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Dispersion in a Non-Linear Non-Darcy Flow of a Variable Viscosity Liquid

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Abstract

An infinite horizontally extended sparcely packed chemically inert porous channel flow of a Newtonian liquid is considered. The walls of the channels are assumed to be at different temperatures so that the viscosity of the fluid varies across the channel. A dimensionless variable viscosity coefficient is introduced in the Darcy–Forchheimer- Brinkman model, along with the Darcy number, Forchheimer number and the Brinkman number. A series solution is obtained for the Darcy-Forchheimer-Brinkman equation using the differential transform method (DTM). Using this solution for the fully developed flow velocity and the convective diffusion equation, the influence of the variable viscosity coefficient and the Darcy, Brinkman and Forchheimer numbers on the all-time valid dispersion coefficient is analyzed. An increase the variable viscosity parameter is to increase the dispersion coefficient while an increase in all the other parameters will decrease the dispersion coefficient. The Light hill, Taylor-Aris and Taylor dispersion coefficients are obtained as the limiting cases of the generalized dispersion coefficient.

Keywords: Variable viscosity, non-Darcy flow, porous media, dispersion.

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1. Introduction

The first analytical description of dispersion of a soluble material in a fluid flow was given by Taylor [1] and is applicable at asymptotically long times. Aris [2] extended Taylor's theory to include longitudinal diffusion. Barton [3] resolved certain technical difficulties in the Aris method of moments and obtained a dispersion coefficient that is valid for all times. Gill and Sankarasubramanian [4] proposed the generalized dispersion to study dispersion of passive solutes in Newtonian fluid flows.

Dispersion in non-Darcy flows through a porous medium is of practical importance ([5], [6]). Most of the studies on dispersion ([7], [8]) failed to achieve good correlation with the experimental data ([9], [10]). The primary reason for the poor correlation was because of not taking cognizance of non-Darcy effects, namely, boundary and super-linear drag effects. Using the generalized dispersion model of Gill and Sankarasubramanian [4], Shiva Kumar et al. [11] and Rudraiah et al. [12] studied dispersion using these non-Darcy effects. They have used a moving coordinate system moving with the mean speed of the flow and hence they needed to obtain only the dispersion coefficient. Jagadeesh et al. [13] has studied the effect of the boundary layer viscous shear and the porous parameters on the dispersion of a solute in a porous channel flow, using the all-time approach of Gill and Sankarasubramanian [4]. The bounding walls of the channel are assumed to be of the same temperature .But in most of the real lifetime problems it is observed that the bounding walls of the flow system remains in different temperature causes a temperature gradient across the walls and this causes a variation in the viscosity of the fluid [14]. Ling and Dybbs [15] observed that the viscosity of the water decreases by about 240 percent when temperature increases from $10^0 C$ to $50^0 C$. This viscosity variation of the fluid plays an important role in the dispersion mechanism within the system. In this paper such a flow is considered to analyze the influence of the temperature dependent variable viscosity, the inertia and the friction of porous matrix and the boundary layer viscous shear on the all-time valid dispersion coefficient. We assume a moving coordinate system moving with the mean speed of the flow [13]. We have used the Differential Transform Method (DTM) [16], to obtain a series solution for the Darcy-Forchheimer-Brinkman momentum equation. The convective-diffusion equation is also solved to analyze the influence of Darcy, Brinkman and Forchheimer numbers on the dispersion coefficients.

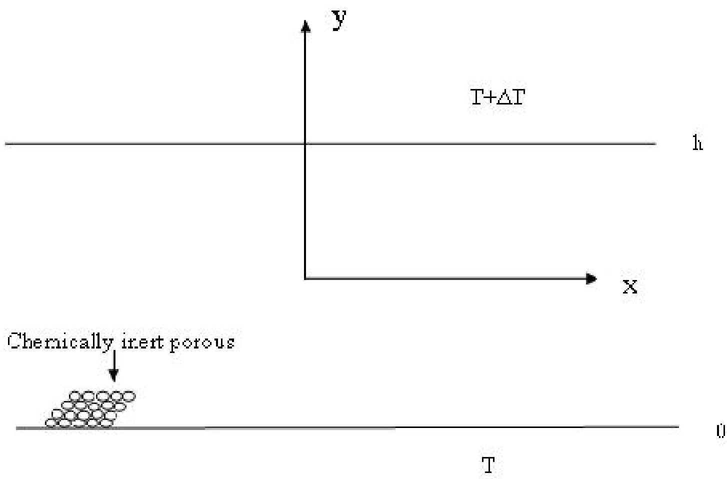

Figure-1: Schematic of the flow diagram

2. Mathematical Formulation

We consider an infinite horizontally extended sparsely packed chemically inert porous medium bounded by solvent-impermeable walls of width h. The walls of the channel are kept at temperatures T_0 and $T_0 + \Delta T$ as shown in figure (1). It is assumed that under fully developed flow conditions with uniform pressure gradient, the velocity of the solvent saturated medium is describable by the Darcy-Forchheimer-Brinkman (DFB) model with temperature dependent viscous coefficient. Hence we use the governing equation

$$\frac{d}{dy}\left[\mu'(T)\frac{du}{dy}\right] - \frac{\mu'(T)}{K}u - \frac{\rho C_b}{\sqrt{K}}u^2 = \frac{dp}{dx} \tag{1}$$

where ρ is the density, $u(y)$ is the axial filter velocity, $p(x)$ is the axial pressure, $\mu(t) = \dfrac{\mu_0}{1+\delta(T-T_0)}$ is the dynamic viscosity, $\mu'(t) = \dfrac{\mu'_0}{1+\delta(T-T_0)}$ is the effective viscosity, δ is the thermo-rheological constant, K is the permeability of the porous medium and C_b is the dimensionless quadratic drag coefficient and μ_0 and μ'_0 are the initial values of $\mu(T)$ and $\mu'(T)$ respectively. In writing equation (1), we have used the "Dupuit-Forchheimer" relation $u = \phi u'$, where ϕ is the porosity, u is the average of fluid velocity over an elemental volume of solid matrix and fluid and the intrinsic velocity u' is the average over the fluid volume only. A slug of initial input concentration c_0 is introduced in the above non-Darcy flow. The concentration $c(t,x,y)$ of the solute in the flow satisfies the convective-diffusion equation

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) \tag{2}$$

where D is the diffusion coefficient. For solving the equations (1) and (2), we use the boundary conditions

$$u = 0 \;\; at \;\; y = \pm h, \tag{3}$$

$$c(0,x,y) = c_0 \quad \text{for} \quad |x| \le \frac{x_s}{2}, \tag{4}$$

$$c(0,x,y) = 0 \quad \text{for} \quad |x| > \frac{x_s}{2}, \tag{5}$$

$$c(t,\infty,y) = 0, \tag{6}$$

$$\frac{\partial c}{\partial y}(t,x,-h) = \frac{\partial c}{\partial y}(t,x,h) = 0\,. \tag{7}$$

Equation (3) is the non slip condition; equation (4) and (5) are the initial concentration of the input slug, equation (6) is due to the zero wall concentration flux at the horizontal extents and the equation (7) is consistent with no mass transfer at the channel walls. We use the following definitions to make equations (1)-(7) dimensionless:

$$\left.\begin{array}{l} \theta = \dfrac{c}{c_0}, \quad \tau = \dfrac{Dt}{h^2}, \quad \xi = \dfrac{x}{hPe}, \quad Y = \dfrac{y}{h} \\[2ex] X_s = \dfrac{x_s}{hPe}, \quad U = \dfrac{u}{\left(-\bar{U}\dfrac{dP}{d\xi}\right)}, \quad P = \dfrac{p}{\left(-\dfrac{\mu\bar{U}}{h}\dfrac{dP}{d\xi}\right)} \end{array}\right\} \tag{8}$$

where $\overline{U} = \int_0^1 U(\eta)d\eta$ is the mean velocity of the flow and $Pe = \dfrac{\overline{U}h}{D}$ is the Peclet number. Using the one dimensional heat flow equation $\dfrac{d^2T}{dy^2} = 0$ and the boundary conditions $T = T_0$ at $y = 0$ and $T = T_0 + \Delta T$ at $y = h$, we will get an expression for the temperature profile across the walls of the channel as $T = T_0 + \dfrac{\Delta T\, y}{h}$ and use this to simplify equation (1). The non dimensional form of this simplified equation and equation (3) using the equations (8) are

$$\frac{d^2U}{dY^2} - \frac{V}{(1+VY)}\frac{dU}{dY} - AU - F(1+VY)U^2 = -\Lambda(1+VY), \tag{9}$$

$$U = 0 \quad \text{at} \quad Y = 0 \quad \text{and} \quad Y = 1, \tag{10}$$

where

$Da = \dfrac{h}{\sqrt{K}}$ is the Darcy number ,

$\Lambda = \dfrac{\mu}{\mu'}$ is the Brinkman number,

$\mathrm{Re} = \dfrac{h\rho\bar{U}}{\mu'}$ is the Reynolds number,

$F = \Lambda\,\mathrm{Re}\,C_b\,Da$ is the Forchheimer number

and $A = \Lambda Da^2$. In the formulation of equation (9) we absorbed the constant pressure gradient into the dimensionless velocity U.

A series solution of equations (9) and (10) using the DTM [16] is given by Jagadeesh et al. [17] as follows:

$$U_6(Y) = a_0 + a_1 Y + a_2 Y^2 + a_3 Y^3 + a_4 Y^4 + a_5 Y^5 \tag{11}$$

where $a_0 = 0$ is the maximum velocity of the flow and

$$a_1 = \alpha\,,\; a_2 = \frac{-1}{2}\left[\Lambda - \alpha V\right], a_3 = \frac{1}{6}\left[A\alpha - 2\Lambda V\right]$$

$$a_4 = \frac{1}{12}\left[(F\alpha^2 + A\alpha V - \Lambda V^2) + \frac{1}{2}A(-\Lambda + \alpha V) - \frac{1}{2}V(A\alpha - 2\Lambda V)\right],$$

$$a_5 = \frac{1}{20}\left[2F\alpha^2 V + F\alpha(-\Lambda + \alpha V) + \frac{1}{2}AV(-\Lambda + \alpha V) + \frac{1}{6}A(A\alpha - 2\Lambda V)\right] +$$
$$\frac{1}{20}\left[-\frac{2}{3}V(F\alpha^2 + A\alpha V - \Lambda V^2) + \frac{1}{2}A((-\Lambda + \alpha V) - \frac{1}{2}V(A\alpha - 2\Lambda V)\right].$$

The convergence criteria for arriving at an approximated six term solution is $|U_{N+1} - U_N| < 10^{-4}$. We now proceed with the mass balance equation for the solute concentration as described by equations (2), (4), (5), (6), (7). Introduce a new axial coordinate x' moving with the average velocity $\overline{U}$ of the flow by defining

$$x' = x - \overline{U}t \tag{12}$$

By defining $X = \dfrac{x'}{hPe}$ and using equation (8), the dimensionless form of equation (12) is

$$X = \xi - t \tag{13}$$

Using equation (8) and equation (13), the non dimensional form of equations (2), (4), (5), (6) and (7) are:

$$\frac{\partial\theta}{\partial\tau} + U^*(Y)\frac{\partial\theta}{\partial X} = \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}, \tag{14}$$

$$\theta(0, X, Y) = 1 \text{ for } |X| \leq \frac{X_s}{2}, \tag{15}$$

$$\theta(0, X, Y) = 0 \text{ for } |X| > \frac{X_s}{2}, \tag{16}$$

$$\theta(\tau, \infty, Y) = 0, \tag{17}$$

$$\frac{\partial\theta}{\partial Y}(\tau, X, -1) = \frac{\partial\theta}{\partial Y}(\tau, X, 1) = 0, \tag{18}$$

where

$$U^*(Y) = \frac{U - \overline{U}}{\overline{U}}, \tag{19}$$

$$\int_0^1 U^*(\eta)\, d\eta = 0 \tag{20}$$

We now proceed to the derivation of the dispersion model.

3. Dispersion Model

Based on an observation made by Taylor [1] Gill and Sankarasubramanian [4] assumed the solution of equation (14) subjected to equations (15)-(18) as a series expansion in $\frac{\partial k \theta_m}{\partial X^k}$ such that

$$\theta = \theta_m(\tau, X) + \sum_{k=1}^{\infty} f_k(\tau, Y) \frac{\partial^k \theta_m}{\partial X^k}, \tag{21}$$

where

$$\theta_m = \int_0^1 \theta dY. \tag{22}$$

We now introduce the general dispersion model of Gill and Sankarasubramanian with time dependent dispersion coefficient as:

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial X^i}. \tag{23}$$

Substituting (21) in (14) and using equation (23) and its derivatives with respect to X in the equation obtained and equating the coefficient of $\frac{\partial^i \theta_m}{\partial X^i}$ $(i = 1,2,3...)$ to 0, we will get an infinite set of differential equation satisfied by f_i (i=1,2,3…) of which the first two follows:

$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial Y^2} - U^*(Y) - K_1(\tau), \tag{24}$$

$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial Y^2} - U^*(Y) f_1 - K_2(\tau) + \frac{1}{Pe^2}. \tag{25}$$

Now θ_m will be chosen to satisfy the initial condition on θ given by equation (17) and hence the conditions on $f_k(\tau, Y)$ $((f_0 = 1)$ are

$$f_k(0,Y) = 0, \quad \frac{\partial f_k}{\partial Y} = 0 \text{ at } Y = 0 \text{ and } Y = 1, \tag{26}$$

$$\int_0^1 f_k \, dY = 0, \quad k = 1, 2. \tag{27}$$

Integrating (24) with respect to Y from 0 to 1 and using equations (20), (26) and (27), we get $K_1(\tau) = 0$ and hence equation (24) becomes

$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial Y^2} - U^*(Y).$$

To solve the above equation, we have substituted the value of U^* from equation (19) and assumed that

$$f_1(\tau, Y) = f_{10}(Y) + f_{11}(\tau, Y)$$

where $f_{10}(Y)$ corresponds to an infinitely wide slug which is independent of τ. The boundary conditions on f_{10} and f_{11} will follow from equations (26) and (27). Equating the respective terms, we will have an ordinary differential equation satisfied by $f_{10}(Y)$ and a partial differential equation satisfied by $f_{11}(\tau, Y)$. The solution of the former will get by successive integration and the solution of the latter one will get by the method of separation of variables.

To find the time dependent diffusion coefficient, we now proceed to the solution of equation (25) by integrating the same with respect to Y from −1 to 1 and using equations (26) and (27). The expression obtained for $K_2(\tau)$ is

$$K_2(\tau) = \frac{1}{Pe^2} - \int_0^1 U^*(Y) f_1 dY. \tag{28}$$

Having the expressions for f_1 and U (and hence U^*) as a series in equation (28) a direct integration will lead to an expression for the dispersion coefficient as a function of time as given in the appendix.

4. Results and Discussion

In this paper we analyses the dispersion of solute in a non linear, non-Darcy flow through sparsely packed chemically inert porous medium with temperature dependent variable viscosity. A series solution obtained for the Darcy-Forchheimer-Brinkman momentum equation is used to analyze the dispersion. As we consider a nonreactive flow and used a moving coordinate moving with the average velocity of the flow, the dispersion is described by $K_2(\tau)$. Jagadeesh et al. [17] observed that the porous parameters namely the Darcy number and the Forchheimer number inhibits the flow while the Brinkman parameter and the variable viscosity parameter will enhance the flow. In view of this observation we now proceed to discuss the results on dispersion.

Figures (2) - (4) gives the plots of $K_2(\tau) - Pe^{-2}$ against dimensionless time for various values of Da, Λ and F respectively and it is seen that for small values of V and for small values of Da, Λ and F, $K_2(\tau)$ is essentially a constant after $\tau = 0.2$. Beyond this value of τ, the Taylor-Aris theory applies to the slugs which were originally uniformly distributed across the stream at $\tau = 0$. The dispersion coefficient varies rapidly with τ in the region $0 \le \tau \le 1$ (approximately) and then it changes more slowly until $K_2(\tau)$ becomes constant after $\tau = 0.2$. For higher values of V and small values of Da, Λ and F, $K_2(\tau)$ increases rapidly initially up to 0.2 (approximately) and there after increases slowly and become constant after $\tau = 0.5$. It is evident from all these plots that the dispersion coefficient as well as the interval of rapid variation increase with increase in V.

Figure (4) shows that $K_2(\tau)$ decreases as Da increases. This is due to the decrease in the permeability of the porous medium due to the increase in Da. Figure (5) shows that $K_2(\tau)$ decreases as Λ increases. This can be interpreted as follows. An increase in Λ is caused by the decrease in the effective viscosity which means that the boundary effect is less in the case of large values of Λ. This causes a decrease in the axial dispersion and hence in the dispersion coefficient. Figure (6) shows that the dispersion coefficient $K_2(\tau)$ decrease with the increase of Forchheimer number. Thus the boundary layer friction, the friction and inertia of the porous matrix has an adverse effect on dispersion coefficient. A limiting case of this when $V = 0$ is discussed in Jagadeesh et al. (2012).

5. Conclusions

The results of the study can be concluded as follows. An increase the variable viscosity parameter is to increase the dispersion coefficient while an increase in all the other parameters viz., Da, Λ and F will decrease the dispersion coefficient. For small values of V and for small values of Da, Λ and F $K_2(\tau)$ is a constant beyond $\tau = 0.2$. The dispersion coefficient varies rapidly with τ in the region $0 \le \tau \le 1$ (approximately) and then it changes more slowly until $K_2(\tau)$ becomes constant after $\tau = 0.2$. For higher values of V and small values of Da, Λ and F, $K_2(\tau)$ increases rapidly initially up to 0.2 (approximately) and there after increases slowly and become constant after $\tau = 0.5$.

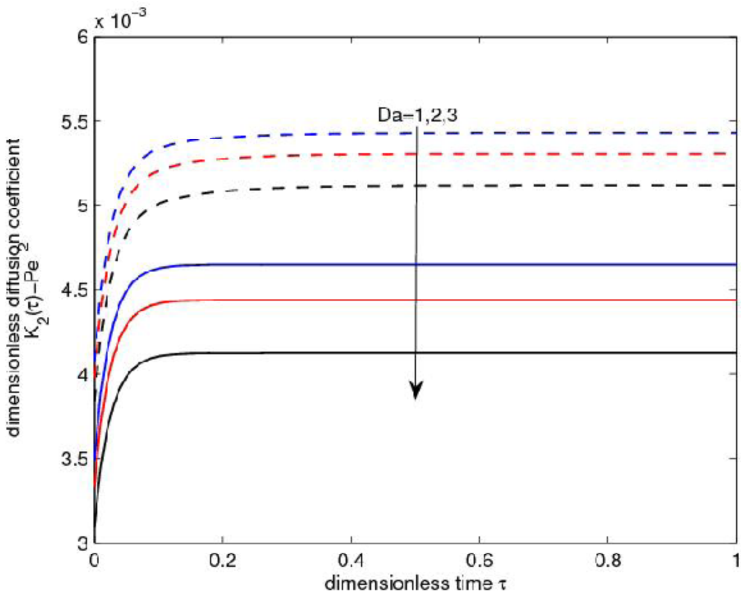

Figure 2: Plots of dimensionless dispersion coefficient $K_2(\tau) - Pe^{2}$ against dimensionless time τ for various values of Da, with Pe = 1000, V = 0(-), V = 1(- -), Λ = 1 and F = 3.

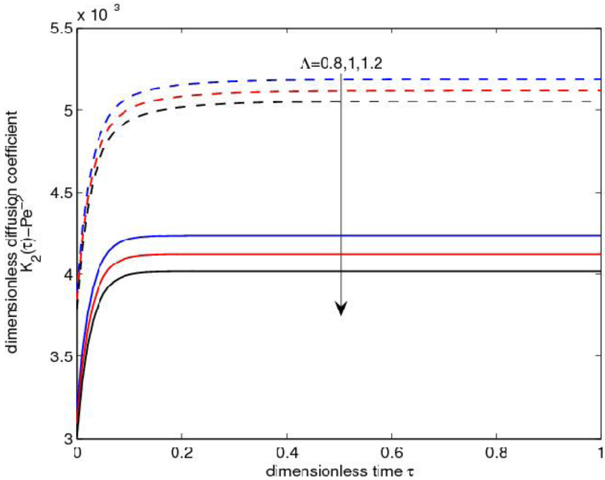

Figure 3: Plots of dimensionless dispersion coefficient $K_2(\tau) - Pe^{-2}$ against dimensionless time τ for various values of Λ with Pe = 1000, V = 0 (-), V = 1(- -), Da = 3 and $F = 3$.

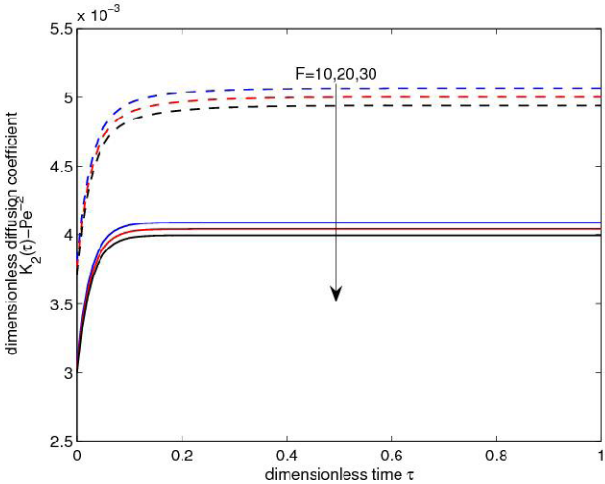

Figure 4: Plots of dimensionless dispersion coefficient $K_2(\tau) - Pe^{-2}$ against dimensionless time τ for various values of F with Pe = 1000, V = 0 (-), V = 1(- -), Da = 3 and $\Lambda = 1$.

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Appendix

$$f_1 = \frac{6F\alpha^2 - \Lambda(49+3A+28V) + \alpha(105+14A+49V+6AV)}{21B} +$$

$$\frac{Y^2(-2F\alpha^2 + \Lambda(20+A+10V) - \alpha(60+5A+20V+2AV))}{2B} -$$

$$\frac{20Y^2\alpha}{B} + \frac{5Y^4(\Lambda - \alpha V)}{B} + \frac{Y^5(A\alpha - 2\Lambda V)}{B} + \frac{Y^6(2F\alpha^2 - A\Lambda + 2A\alpha V)}{6B} +$$

$$\sum_{n=1}^{\infty} A_n e^{-\lambda_n \tau} Cos(\lambda_n Y)]$$

$$K_2(\tau) = \frac{1}{Pe^2} - \frac{1}{693B^2}(-38F^2\alpha^4 - 2F\alpha^3(5445 + 2816V + 6A(143 + 64V)) -$$
$$2\Lambda^2(48A^2 + 22A(64 + 39V) + 55(192 + 231V + 70V^2)) +$$
$$2\alpha\Lambda(3A^2(143 + 64V) + 165(252 + 278V + 77V^2) +$$
$$11A(825 + 734V + 156V^2)) - \alpha^2(A^2(1925 + 1716V + 384V^2) +$$
$$A(24750 - 384F\Lambda + 23595V + 5632V^2) -$$
$$88(F\Lambda(64 + 39V) - 15(63 + 63V + 16V^2)))) + \sum_{n=1}^{\infty} A_n B_n e^{-\lambda_n \tau}$$

where

$$A_n = \frac{-1}{2B\lambda_n^6}(-120A\alpha + 120\lambda_n{}^2\alpha + 240\Lambda V + 120A\alpha\, Cos\lambda_n + 240F\alpha^2\, Cos\lambda_n -$$
$$120\lambda_n{}^2\alpha Cos\lambda_n - 60\lambda_n{}^2 A\alpha\, Cos\lambda_n - 40\lambda_n{}^2 F\alpha^2 Cos\lambda_n - 120A\Lambda Cos\lambda_n +$$
$$120\lambda_n{}^2\Lambda Cos\lambda_n + 20\lambda_n{}^2 A\Lambda Cos\lambda_n + 240A\alpha V Cos\lambda_n - 120\lambda_n{}^2\alpha V Cos\lambda_n -$$
$$-40\lambda_n{}^2 A\alpha V Cos\lambda_n - 240\Lambda V Cos\lambda_n)$$

$$B = 2F\alpha^2 - \Lambda(A + 10(2 + V)) + \alpha(20(3 + V) + A(5 + 2V))$$

$$B_n = -\frac{-Cos\lambda_n}{6B}(4F\alpha^2 - 2\Lambda(15 + A + 9V) + \alpha(30(2 + V) + A(9 + 4V))$$