution of steady mixed convection flow over a vertical cone has been obtained for both assisting and opposing flows with an applied magnetic field. The non-linear two point boundary value problem of partial differential equations has been solved by an implicit finite difference scheme along with quasilinearization technique. It is found that the magnetic parameter enhances the skin friction and heat transfer coefficients in both the flows.

## Acknowledgement

One of the authors (KRJ) expresses his thanks to Principal and the Management of K S School of Engineering and Management, Bangalore for their kind support. The authors are also grateful to PETRF, PES College of Engineering, Mandya–571401 for providing excellent facilities for research.

## References

[1] R G Hering and R J Grosh, Laminar combined convection from a rotating cone, *ASME J. Heat Transfer*, vol. 85, pp. 29-34, 1963.

[2] K Himasekhar, P K Sarma and K Janardhan, Laminar mixed convection from a vertical cone, *Int. Comm. Heat Mass Transfer*, vol. 16, pp. 99-106, 1989.

[3] M Kumari, I Pop and G Nath, Mixed convection along a vertical cone, *Int. Comm. Heat Mass Transfer*, vol. 16, pp. 247–255, 1989.

[4] A Bejan, *Convective Heat Transfer*, Wiley Interscience, 1994.

[5] K Inouye and A Tate, Finite difference version of quasilinearization applied to boundary layer equations. *AIAA J.*, vol. 12, pp. 558-560, 1974.