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On the Combined Role of Strong and Electroweak Interactions in Understanding Nuclear Binding Energy Scheme

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Abstract

An attempt is made to model the atomic nucleus as a combination of bound and free or unbound nucleons. Due to strong interaction, bound nucleons help increase nuclear binding energy, and due to electroweak interaction, free or unbound nucleons help decrease nuclear binding energy. In this context, concerning the proposed 4G model of final unification and strong interaction, we have recently developed a unified nuclear binding energy scheme with four simple terms: one energy coefficient of 10.1 MeV and two small numbers, 0.0016 0.0019. In this paper, by eliminating the number 0.0019, we try to fine-tune the estimation procedure of number of free or unbound nucleons pertaining to the second term with an energy coefficient of 11.9 MeV. It seems that some kind of electroweak interaction is playing a strange role in maintaining free or unbound nucleons within the nucleus. It is possible to say that strong interaction plays a vital role in increasing nuclear binding energy and electroweak interaction plays a vital role in reducing nuclear binding energy. An interesting observation is that Z can be considered a characteristic representation of a range of bound isotopes of Z. For medium, heavy and super-heavy atoms, beginning and

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ending mass numbers pertaining to bound states can be understood with 2Z+0.004Z^2 and 3Z+0.004Z^2, respectively. With further study, neutron drip lines can be understood. Based on this kind of data fitting procedure and by considering the mass ratio of pions and electroweak bosons, existence of our 4G model of electroweak fermion of rest energy 584.725 GeV can be confirmed confidently.

Keywords: 4G model of final unification; Four gravitational constants; Unified nuclear binding energy scheme; Free or unbound nucleons; Strong interaction; Electroweak interaction;

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1. Introduction

An attempt is made to model the atomic nucleus as a combination of bound and free or unbound nucleons. Due to strong interaction, bound nucleons help in increasing nuclear binding energy, and due to electroweak interaction, free or unbound nucleons help in decreasing nuclear binding energy. We would like to emphasize the fact that physics and mathematics associated with the fixing of the energy coefficients of semi-empirical mass formulae (S.E.M.F.) [1,2,3,4,5] are neither connected with residual strong nuclear force nor connected with a strong coupling constant α_s. Since nuclear force is mediated via quarks and gluons, it is necessary and compulsory to study the nuclear binding energy scheme in terms of nuclear coupling constants. In this direction, N. Ghahramany and team members have taken a great initiative in exploring the secrets of nuclear binding energy and magic numbers [6,7] with reference to quarks. A very interesting point of their study is that - nuclear binding energy can be understood with two or three terms having a single variable energy coefficient. In this direction, based on three unified assumptions connected with gravity and atomic interactions, in a semi-empirical approach, recently, we have developed a very simple formula for nuclear binding energy with a single energy coefficient having four simple terms [8-15]. Corresponding relations can be expressed in the following way. Starting from Z=3 to 118,

$$A_s \cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \tag{1}$$
$\cong$ Estimated mass number close to proton-neutron mean stability line.

$$BE \cong \left\{ A - A_{fg} - A^{1/3} - \frac{(A_s - A)^2}{A_s} \right\} (B_0 \cong 10.1 \text{ MeV}) \tag{2}$$
$\cong$ Estimated nuclear binding energy

Here, we would like to appeal that

1. $A_{fg} \cong \left(1 + 0.0019A\sqrt{ZN}\right)$ can be called the geometric number of free or unbound nucleons.
2. $A^{1/3}$ can be called a radial factor associated with nucleons.
3. $\frac{(A_s - A)^2}{A_s}$ can be called an isotopic asymmetric term associated with mean stable mass number.
4. The binding energy coefficient $B_0 \cong \frac{1}{\alpha_s}\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong 10.1 \text{ MeV}$ seems to be associated with nuclear radius R_0, strong coupling constant α_s and fine structure ratio α.

2. List of symbols

Newtonian gravitational constant = G_N	Mass of proton = m_p
Electromagnetic gravitational constant= G_e	Mass of neutron = m_n
Nuclear gravitational constant = G_s	Mass of electron = m_e
Weak gravitational constant = G_W	Charge radius of nucleus= R_0
Fermi's weak coupling constant = G_F	Proton number = Z
Strong coupling constant = α_s	Neutron number = N
Fine structure ratio = α	Mass number = A
Mass of electroweak fermion = M_w	Estimated mass number close to stability = A_s
Reduced Planck's constant = $\hbar$	Nuclear binding energy coefficient = B_0
Speed of light = c	Mass of pions = $(m_\pi)^0, (m_\pi)^\pm$

Elementary charge = e

Mass of weak bosons $= (m_z)^0, (m_w)^{\pm}$

Strong nuclear charge = e_s

3. Basic assumptions

1. There exists a characteristic electroweak fermion of rest energy, $M_w c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2. There exists a strong interaction elementary charge (e_s) in such a way that, its squared ratio with normal elementary charge is close to the reciprocal of the strong coupling constant.
3. Each atomic interaction is associated with a characteristic gravitational coupling constant.

It may be noted that when the mass of any elementary particle is extremely small/negligible compared to macroscopic bodies, highly curved microscopic space-time can be addressed with large gravitational constants and the magnitude of elementary gravitational constant seems to increase with decreasing mass and increasing interaction range. Based on this logic, we consider the possibility of the existence of three large gravitational constants assumed to be associated with the electromagnetic, strong and weak interactions. Approximate background relation is, $G_x m_x^2 \approx \hbar c$. Based on these assumptions, in our recently published paper [15], we have developed a semi-empirical scheme for deriving the important results. Readers are encouraged to refer to it for further analysis. Quantitatively,

$$\left.\begin{array}{l} G_e \cong 2.374335\times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_s \cong 3.329561\times 10^{28}\text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_w \cong 2.909745\times 10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_N \cong 6.679855\times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_F \cong 1.4402105\times 10^{-62}\ \text{J.m}^3 \\ \alpha_s \cong 0.1151937 \text{ and } e_s \cong 2.9463591e \end{array}\right\}$$

As our model is associated with 3 atomic gravitational constants and one celestial gravitational constant, we call it as 4G model of Final Unification. Important results pertaining to nuclear physics are [15,16,17,18],

$$\alpha_s \cong \left(\frac{e}{e_s}\right)^2 \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.1151937$$

(3) $R_0 \cong \dfrac{2G_s m_p}{c^2} \cong 1.23929\ \text{fm}$

(4)

$\hbar c \cong G_w M_w^2$

(5)

$$\left(\frac{e_s}{m_p}\right) \div \left(\frac{e}{m_e}\right) \cong \frac{G_s m_p m_e}{G_w M_w^2} \cong \frac{G_w M_w^2}{G_e m_e^2} \cong \frac{m_p}{M_w} \cong 0.001605 \tag{6}$$

$$\begin{array}{l} G_F \cong G_w M_w^2 R_w^2 \\ \text{where, } R_w \cong \left(2G_w M_w / c^2\right) \end{array} \tag{7}$$

4. A review of the second term and fine-tuning of the number of free or unbound nucleons

In our recent paper [14], we proposed that, starting from Z=3 to 118,

1. All the nucleons are not involved in the nuclear binding energy scheme.

2. Nucleons that are not involved in the nuclear binding energy scheme can be called 'free nucleons'.
3. The number of free nucleons increases with increasing $A\sqrt{ZN}$
4. Nucleons that involve in the nuclear binding energy scheme can be called 'active nucleons'.
5. In finding the free nucleon number, with trial–error solutions, a number close to 0.0019 could be arrived at.
6. Z= 3 to 118, the minimum number of free or unbound nucleons is 1.
7. For Z=2, the minimum number of free or unbound nucleons is 'Zero'.
8. The number of free or unbound protons can be expressed with a relation of the form,

$$A_{fp} \cong 0.0019AZ \tag{8}$$

9. The number of free or unbound neutrons can be expressed with a relation of the form,

$$A_{fn} \cong 0.0019AN \tag{9}$$

10. The geometric number of free nucleons can be expressed with a relation of the form,

$$A_{fg} \cong 0.0019A\sqrt{ZN} \tag{10}$$

11. Active nucleon number can be expressed with a relation of the form,

$$A_a \cong A - A_{fg} \tag{11}$$

12. As the number 0.0019 is very close to 0.0016, in this paper, we try to eliminate 0.0019 with $\left(\frac{m_p}{M_w}\right)$.

Starting from Z=3 to 118,

1. The number of free or unbound protons can be re-expressed with a relation of the form,

$$A_{fp} \approx \left(\frac{x_p m_p}{M_w}\right) \tag{12}$$

where x_p is a characteristic number associated with the mass number and proton number.

2. The number of free or unbound neutrons can be re-expressed with a relation of the form,

$$A_{fn} \approx \left(\frac{x_n m_n}{M_w}\right) \tag{13}$$

where x_n is a characteristic number associated with the mass number and neutron number.

3. The geometric number of free nucleons can be re-expressed with a relation of the form,

$$A_{fg} \approx \sqrt{\left(\frac{x_p m_p}{M_w}\right)\left(\frac{x_n m_n}{M_w}\right)} \approx \left(\frac{\sqrt{m_p m_n}}{M_w}\right)\sqrt{x_p x_n} \tag{14}$$

$\left(x_p, x_n\right)$ have been chosen in such a way that,

$$\left.\begin{array}{l} \sqrt{x_p + x_n} \cong A \\ \rightarrow x_p = AZ \text{ and } x_n = AN \end{array}\right\} \tag{15}$$

Hence, $A_{fg} \approx \left(\frac{\sqrt{m_p m_n}}{M_w}\right)\sqrt{x_p x_n} \approx \left(\frac{\sqrt{m_p m_n}}{M_w}\right) A\sqrt{ZN} \approx 0.001606 A\sqrt{ZN}$ (16)

By considering the minimum number of free nucleons as 1, starting from Z= 3 to 118,

$$A_{fg} \cong 1 + \left\{\left(\frac{\sqrt{m_p m_n}}{M_w}\right) A\sqrt{ZN}\right\} \tag{17}$$

Mass number close to mean stability can be expressed in the following way.

$$A_s \cong 2Z + \left(\frac{\sqrt{m_p m_n}}{M_w}\right)(2Z)^2 \cong 2Z + \left(\frac{\sqrt{m_p m_n}}{M_w}\right)(2Z)^2 \cong 2Z + 0.006423Z^2 \quad (18)$$

Important points to be noted are,

1. The number of free protons are $A_{fp} \approx \left(\frac{AZm_p}{M_w}\right)$, and the number of free neutrons are $A_{fn} \approx \left(\frac{AZm_n}{M_w}\right)$.
2. Characteristic electroweak fermion of rest energy $M_w c^2 \cong 584.725$ GeV seems to play a vital role in estimating the number of free protons and free neutrons. This is the essence of this review.With reference to the data presented in Table 1, it can be confirmed. With this, indirectly, existence of 584.725 GeV can be confirmed. We would like to appeal that some kind of electroweak interaction is playing a strange role in maintaining free or unbound nucleons within the nucleus. It needs further study.
3. The ratio $\left(\frac{\sqrt{m_p m_n}}{M_w}\right) \cong 0.001606$ seems to play an interesting role in estimating the geometric number of free nucleons and proton-neutron mean stability.
4. It is generally believed that, $(m_\pi)^0, (m_\pi)^\pm$ are the force carriers of strong interaction and $(m_z)^0, (m_w)^\pm$ are the force carriers of weak interaction. Considering Pions and electroweak bosons, to a great surprise, we noticed that [19], $\left(\frac{\sqrt{m_p m_n}}{M_w}\right) \cong 0.001606 \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}}\right) \cong \left(\frac{\sqrt{134.98 \times 139.57} \text{ MeV}}{\sqrt{80379.0 \times 91187.6} \text{ MeV}}\right) \cong 0.0016032.$

5. It is also very interesting to note that, $\frac{\sqrt{m_p m_n}}{\sqrt{(m_\pi)^0 (m_\pi)^\pm}} \cong 6.84 \cong \frac{M_w}{\sqrt{(m_z)^0 (m_w)^\pm}} \cong 6.83.$

6. As neutron's weak decay is directly responsible for nuclear stability associated with beta emission, based on the two numerical coincidences, i.e. 0.0016 and 6.83, the existence of our assumed 584.725 GeV weak fermion can be confirmed, and it is also possible to have a relation of the form, $M_w \cong \left(\frac{\sqrt{(m_z)^0 (m_w)^\pm}}{\sqrt{(m_\pi)^0 (m_\pi)^\pm}} \right) m_p \cong 585.244 \text{ GeV}/c^2.$

7. With reference to nucleons and pions, it is reasonable to argue that, if one is willing to consider $\left(m_z c^2\right)^0 \& \left(m_w c^2\right)^\pm$ as the force carriers of weak interaction [20,21,22], then one should not ignore the possibility of considering the proposed weak fermion of rest energy 584.725 GeV as the characteristic field generator of weak interaction. Clearly speaking, weak force carriers cannot exist without the existence of their weak field generating fermion.

8. When $(A-2Z) \to A_{fP}$ bound states of (A,Z) seem to have possible stability on the lower side of A. This peculiar condition seems to be satisfied at $A_{low} \cong 2Z + 0.004Z^2$ where $\left(\frac{m_n - m_p}{m_e}\right) \times 0.001606 = 2.531 \times 0.001606 = 0.004$. For medium, heavy and super-heavy atomic nuclides, this type of condition can be considered as a clue [22, 23]. See Table 2.

9. Similarly, when, $A_{up} \cong 3Z + 0.004Z^2$, binding energy seems to start reducing. If one is willing to consider A_{up} as an upper limit of a bound state of (A,Z), then Z can be considered as a characteristic representation of a range of number of bound states of Z. These isotopes may or may not be stable. Clearly speaking, $(A_{\text{low}}, A_{\text{up}})$ seems to represent the starting and ending

points of the probability of forming of bound states of Z. Proceeding further, neutron drip lines can be understood.

10. Energy coefficient for the second term becomes, $\left(\frac{0.0019}{0.001606}\right)10.1 \cong 1.1831\times 10.1 \cong 11.95$ MeV. For data fitting purpose, we consider it as 11.90 MeV. Now, binding energy can be estimated with the following relation having two energy coefficients.

$$BE \cong \left(A - A^{1/3} - \frac{(A_s - A)^2}{A_s}\right) 10.1 \text{ MeV} - \left[1 + \left(0.001606 A\sqrt{ZN}\right)\right] 11.9 \text{ MeV} \quad (19)$$

11. Based on the relation (19), it is possible to say that strong interaction plays a vital role in increasing nuclear binding energy and electroweak interaction plays a vital role in reducing nuclear binding energy.

5. Discussion

The binding energy coefficient can be understood with the following relations.

$$B_0 \cong \frac{1}{\alpha_s}\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong \left(\frac{e_s^2}{4\pi\varepsilon_0 R_0}\right) \cong 10.08 \text{ MeV} \quad (20)$$

where, $\alpha_s \cong 0.1152$ and $R_0 \cong 1.24$ fermi

Based on relations (3) and (4),

$$B_0 \cong \frac{1}{2}\left(\frac{e e_s}{4\pi\varepsilon_0 \hbar c}\right)(m_p c^2) \cong \frac{1}{2}\sqrt{\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\left(\frac{e_s^2}{4\pi\varepsilon_0 \hbar c}\right)}(m_p c^2) \cong 10.09 \text{ MeV} \quad (21)$$

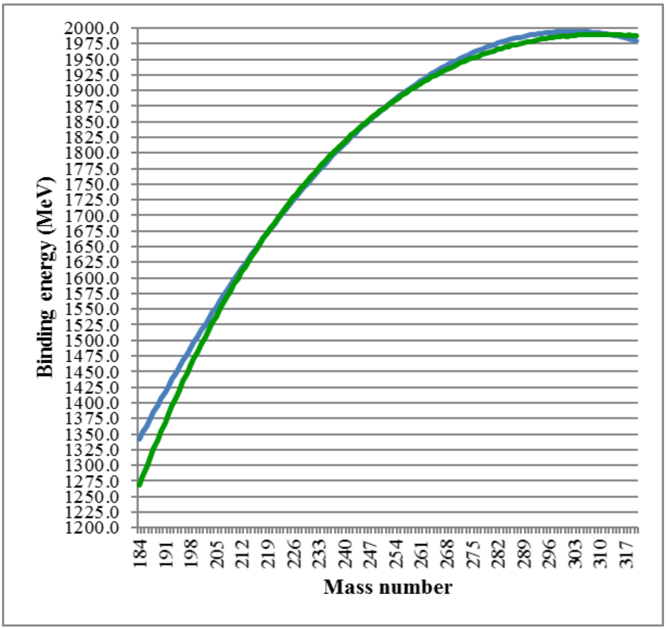
where, $\left(\frac{e_s^2}{4\pi\varepsilon_0 \hbar c}\right) \cong 0.06334854$ can be called as 'nuclear fine structure ratio'.

$\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right) \cong \alpha$ is the 'fine structure ratio'.

Considering the following reference semi-empirical mass formula (S.E.M.F.) [5,14], we have prepared Table 1 and Figure 1. Readers are encouraged to refer to other S.E.M.F. having different sets of energy coefficients.

$$BE_{\text{Ref}} \cong \left\{ \begin{array}{c} \left[(A\times 15.36)\right]-\left[\left(A^{2/3}\times 16.32\right)\right]-\left[\left(\frac{Z^2}{A^{1/3}}\right)0.6929\right] \\ -\left[\frac{\left((A/2)-Z\right)^2}{A}\times 90.46\right]\pm\left(\frac{11.32}{\sqrt{A}}\right) \end{array} \right\} \text{MeV} \tag{22}$$

By correlating the relations (16 to 22) and with a systematic study, in a microscopic approach, hidden physics can be explored in a unified approach. In Fig. 1 blue curve indicates our estimated binding energy, and the green curve indicates reference binding energy. Estimated binding energy needs a review for mass numbers close to A=2Z. The point to be noted is that, error in binding energy for the estimated range of lower (218) and upper (310) mass limits of Z=92 is on the minimum side.



1: Estimated binding energy of isotopes of Z=92

Table 1: Estimated nuclear binding energy of isotopes of Z=92

Proton number	Estimated Mean mass number	Mass number	Neutron number	Excess neutron number	Free proton number	Free neutron number	Estimated B.E. (MeV)	Reference B.E. (MeV)	Difference in Binding energy (MeV)
92	238	184	92	0	27	27	1341.8	1268.0	-73.8
92	238	185	93	1	27	28	1352.8	1282.4	-70.4
92	238	186	94	2	27	28	1363.7	1298.1	-65.6
92	238	187	95	3	28	29	1374.6	1312.0	-62.6
92	238	188	96	4	28	29	1385.3	1327.3	-58.0
92	238	189	97	5	28	29	1395.9	1340.6	-55.3
92	238	190	98	6	28	30	1406.5	1355.4	-51.1
92	238	191	99	7	28	30	1416.9	1368.3	-48.6
92	238	192	100	8	28	31	1427.3	1382.7	-44.6
92	238	193	101	9	29	31	1437.5	1395.1	-42.4
92	238	194	102	10	29	32	1447.7	1409.0	-38.7
92	238	195	103	11	29	32	1457.8	1421.0	-36.8
92	238	196	104	12	29	33	1467.8	1434.5	-33.3
92	238	197	105	13	29	33	1477.7	1446.1	-31.6
92	238	198	106	14	29	34	1487.4	1459.1	-28.4
92	238	199	107	15	29	34	1497.1	1470.3	-26.9
92	238	200	108	16	30	35	1506.8	1482.9	-23.9
92	238	201	109	17	30	35	1516.3	1493.7	-22.6
92	238	202	110	18	30	36	1525.7	1505.9	-19.8
92	238	203	111	19	30	36	1535.0	1516.3	-18.7
92	238	204	112	20	30	37	1544.2	1528.1	-16.1
92	238	205	113	21	30	37	1553.4	1538.1	-15.2
92	238	206	114	22	30	38	1562.4	1549.6	-12.9
92	238	207	115	23	31	38	1571.4	1559.2	-12.1
92	238	208	116	24	31	39	1580.2	1570.3	-9.9
92	238	209	117	25	31	39	1589.0	1579.6	-9.4
92	238	210	118	26	31	40	1597.6	1590.3	-7.3
92	238	211	119	27	31	40	1606.2	1599.3	-6.9
92	238	212	120	28	31	41	1614.7	1609.7	-5.0
92	238	213	121	29	31	41	1623.1	1618.3	-4.8
92	238	214	122	30	32	42	1631.4	1628.3	-3.0
92	238	215	123	31	32	42	1639.5	1636.6	-2.9
92	238	216	124	32	32	43	1647.6	1646.3	-1.3
92	238	217	125	33	32	44	1655.7	1654.3	-1.3
92	238	218	126	34	32	44	1663.6	1663.7	0.2
92	238	219	127	35	32	45	1671.4	1671.4	0.0
92	238	220	128	36	33	45	1679.1	1680.5	1.4
92	238	221	129	37	33	46	1686.7	1687.9	1.1
92	238	222	130	38	33	46	1694.3	1696.7	2.4
92	238	223	131	39	33	47	1701.7	1703.8	2.0
92	238	224	132	40	33	47	1709.1	1712.2	3.2
92	238	225	133	41	33	48	1716.3	1719.1	2.7
92	238	226	134	42	33	49	1723.5	1727.3	3.8
92	238	227	135	43	34	49	1730.6	1733.8	3.2

92	238	228	136	44	34	50	1737.5	1741.7	4.2
92	238	229	137	45	34	50	1744.4	1748.0	3.6
92	238	230	138	46	34	51	1751.2	1755.6	4.4
92	238	231	139	47	34	52	1757.9	1761.7	3.8
92	238	232	140	48	34	52	1764.5	1769.0	4.5
92	238	233	141	49	34	53	1771.0	1774.8	3.8
92	238	234	142	50	35	53	1777.4	1781.9	4.5
92	238	235	143	51	35	54	1783.8	1787.4	3.7
92	238	236	144	52	35	55	1790.0	1794.3	4.3
92	238	237	145	53	35	55	1796.1	1799.6	3.5
92	238	238	146	54	35	56	1802.2	1806.2	4.1
92	238	239	147	55	35	56	1808.1	1811.3	3.2
92	238	240	148	56	35	57	1813.9	1817.6	3.7
92	238	241	149	57	36	58	1819.7	1822.4	2.7
92	238	242	150	58	36	58	1825.4	1828.6	3.2
92	238	243	151	59	36	59	1830.9	1833.2	2.2
92	238	244	152	60	36	60	1836.4	1839.1	2.7
92	238	245	153	61	36	60	1841.8	1843.5	1.7
92	238	246	154	62	36	61	1847.1	1849.2	2.1
92	238	247	155	63	36	61	1852.3	1853.3	1.0
92	238	248	156	64	37	62	1857.4	1858.8	1.4
92	238	249	157	65	37	63	1862.4	1862.8	0.4
92	238	250	158	66	37	63	1867.3	1868.0	0.7
92	238	251	159	67	37	64	1872.1	1871.8	-0.4
92	238	252	160	68	37	65	1876.9	1876.9	0.0
92	238	253	161	69	37	65	1881.5	1880.4	-1.1
92	238	254	162	70	38	66	1886.1	1885.3	-0.8
92	238	255	163	71	38	67	1890.5	1888.6	-1.9
92	238	256	164	72	38	67	1894.9	1893.3	-1.6
92	238	257	165	73	38	68	1899.1	1896.5	-2.7
92	238	258	166	74	38	69	1903.3	1900.9	-2.3
92	238	259	167	75	38	69	1907.4	1903.9	-3.4
92	238	260	168	76	38	70	1911.3	1908.2	-3.1
92	238	261	169	77	39	71	1915.2	1911.0	-4.2
92	238	262	170	78	39	72	1919.0	1915.1	-3.9
92	238	263	171	79	39	72	1922.7	1917.7	-5.0
92	238	264	172	80	39	73	1926.3	1921.7	-4.7
92	238	265	173	81	39	74	1929.9	1924.1	-5.7
92	238	266	174	82	39	74	1933.3	1927.9	-5.4
92	238	267	175	83	39	75	1936.6	1930.2	-6.5
92	238	268	176	84	40	76	1939.8	1933.7	-6.1
92	238	269	177	85	40	76	1943.0	1935.8	-7.1
92	238	270	178	86	40	77	1946.0	1939.3	-6.8
92	238	271	179	87	40	78	1949.0	1941.2	-7.8
92	238	272	180	88	40	79	1951.9	1944.5	-7.4
92	238	273	181	89	40	79	1954.6	1946.3	-8.4
92	238	274	182	90	40	80	1957.3	1949.4	-7.9
92	238	275	183	91	41	81	1959.9	1951.0	-8.9
92	238	276	184	92	41	82	1962.4	1953.9	-8.4
92	238	277	185	93	41	82	1964.8	1955.4	-9.3
92	238	278	186	94	41	83	1967.1	1958.2	-8.9
92	238	279	187	95	41	84	1969.3	1959.6	-9.7

92	238	280	188	96	41	85	1971.4	1962.2	-9.2
92	238	281	189	97	42	85	1973.4	1963.4	-10.0
92	238	282	190	98	42	86	1975.4	1965.9	-9.5
92	238	283	191	99	42	87	1977.2	1966.9	-10.3
92	238	284	192	100	42	88	1978.9	1969.3	-9.7
92	238	285	193	101	42	88	1980.6	1970.2	-10.4
92	238	286	194	102	42	89	1982.2	1972.4	-9.8
92	238	287	195	103	42	90	1983.6	1973.2	-10.5
92	238	288	196	104	43	91	1985.0	1975.2	-9.8
92	238	289	197	105	43	91	1986.3	1975.9	-10.4
92	238	290	198	106	43	92	1987.5	1977.8	-9.7
92	238	291	199	107	43	93	1988.6	1978.3	-10.2
92	238	292	200	108	43	94	1989.6	1980.1	-9.5
92	238	293	201	109	43	95	1990.5	1980.5	-10.0
92	238	294	202	110	43	95	1991.3	1982.2	-9.1
92	238	295	203	111	44	96	1992.0	1982.5	-9.6
92	238	296	204	112	44	97	1992.6	1984.0	-8.7
92	238	297	205	113	44	98	1993.2	1984.1	-9.0
92	238	298	206	114	44	99	1993.6	1985.5	-8.1
92	238	299	207	115	44	99	1994.0	1985.6	-8.4
92	238	300	208	116	44	100	1994.2	1986.9	-7.4
92	238	301	209	117	44	101	1994.4	1986.8	-7.6
92	238	302	210	118	45	102	1994.5	1987.9	-6.5
92	238	303	211	119	45	103	1994.5	1987.7	-6.7
92	238	304	212	120	45	104	1994.3	1988.8	-5.6
92	238	305	213	121	45	104	1994.1	1988.5	-5.7
92	238	306	214	122	45	105	1993.8	1989.4	-4.4
92	238	307	215	123	45	106	1993.5	1989.0	-4.5
92	238	308	216	124	46	107	1993.0	1989.8	-3.2
92	238	309	217	125	46	108	1992.4	1989.3	-3.1
92	238	310	218	126	46	109	1991.7	1990.0	-1.7
92	238	311	219	127	46	109	1991.0	1989.4	-1.6
92	238	312	220	128	46	110	1990.1	1990.0	-0.2
92	238	313	221	129	46	111	1989.2	1989.2	0.1
92	238	314	222	130	46	112	1988.1	1989.7	1.6
92	238	315	223	131	47	113	1987.0	1988.9	1.9
92	238	316	224	132	47	114	1985.8	1989.3	3.5
92	238	317	225	133	47	115	1984.4	1988.3	3.9
92	238	318	226	134	47	115	1983.0	1988.6	5.6
92	238	319	227	135	47	116	1981.5	1987.6	6.0
92	238	320	228	136	47	117	1979.9	1987.7	7.8

Table-2: Lower and upper mass limits of heavy and super heavy atomic nuclides

Proton number	Estimated mean mass number	Estimated lower mass number	Estimated upper mass number
118	325	292	410
117	322	289	406
116	318	286	402
115	315	283	398

114	311	280	394
113	308	277	390
112	304	274	386
111	301	271	382
110	297	268	378
109	294	266	375
108	291	263	371
107	287	260	367
106	284	257	363
105	281	254	359
104	277	251	355
103	274	248	351
102	271	246	348
101	267	243	344
100	264	240	340
99	261	237	336
98	257	234	332
97	254	232	329
96	251	229	325
95	248	226	321
94	245	223	317
93	241	221	314
92	238	218	310
91	235	215	306
90	232	212	302
89	229	210	299
88	226	207	295
87	222	204	291
86	219	202	288
85	216	199	284
84	213	196	280
83	210	194	277
82	207	191	273
81	204	188	269
80	201	186	266

6. Conclusion

Considering the proposed relations (1 to 22), our unified binding energy scheme assumed to be associated with free protons and free neutrons can be recommended for further research. We would like to appeal that some kind of electroweak interaction is playing a strange role in maintaining free or unbound nucleons within the nucleus. With further study, lower and upper mass limits of bound states of medium and heavy atomic nuclides and corresponding neutron drip lines can be explored. Proceeding further, the existence of our 4G model of electroweak fermion of rest energy 584.725 GeV can be confirmed indirectly.

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Conflict of Interest

The authors declare no conflict of interest in this paper.

Authors' contribution

This work was carried out in collaboration among the two authors. Author U.V.S.S. designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author S.L managed the analyses of the study. Both authors read and approved the final manuscript.