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Effect of Non-Uniform Temperature Gradient on the Onset of Rayleigh–Bénard–Magnetoconvection in Micropolar Fluid with Maxwell–Cattaneo Law

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Abstract

The effect of non-uniform temperature gradient on the onset of Rayleigh-Bénard magnetoconvection in a Micropolar fluid with Maxwell-Cattaneo law is studied using the Galerkin technique. The eigenvalue is obtained for free-free, rigid-free and rigid-rigid velocity boundary combinations with isothermal condition on the spin-vanishing boundaries. A linear stability analysis is performed. The influence of various parameters on the onset of convection has been analyzed. One linear and five non-linear temperature profiles are considered and their comparative influence on onset of convection is discussed. The classical approach predicts an infinite speed for the propagation of heat. The present non-classical theory involves a wave type heat transport (Second Sound) and does not suffer from the physically unacceptable drawback of infinite heat propagation speed.

Keywords: Rayleigh-Bénard Convection, Non-uniform basic temperature, magnetic field, Maxwell-Cattaneo law and Galerkin technique.

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1. Introduction

The instability of Rayleigh-Bénard convection is due to the effect of thermal buoyancy. Theoretical studies of the onset of convection in classical viscous fluids with non-uniform heating have been made by Currie [1] with isothermal boundaries and by Nield [2] with adiabatic boundaries and showed that in the case of piecewise linear temperature profile the onset of convection could occur at a smaller Rayleigh number than of uniform heating or cooling. The non-uniform temperature gradient finds its origin in the transient heating or cooling at the boundaries and as a result the basic temperature profile depends explicitly on position and time. This has to be determined by solving the coupled momentum and energy equations. This coupling makes the problem very complicated. In the present study, therefore, we adopt a series of temperature profiles based on a simplification in the form of a quasi – static approximation (Currie [1], Lebon and Cloot [3]) that consists of freezing the temperature distribution at a given instant of time. In this method, we assume that the perturbation grows much faster than the initial state and hence freeze the initial state into some spatial distribution. This hypothesis is sufficient for our purpose because we are interested only in finding the conditions for the onset of convection. Even with these simplifications, the solutions to the variable-coefficients stability equation pose a problem because the temperature gradient varies with depth.

Convection in Micropolar fluid has been the subject of intensive study because of the remarkable physical properties of the fluid as well as its practical applications (see Power [4], Lukaszewicz [5] and Eringen [6]). Rayleigh-Bénard/Marangoni convection in Micropolar fluid with and without non-uniform temperature gradient has been investigated by many authors (Datta and Sastry [7], Bhattacharya and Jena [8], Siddheshwar and Pranesh [9, 10, 11, 12, 13, and 14] and Pranesh and Riya Baby [15]). The main results from all these studies are that for heating from below stationary convection is the preferred mode. All the above reported works are with classical Fourier heat flux law.

A well known consequence of Classical Fourier heat conduction law is that heat perturbations propagate with an infinite velocity.

This drawback of the classical law motivated Maxwell [16], Cattaneo [17], Lindsay and Stranghan [18], Straughan and Franchi [19], Pranesh and Kiran [20] and Pranesh and Smita [21] to adopt a non-classical heat flux Maxwell-Cattaneo law in studying Rayleigh-Bénard / Marangoni convection to get rid of this unphysical results. This Maxwell-Cattaneo equation contains an extra inertial term with respect to the Fourier law

$$\tau \frac{d\vec{Q}}{dt} + \vec{Q} = -\kappa \nabla T$$

where, $\vec{Q}$ is the heat flux, τ is a relaxation time and κ is the heat conductivity. This heat conductivity equation and the conservation of energy equation introduce the hyperbolic equation, which describes heat propagation with finite speed. Puri and Jordan [22, 23], Puri and Kythe [24, 25] and Straughan [26] have studied other fluid mechanics problems by employing the Maxwell-Cattaneo heat flux law.

The objective of this paper is to replace the classical parabolic heat equation by non-classical Maxwell-Cattaneo Law and study the effect of non-uniform basic temperature gradients on the onset of Rayleigh-Bénard magnetoconvection in Micropolar fluids.

2. Mathematical Formulation

Consider an infinite horizontal layer of a Boussinesquian, electrically conducting fluid, with non-magnetic suspended particle of depth ‘d’ permeated by an externally applied uniform magnetic field normal to the fluid layer. Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards. Let ΔT be the temperature difference between the upper and lower boundaries. (See Figure (1)).

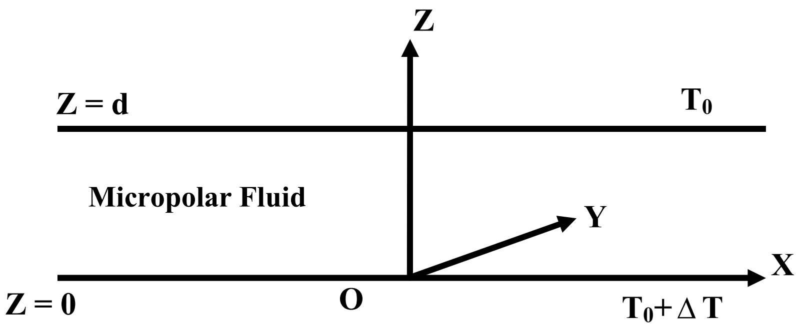


Figure 1. Schematic diagram of the Rayleigh-Bénard situation for a Micropolar fluid.

The governing equations for the Rayleigh-Bénard situation in a Boussinesquian fluid with suspended particles are

Continuity equation

$$\nabla.\vec{q} = 0, \tag{1}$$

Conservation of linear momentum

$$\rho_o\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g\hat{k} + (2\zeta + \eta)\nabla^2\vec{q} + \zeta\nabla\times\vec{\omega} + \mu_m(\vec{H}.\nabla)\vec{H}, \tag{2}$$

Conservation of angular momentum

$$\rho_o I\left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + (\eta'\nabla^2\vec{\omega}) + \zeta(\nabla\times\vec{q} - 2\vec{\omega}), \tag{3}$$

Conservation of energy

$$\frac{\partial T}{\partial t} + \left(\vec{q} - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}\right).\nabla T = -\nabla.\vec{Q}, \tag{4}$$

Maxwell – Cattaneo heat flux law

$$\tau\left[\dot{\vec{Q}} + \vec{\omega}_1 \times \vec{Q}\right] = -\vec{Q} - \kappa\nabla T, \tag{5}$$

Magnetic Induction equation

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = (\vec{H}.\nabla)\vec{q} + \gamma_m \nabla^2 \vec{H}, \tag{6}$$

Equation of state

$$\rho = \rho_o[1 - \alpha(T - T_o)]. \tag{7}$$

where, $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, P is the hydromagnetic pressure, ρ is the density, ρ_o is the density of the fluid at reference temperature $T = T_o$, $\gamma_m = \frac{1}{\mu_m \sigma}$, μ_m is magnetic permeability, g is the acceleration due to gravity, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are the bulk and shear spin viscosity coefficient, β is the Micropolar heat conduction coefficient, C_v is the specific heat, κ is the thermal conductivity, α is the co – efficient of thermal expansion, $\vec{\omega}_1 = \frac{1}{2}\nabla \times \vec{q}$, $\vec{Q}$ is the heat flux vector and τ is the constant relaxation time.

3. Basic State

The basic state of the fluid being quiescent is described by

$$\left.\begin{array}{l} \vec{q}_b = 0,\ \vec{\omega}_b = 0,\ P = P_b(z),\ \rho = \rho_b(z),\ \vec{\omega} = \vec{\omega}_b(z), \\ \vec{H} = H_0\hat{k},\ \vec{Q} = \left(0, 0, Q_b(z)\right),\ \dfrac{dT_b}{dz} = \dfrac{-\Delta T}{d} f(z). \end{array}\right\} \tag{8}$$

The monotonic, non-dimensional basic temperature gradient $f(z)$which is non-negative satisfies the condition $\int_0^1 f(z)\,dz = 1$. We have considered various steady state temperature gradients in this paper and these are defined below.

Table (1): Basic-State Temperature Gradients

Model	Basic temperature gradients	f(z)
1.	Linear	1
2.	Heating from below	$\begin{cases} \varepsilon^{-1} & 0 \le z < \varepsilon \\ 0 & \varepsilon < z \le 1 \end{cases}$
3.	Cooling from above	$\begin{cases} 0 & 0 \le z < 1-\varepsilon \\ \varepsilon^{-1} & 1-\varepsilon < z \le 1 \end{cases}$
4.	Step function	$\delta(z-\varepsilon)$
5.	Inverted Parabolic	$2(1-z)$
6.	Parabolic	$2z$

Equations (2), (4), (5) and (7) in the basic state specified by equation (8) respectively become

$$\left.\begin{aligned} &\frac{dP_b}{dz} = -\rho_o g\hat{k}, \quad \frac{d\vec{Q}_b}{dz} = 0, \quad \vec{Q}_b = -\kappa\frac{dT_b}{dz}, \\ &\rho_b = \rho_o[1-\alpha(T_b - T_o)], \quad \frac{d^2T_b}{dz^2} = 0. \end{aligned}\right\} \tag{9}$$

4. Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}', \ \vec{\omega} = \vec{\omega}_b + \vec{\omega}', \ P = P_b + P', \ \vec{Q} = \vec{Q}_b + \vec{Q}', \\ &\rho = \rho_b + \rho', \ T = T_b + T', \ \vec{H} = \vec{H}_0 + \vec{H}'. \end{aligned}\right\} \tag{10}$$

The primes indicate that the quantities are infinitesimal perturbations and subscript 'b' indicates basic state value.

Substituting equation (10) into equations (1) – (7) and using the basic state (9), we get linearised equation governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q}' = 0, \tag{11}$$

$$\begin{aligned}\rho_o\left[\frac{\partial \vec{q}'}{\partial t}\right] &= -\nabla P' - \rho' g\hat{k} + (2\zeta + \eta)\nabla^2\vec{q}' \\ &+ (\zeta\nabla\times\vec{\omega}') + \mu_m(H_0\hat{k}.\nabla)\vec{H}',\end{aligned} \tag{12}$$

$$\rho_o I\left[\frac{\partial \vec{\omega}'}{\partial t}\right] = (\lambda' + \eta')\nabla(\nabla\vec{\omega}') + (\eta'\nabla^2\vec{\omega}') + \zeta(\nabla\times\vec{q}' - 2\vec{\omega}'), \tag{13}$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d} f(z)\left[\vec{q}' - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}'\right] - \nabla.\vec{Q}', \tag{14}$$

$$\left[1 + \tau\frac{\partial}{\partial t}\right]\vec{Q}' = -\frac{1}{2}\chi_1\frac{\Delta T}{d}\left(\frac{\partial \vec{q}'}{\partial z} - \nabla W'\right) - \kappa\,\nabla T', \tag{15}$$

$$\frac{\partial \vec{H}'}{\partial t} = \gamma_m\nabla^2\vec{H}' + H_0\frac{\partial \vec{q}'}{\partial z}, \tag{16}$$

$$\rho' = -\alpha\rho_o T'. \tag{17}$$

where $\chi_1 = \tau\kappa$.

Operating divergence on the equation (15) and substituting in equation (14), on using equation (11), we get

$$\begin{aligned}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial T'}{\partial t} &= \left(1+\tau\frac{\partial}{\partial t}\right)\frac{\Delta T}{d}f(z)\left(W' - \frac{\beta}{\rho_0 C_v}\Omega_z\right) \\ &+ \kappa\nabla^2 T' - \frac{1}{2}\chi_1\frac{\Delta T}{d}f(z)\left(\nabla^2 W'\right),\end{aligned} \tag{18}$$

where $\Omega = \nabla\times\vec{\omega}'$.

The perturbation equation (12), (13), (16) and (18) are non – dimensionalised using the following definition:

$$\left.\begin{aligned}&(x^*,y^*,z^*)=\frac{(x,y,z)}{d},\ W^*=\frac{W'}{\left(\kappa/d\right)},\ \vec{\omega}^*=\frac{\omega'}{\left(\kappa/d^2\right)},\ t^*=\frac{t}{\left(d^2/\kappa\right)},\\ &T^*=\frac{T'}{\Delta T},\ \Omega^*=\frac{\Omega_z}{\left(\kappa/d^3\right)},\ H^*=\frac{H'}{H_0}.\end{aligned}\right\}\tag{19}$$

Using equation (17) in (12), operating curl twice on the resulting equation, operating curl once on equation (13) and non-dimensionalising the two resulting equation and also equations (16) and (18), we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W)=R\nabla_1^2 T+(1+N_1)\nabla^4 W+N_1\nabla^2\Omega_z+Q\frac{Pr}{Pm}\nabla^2\left(\frac{\partial H_z}{\partial z}\right),\tag{20}$$

$$\frac{N_2}{Pr}\frac{\partial}{\partial t}(\Omega_z)=N_3\nabla^2\Omega_z-N_1\nabla^2 W-2N_1\Omega_z,\tag{21}$$

$$\frac{\partial H_z}{\partial t}=\frac{\partial W}{\partial z}+\frac{Pr}{Pm}\nabla^2 H_z,\tag{22}$$

$$\begin{aligned}\left(1+2C\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=\left(1+2C\frac{\partial}{\partial t}\right)f(z)W-\left(1+2C\frac{\partial}{\partial t}\right)N_5\,f(z)\Omega_z\\ +\nabla^2 T-C\,f(z)\nabla^2 W,\end{aligned}\tag{23}$$

where the asterisks have been dropped for simplicity and the non-dimensional parameters N_1, N_3, N_5, R, Q, Pr, Pm and C are as defined as

$$N_1=\frac{\zeta}{\zeta+\eta}\qquad\text{(coupling parameter)},$$

$$N_3=\frac{\eta'}{(\zeta+\eta)d^2}\qquad\text{(couple stress parameter)},$$

$$N_5=\frac{\beta}{\rho_o C_v d^2}\qquad\text{(micropolar heat conduction parameter)},$$

$$R = \frac{\rho_o \alpha g \Delta T d^3}{\kappa(\zeta+\eta)} \quad \text{(Rayleigh number)},$$

$$Q = \frac{\mu_m \vec{H}_o d^2}{\gamma_m(\zeta+\eta)} \quad \text{(Chandrasekhar Number)},$$

$$\Pr = \frac{\zeta+\eta}{\rho_o \kappa} \quad \text{(Prandtl Number)},$$

$$Pm = \frac{\zeta+\eta}{\gamma_m} \quad \text{(Magnetic Prandtl Number)},$$

$$C = \frac{\tau\kappa}{2d^2} \quad \text{(Cattaneo number)}.$$

The infinitesimal perturbation W, Ω_z, H_z and T are assumed to be periodic waves and hence these permit a normal mode solution in the form

$$\begin{bmatrix} W \\ \Omega_z \\ H_z \\ T \end{bmatrix} = \begin{bmatrix} W(z)e^{i(lx+my)} \\ G(z)e^{i(lx+my)} \\ H_z(z)e^{i(lx+my)} \\ T(z)e^{i(lx+my)} \end{bmatrix} \tag{24}$$

where, l and m are horizontal components of the wave number $\vec{a}$.

Substituting equation (24) into equations (20)-(23), we get

$$(1+N_1)(D^2-a^2)^2 W + N_1(D^2-a^2)G - Ra^2 T + Q\frac{\Pr}{Pm}(D^2-a^2)(DH_z) = 0, \tag{25}$$

$$2N_1 G - N_3(D^2-a^2)G + N_1(D^2-a^2)W = 0, \tag{26}$$

$$DW + \frac{\Pr}{Pm}(D^2-a^2)H_z = 0, \tag{27}$$

$$f(z)(W - N_5 G) + (D^2-a^2)T - C\,f(z)(D^2-a^2)W = 0. \tag{28}$$

where $D = \frac{d}{dz}$.

Eliminating H_z between equations (25) and (27), we get

$$(1+N_1)(D^2-a^2)^2W+N_1(D^2-a^2)G-Ra^2T-QD^2W=0, \qquad (29)$$

We now apply the single-term Galerkin method to equations (26), (28) and (29) that gives general results on the eigen value of the problem for various basic temperature gradients using simple, polynomial, trial functions for the lowest eigen value. Now we multiplying equation (29) by W, equation (26) by G and equation (28) by T, integrating the resulting equation by parts with respect to z from 0 to 1 and taking $W = AW_1$, $G = BG_1$ and $T = ET_1$ in which A, B and E are constants with W_1, G_1 and T_1 are trial functions. This procedure yields the following equation for the Rayleigh number R.

$$R=\frac{\left\langle T_1(D^2-a^2)T_1\right\rangle\left[Y_1Y_2+N_1^2Y_3\right]}{a^2\left\langle W_1T_1\right\rangle Y_4} \qquad (30)$$

where,

$$Y_1=(1+N_1)\left\langle W_1(D^2-a^2)^2W_1\right\rangle-Q\left\langle W_1D^2W_1\right\rangle,$$

$$Y_2=N_3\left\langle G_1(D^2-a^2)G_1\right\rangle-2N_1\left\langle G_1^2\right\rangle,$$

$$Y_3=\left\langle G_1(D^2-a^2)W_1\right\rangle\left\langle W_1(D^2-a^2)G_1\right\rangle,$$

$$Y_4=N_1N_5\left\langle G_1(D^2-a^2)W_1\right\rangle\left\langle f(z)T_1G_1\right\rangle$$
$$+Y_2\left[C\left\langle T_1\,f(z)(D^2-a^2)W_1\right\rangle-\left\langle f(z)T_1W_1\right\rangle\right].$$

In the equation (30), $\langle ---\rangle$ denotes integration with respect to z between $z=0$ and $z=1$. We note here that R in equation (30) is a functional and the Euler – Lagrange equations for the extremisation of R are equations (26), (28) and (29).

The value of critical Rayleigh number depends on the boundaries. In this paper we consider the following boundary combinations:

1. Free-free isothermal, no-spin
$$W = D^2W = T = G = 0, \quad \text{at} \quad z = 0, 1.$$
2. Rigid-rigid isothermal, no-spin
$$W = DW = T = G = 0, \quad \text{at} \quad z = 0, 1.$$
3. Rigid-free isothermal, no-spin
$$W = DW = T = G = 0 \quad \text{at} \quad z = 0,$$
$$W = D^2W = T = G = 0 \quad \text{at} \quad z = 1.$$

Trial functions satisfying the boundary conditions are given below.

Free-free condition $W_1 = z^4 - 2z^3 + z$,

Rigid-rigid condition $W_1 = z^4 - 2z^3 + z^2$,

Rigid-free condition $W_1 = 2z^4 - 5z^3 + 3z^2$,

Isothermal condition $T_1 = z(1-z)$,

no-spin condition $G_1 = z(1-z)$.

5. Results and Discussion

In this paper, we study the classical Rayleigh-Bénard magnetoconvection in Micropolar fluids in presence of non-uniform temperature gradients by replacing the classical Fourier heat flux law by a non-classical Maxwell-Cattaneo heat flux law. Keeping in mind the laboratory and geophysical problem, the following types of boundaries have been investigated:

(i) Free-free isothermal, no-spin condition,

(ii) Rigid-rigid isothermal, no-spin condition and

(iii) Rigid-free isothermal, no-spin condition.

One uniform and five non-uniform basic temperature gradients are chosen for study. We find that $R_{C1} = R_{C5} = R_{C6} \neq R_{C4}$ and $R_{C2} = R_{C3} \neq R_{C4}$ for the symmetric boundary combination. On the basis of this following grouping of non-uniform temperature profile can be made.

Group 1	Group 2	Group 3
Linear (R_{C1}) Inverted parabolic (R_{C5}) Parabolic (R_{C6})	Piecewise linear heating from below (R_{C2}) Piecewise linear cooling from above (R_{C3})	Step function (R_{C4})

R_{Ci} (i=1 to 6) in the table are the critical Rayleigh numbers corresponding to the six basic temperature gradients. In the case of rigid-free boundaries (non-symmetric boundary combinations) no two R_{Ci} are the same. In the non-symmetric case we find that,

$$R_{C4} < R_{C3} < R_{C2} < R_{C6} < R_{C1} < R_{C5}$$

For symmetric/non-symmetric boundaries we find that the step function is the most destabilizing basic temperature and inverted parabolic is the most stabilizing basic temperature distribution.

In the case of piecewise linear and step function profiles, the critical Rayleigh number R_C depends on the thermal depth ε_c, in addition to depending on the parameters of the problem.

Table (2): Isothermal: $N_1 = 0.1$, $N_3 = 2.0$, $N_5 = 1.5$, $C = 0.01$, $Q = 10$.

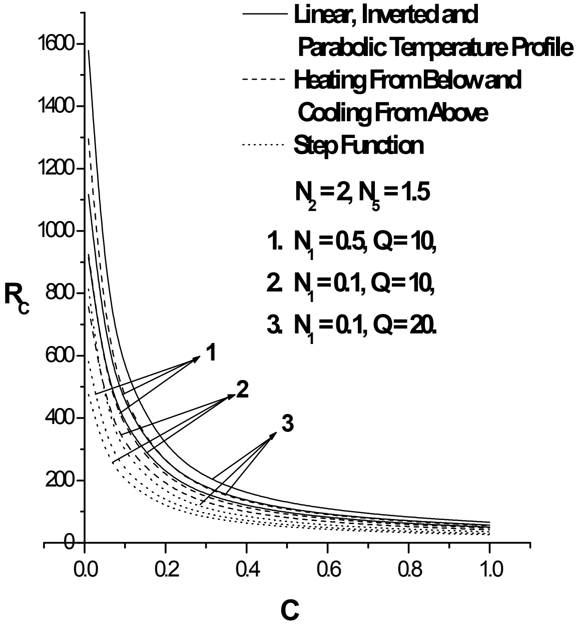
Boundary	**Free-Free**		**Rigid-Rigid**		**Rigid-Free**	
Profile	ε_c	Ratio of R_{Ci}	ε_c	Ratio of R_{Ci}	ε_c	Ratio of R_{Ci}
Heating from below	0.72	$R_{C2}=R_{C1}/1.1364$	0.70	$R_{C2}=R_{C1}/1.3498$	0.76	$R_{C2}=R_{C1}/1.1606$
Cooling from above	0.72	$R_{C3}=R_{C1}/1.1364$	0.70	$R_{C3}=R_{C1}/1.3498$	0.65	$R_{C3}=R_{C1}/1.3416$
Step function	0.52	$R_{C4}=R_{C1}/1.9221$	0.50	$R_{C4}=R_{C1}/2.3350$	0.54	$R_{C4}=R_{C1}/2.1317$
Inverted Parabolic	--	$R_{C5}=R_{C1}$	--	$R_{C5}=R_{C1}$	--	$R_{C5}=R_{C1}/0.8957$
Parabolic	--	$R_{C6}=R_{C1}$	--	$R_{C6}=R_{C1}$	--	$R_{C6}=R_{C1}/1.1042$

Tables (2) provide information on the critical thermal depth ε_c that yields the critical eigenvalues for different boundary combinations in respect of all relevant basic temperature gradients. Figures (2)-(4) are the plot of critical Rayleigh number R_C versus Cattaneo number C, for different values of (a) coupling parameter N_1, (b) couples stress parameter N_3, (c) Micropolar heat condition parameter N_5 and for different Chandrasekhar number Q and different basic temperature gradient for free-free, rigid-rigid and rigid-free boundaries respectively.

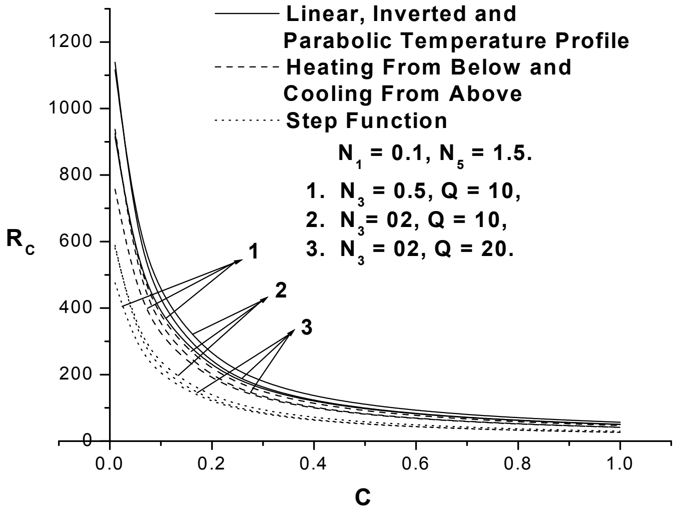
From these figures following observation are made:

1. As C increases R_C decreases, C is the scaled relaxation time and it accelerates the onset of convection. Increase in C leads to narrowing of the convection cells and thus lowering of the critical Rayleigh number. It is also observed from the figures that influence of C is dominant for small values because the convection cells have fixed aspect ratio.
2. The increase in N_1 increases R_C. Increase in N_1 indicates the increase in the concentration of microelements. These elements consume the greater part of the energy in forming the gyrational velocities and as a result the onset of convection is delayed. From these we conclude that increase N_1 stabilize the system.
3. As N_3 increases R_C decreases, because when N_3 increases the couple stress of the fluid increases, which causes the microroation to decrease. Therefore, increase in N_3 destabilizes the system.
4. When N_5 increases the heat induced in to the fluid due to these microelements also increases, thus reducing the heat transfers from bottom to top. The decrease in heat transfer is responsible for delaying onset of instability. Therefore, increase in N_5 increase R_C and thereby stabilizes the system.
5. Increase in Q increases the R_C. When the magnetic field strength permeating the medium is considerably strong, it induces viscosity into the fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder

the growth of disturbances, leading to the delay in the onset of instability. However, the viscosity produced by the magnetic field lessens the rotation of the fluid particles.



(a)



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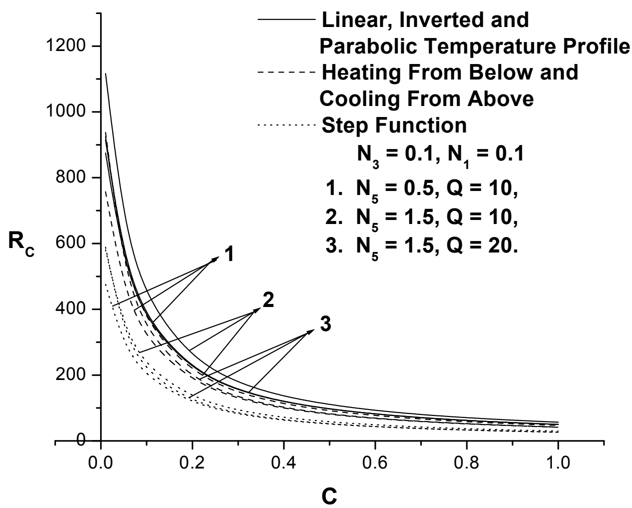
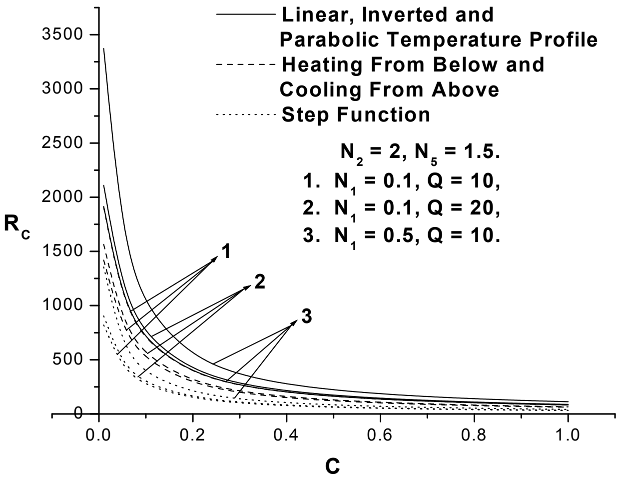


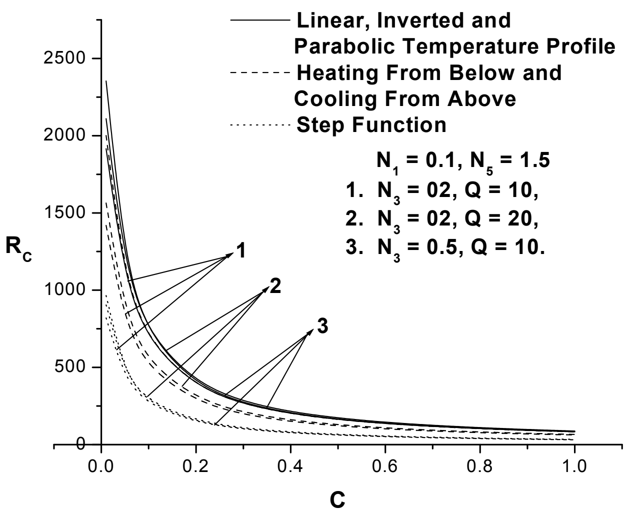
(c)

Figure 2. Plot of critical Rayleigh number R_C Vs. Cattaneo number C with respect to free-free isothermal no-spin boundary condition for different values of Q for (a) N_1, (b) N_3, (c) N_5 and for different non-uniform basic temperature gradients.



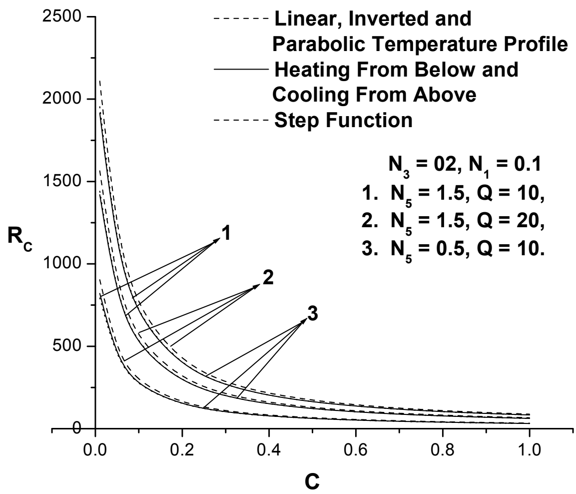
Linear, Inverted and Parabolic Temperature Profile
Heating From Below and Cooling From Above
Step Function
N2 = 2, N5 = 1.5.
1. N1 = 0.1, Q = 10,
2. N1 = 0.1, Q = 20,
3. N1 = 0.5, Q = 10.
Rc
C

(a)



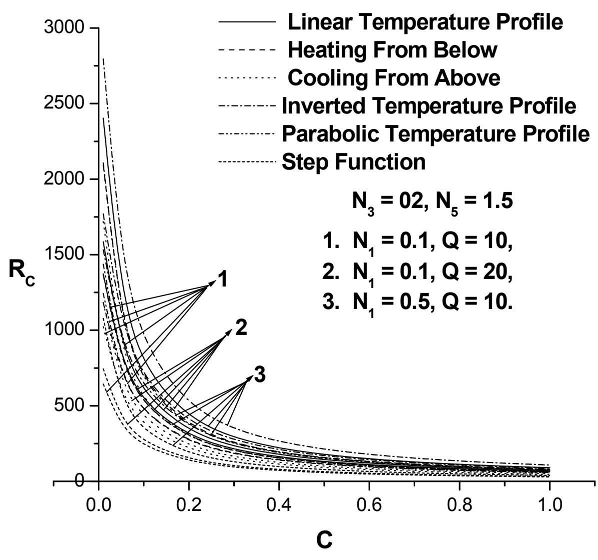
Linear, Inverted and Parabolic Temperature Profile
Heating From Below and Cooling From Above
Step Function
N1 = 0.1, N5 = 1.5
1. N3 = 02, Q = 10,
2. N3 = 02, Q = 20,
3. N3 = 0.5, Q = 10.
Rc
C

(b)

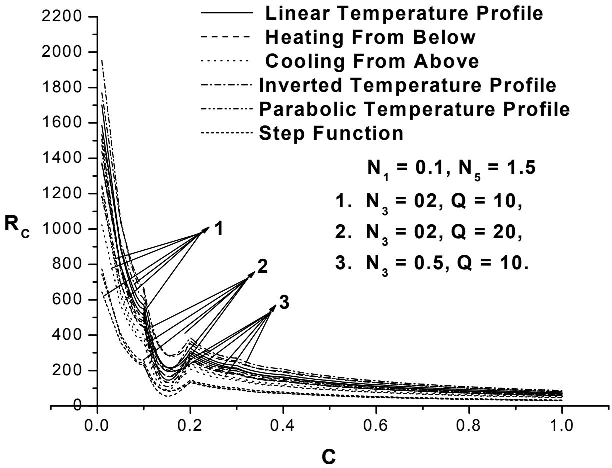


(c)

Figure 3. Plot of critical Rayleigh number R_C Vs. Cattaneo number C with respect to rigid-rigid isothermal no-spin boundary condition for different values of Q for (a) N_1, (b) N_3, (c) N_5 and for different non-uniform basic temperature gradients.



(a)



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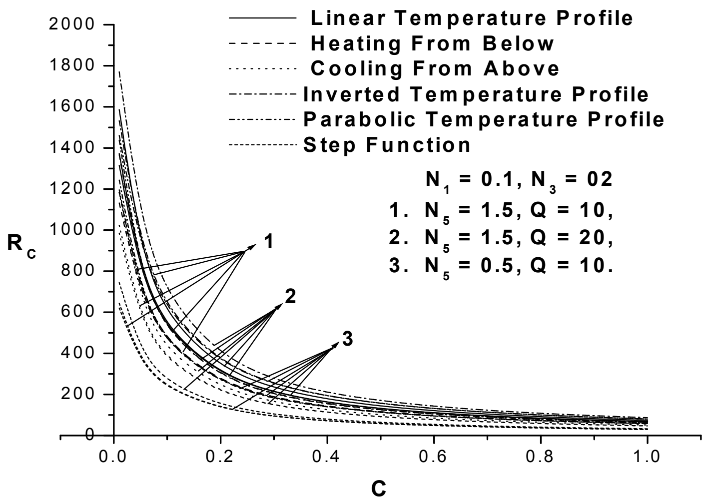

(c)

Figure 4. Plot of critical Rayleigh number R_C Vs. Cattaneo number C with respect to rigid-free isothermal no-spin boundary condition for different values of Q for (a) N_1, (b) N_3, (c) N_5 and for different non-uniform basic temperature gradients.

6. Conclusions

Following conclusions are drawn from the problem:

The inverted parabolic basic temperature profile is most stabilizing temperature profile. The step function basic temperature profile is most destabilizing temperature profile.

1. By creating conditions for appropriate basic temperature gradients we can also make a prior decision on advancing or delaying convection.
2. By adjusting the Chandrasekhar number Q we can control the convection.
3. Rayleigh-Bénard convection in Newtonian fluids may be delayed by adding micron sized suspended particles.
4. $R_C^{RR} > R_C^{RF} > R_C^{FF}$ where, the superscripts correspond to the three different velocity boundary combinations.
5. $R_c^{HHE} < R_c^{PHE}$ where, HHE – Hyperbolic heat equation and PHE – Parabolic heat equation.
6. The non-classical Maxwell-Cattaneo heat flux law involves a hyperbolic type heat transport equation that predicts finite speeds of heat wave propagation. Hence it does not suffer from the physically unacceptable drawback of infinite heat propagation speed predicted by the parabolic heat equation. The classical Fourier flux law overpredicts the critical Rayleigh number compared to that predicted by the non-classical law.

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