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Open Support Strong Efficient Domination Number of Some Standard Graphs Under Addition and Multiplication

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Abstract

Let G = (V, E) be a graph with p points and q nodes. Let S be a γ_{se} - set of G. Let v ϵ S. An open support strong efficient domination number of v under addition is defined by $\sum_{u \epsilon N(v)} \deg(u)$ and it is denoted by supp $\gamma_{se}{}^{+}(v)$. An open support strong efficient domination number of G under addition is defined by $\sum_{v \epsilon S} \text{supp } \gamma_{se}{}^{+}(v)$ and it is denoted by supp $\gamma_{se}{}^{+}(G)$. An open support strong efficient domination number of v under multiplication is defined by $\prod_{u \epsilon N(v)} \deg(u)$ and it is denoted by supp $\gamma_{se}{}^{\times}(v)$. An open support strong efficient domination number of G under multiplication is defined by $\prod_{v \epsilon S} \text{supp } \gamma_{se}{}^{\times}(v)$ and it is denoted by supp $\gamma_{se}{}^{\times}(G)$. In this paper, open support strong efficient domination number of some standard graphs is studied.

Keywords: Strong efficient domination number, open support strong efficient domination number of a point under addition, open support strong efficient domination number of a graph under addition, open support strong efficient domination number of a point under multiplication, and open support strong efficient domination number of a graph under multiplication.

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1. Introduction

Throughout this paper only finite, undirected graphs without loops or multiple nodes are considered. Let G = (V, E) be a graph with p points and q nodes. The degree of any point v of a graph G is the number of nodes incident with v and is denoted by deg (v). A subset S of V(G) is called a dominating set of G if every point in V(G) - S is adjacent to a point in S [6]. The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. Sampath Kumar et al introduced the concept of strong (weak) domination in graphs [11]. A subset S of V(G) is called a strong dominating set of G if for every v $\in V(G) - S$ there exists a point u $\in S$ such that u and v are adjacent and deg(u) $\geq \deg(v)$. A subset S of V(G) is called an efficient dominating set if for every v $\in V(G)$, $|N[v] \cap S| = 1$ [3, 5]. The concept of strong (weak) efficient domination in graphs was introduced by Meena et al [10] and further studied in [7, 8, 9]. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every point $v \in V(G)$ we have $|N_s[v] \cap S| = 1$ ($|N_W[v] \cap S|$1), where $N_s(v)$ =$\{u \in V(G); uv \in E(G), \deg(u) \geq \deg(v)\}$ and $N_s[v]$ = $N_s(v) \cup$ {v}($N_w(v) = \{u \in V(G); uv \in E(G), \deg(u) \leq \deg(v)\}$and $N_w[v]$ = $N_w(v) \cup$ {v}). The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)(\gamma_{we}(G))$. A graph G is strong efficient if there exists a strong efficient dominating set of G. Balamurugan et al introduced the concept of open support of a graph under addition [2] and multiplication [1]. Open support of a point v under addition is defined by $\sum_{u \in N(v)} \deg(u)$ and it is denoted by $supp(v)$. An open support of a graph G under addition is defined by $\sum_{v \in V(G)} supp(v)$ and it is denoted by $supp(G)$. An open support of a point v under multiplication is defined by $\prod_{u \in N(v)} \deg(u)$ and is denoted by $mult(v)$. An open support of a graph G under multiplication is defined by $\prod_{u \in V(G)} mult(u)$ and it is denoted by $mult(G)$. Inspired by the above definitions, the concept of an open support strong efficient domination number of a graph under addition and multiplication is introduced in this paper. For all Graph-theoretic terminologies and notations, Harary [4] is followed. Following previous results are necessary for the present study.

Previous results [9]:

1) For any path P_m, $\gamma_{se}(P_m)\begin{cases} n \text{ if } m = 3n, n \in N \\ n+1 \text{ if } m = 3n+1, n \in N \\ n+2 \text{ if } m = 3n+2, n \in N \end{cases}$
2) For any cycle C_{3n}, $\gamma_{se}(C_{3n})$ = n, n$\in N$
3) $\gamma_{se}(K_{1,n})$ = 1, n$\in N$
4) $\gamma_{se}(K_n)$ = 1, n$\in N$
5) $\gamma_{se}(D_{m,n})$ = m + 1, m>n, m, n $\in N$

2. Main results

Definition 2.1: Let G = (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let vϵ S. An open support strong efficient domination number of v under addition is defined by $\sum_{u\epsilon N(v)} \deg(u)$ and it is denoted by supp $\gamma_{se}^{+}(v)$

Example 2.2: Consider the following graph G.

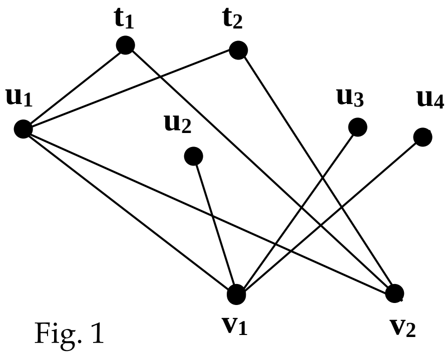

Fig. 1

S = $\{v_1, v_2\}$ is a γ_{se} - set of G.

supp $\gamma_{se}^{+}(v_1) = \sum_{u\epsilon N(v_1)} \deg(u)$=deg$(u_1)$ +deg(u_2)+ deg(u_3) + deg(u_4)=7

supp $\gamma_{se}^{+}(v_2) = \sum_{u\epsilon N(v_2)} \deg(u)$ = deg(u_1) + deg(t_1) + deg(t_2)=8

Definition 2.3: Let G = (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let vϵ S. An open support strong efficient domination number of v under multiplication is defined by$\prod_{u\epsilon N(v)} \deg(u)$ and it is denoted by supp $\gamma_{se}^{\times}(v)$.

Example 2.4: In Fig. 1,

supp $\gamma_{se}{}^{\times}(v_1) = \prod_{u \epsilon N(v_1)} \deg(u)$=deg$(u_1)$ × deg(u_2)× deg(u_3) ×deg(u_4) =4

supp $\gamma_{se}{}^{\times}(v_2) = \prod_{u \epsilon N(v_2)} \deg(u)$=deg$(u_1)$ × deg(t_1) × deg(t_2)=16

Definition 2.5: Let G = (V, E) be a strong efficient graph. Let S be a γ_{se} - set of G. An open support strong efficient domination number of G under addition is defined by $\sum_{v \epsilon S} \text{supp } \gamma_{se}{}^{+}(v)$ and it is denoted by supp $\gamma_{se}{}^{+}(G)$.

Example 2.6: In Fig.1, supp $\gamma_{se}{}^{+}(G)$=supp$\gamma_{se}{}^{+}(v_1)$+supp$\gamma_{se}{}^{+}(v_2)$=7+8=15

Definition 2.7: Let G = (V, E) be a strong efficient graph. Let S be a γ_{se} - set of G. An open support strong efficient domination number of G under multiplication is defined by $\prod_{v \epsilon S} \text{supp } \gamma_{se}{}^{\times}(v)$ and it is denoted by supp $\gamma_{se}{}^{\times}(G)$.

Example 2.8: In Fig.1,supp $\gamma_{se}{}^{\times}(G)$=supp $\gamma_{se}{}^{\times}(v_1)$Xsupp$\gamma_{se}{}^{\times}(v_2)$=4X16=64.

Note 2.9: Open support strong efficient domination number under addition of a graph G is not unique.

Example 2.10: Consider the following graph G.

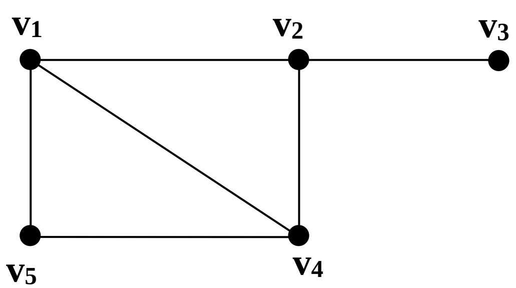

Fig. 2

S_1={v_1, v_3}, S_2={v_2, v_5} and S_3={v_4, v_3}are three γ_{se} - sets of G and γ_{se} (G)=2.

For S_1, supp $\gamma_{se}{}^{+}(v_1)$=$\sum_{u \in N(v_1)} \deg(u)$ = deg(v_2) + deg(v_4) + deg(v_5)=8.

supp $\gamma_{se}{}^{+}(v_3)$= $\sum_{u \in N(v_3)} \deg(u)$ = deg(v_2) = 3.

supp $\gamma_{se}^{+}(G)=\sum_{v\in S_1}$ supp $\gamma_{se}^{+}(v)$=supp $\gamma_{se}^{+}(v_1)$+ supp $\gamma_{se}^{+}(v_3)$= 11.

supp $\gamma_{se}^{+}(G)$= 11.

For S_2, supp $\gamma_{se}^{+}(v_2)=\sum_{u\in N(v_2)}\deg(u)=\deg(v_1)+\deg(v_4)+\deg(v_3)=7$.

supp $\gamma_{se}^{+}(v_5)=\sum_{u\in N(v_5)}\deg(u)=\deg(v_1)+\deg(v_4)=6$.

supp $\gamma_{se}^{+}(G)=\sum_{v\in S_2}$ supp $\gamma_{se}^{+}(v)$=supp $\gamma_{se}^{+}(v_2)$+supp $\gamma_{se}^{+}(v_5)$=13.

supp $\gamma_{se}^{+}(G)$= 13.

For S_3, supp $\gamma_{se}^{+}(v_4)=\sum_{u\in N(v_4)}\deg(u)=\deg(v_2)+\deg(v_1)+\deg(v_5)=8$.

supp $\gamma_{se}^{+}(v_3)=\sum_{u\in N(v_3)}\deg(u)=\deg(v_2)=3$.

supp $\gamma_{se}^{+}(G)=\sum_{v\in S_3}$ supp $\gamma_{se}^{+}(v)$=supp $\gamma_{se}^{+}(v_4)$+supp $\gamma_{se}^{+}(v_3)$=11.

supp $\gamma_{se}^{+}(G)$= 11.

Here min supp $\gamma_{se}^{+}(G)$ = 8 and max supp $\gamma_{se}^{+}(G)$= 11

Note 2.11: Open support strong efficient domination number under multiplication of a graph G is not unique.

Example 2.12: Consider Fig.2,

$S_1=\{v_1, v_3\}$, $S_2=\{v_2, v_5\}$ & $S_3=\{v_4, v_3\}$ are three γ_{se} - sets of G & $\gamma_{se}(G)=2$.

For S_1, supp$\gamma_{se}^{\times}(v_1)=\prod_{u\in N(v_1)}\deg(u)=\deg(v_2)\times\deg(v_4)\times\deg(v_5)=18$.

supp $\gamma_{se}^{\times}(v_3)=\prod_{u\in N(v_3)}\deg(u)=\deg(v_2)=3$.

supp $\gamma_{se}^{\times}(G)=\prod_{v\in S_1}$ *supp* $\gamma_{se}^{\times}(v)$=supp $\gamma_{se}^{\times}(v_1)\times$ supp $\gamma_{se}^{\times}(v_3)$=54.

supp $\gamma_{se}^{\times}(G)$ = 54.

For S_2,supp $\gamma_{se}^{\times}(v_2)=\prod_{u\in N(v_2)}\deg(u)=\deg(v_1)\times\deg(v_4)\times\deg(v_3)=9$.

supp $\gamma_{se}^{\times}(v_5)=\prod_{u\in N(v_5)}\deg(u)=\deg(v_1)\times\deg(v_4)=9$.

supp $\gamma_{se}^{\times}(G)=\prod_{v\in S_2}$ *supp* $\gamma_{se}^{\times}(v)$=supp $\gamma_{se}^{\times}(v_2)\times$ supp $\gamma_{se}^{\times}(v_5)$=81.

supp $\gamma_{se}^{\times}(G)$ = 81.

For S_3,supp$\gamma_{se}^{\times}(v_4)=\prod_{u\in N(v_4)}\deg(u)=\deg(v_2)\times\deg(v_1)\times\deg(v_5)=18$.

supp $\gamma_{se}^{\times}(v_3)=\prod_{u\in N(v_3)}\deg(u)=\deg(v_2)=3$.

supp $\gamma_{se}{}^{\times}(G)=\prod_{v\in S_3} supp\ \gamma_{se}{}^{\times}(v)$=supp $\gamma_{se}{}^{\times}(v_4)\times$supp $\gamma_{se}{}^{\times}(v_3)$=54.
supp $\gamma_{se}{}^{\times}(G)$ = 54.

Here min supp $\gamma_{se}{}^{\times}(G)$ = 54 and max supp $\gamma_{se}{}^{\times}(G)$ = 81

Remark 2.13: **:** Let G be a connected strong efficient graph with a γ_{se} - set S. Since S $\subset$ V(G), supp $\gamma_{se}{}^{+}$(G)<$supp(G)$ and supp $\gamma_{se}{}^{\times}(G)$<$mult(G)$.

Theorem 2.14: Let G = $P_{3n,}$ $n \in N$ Then

supp $\gamma_{se}{}^{+}$ (G) = 4n – 2 and

supp $\gamma_{se}{}^{\times}$ (G) =4^{n-1}

Proof: Let G = $P_{3n,}$ $n \in N$ Let V(G) =$\{v_i;\ 1 \le i \le 3n\}$.
Then S = $\{v_2, v_5, v_{8,,} \dots, v_{3n-4}, v_{3n-1}\}$is the unique γ_{se} - set of G.

$\deg(v_1)$ = $\deg(v_{3n})$ = 1 and $\deg(v_i)$ = 2, 2 $\le$ i $\le$ 3n – 1.

supp $\gamma_{se}{}^{+}(v_2)$ = $\deg(v_1) + \deg(v_3)$ = 3

For i = 5, 8,…, 3n – 4, supp $\gamma_{se}{}^{+}$ (v_i) = $\deg(v_{i-1}) + \deg(v_{i+1})$ = 4

Supp $\gamma_{se}{}^{+}$ (v_{3n-1}) = $\deg(v_{3n-2}) + \deg(v_{3n})$ = 3

Hence supp γ_{se}^{+} (G) = supp $\gamma_{se}{}^{+}$ (v_2) + $\sum_{i=1}^{n-2}$ supp $\gamma_{se}{}^{+}(v_{3i+2})$ + supp $\gamma_{se}{}^{+}$ (v_{3n-1}) = 3+(n – 2)4 +3 = 4n – 2.

supp $\gamma_{se}{}^{\times}$ (v_2) = $\deg(v_1) \times \deg(v_3)$ = 2

For i = 5, 8,…, 3n – 4, supp $\gamma_{se}{}^{\times}$ (v_i) = $\deg(v_{i-1}) \times \deg(v_{i+1})$ = 4

Supp $\gamma_{se}{}^{\times}$ (v_{3n-1}) = $\deg(v_{3n-2}) \times \deg(v_{3n})$ = 2

Hence supp $\gamma_{se}{}^{\times}$ (G) = supp $\gamma_{se}{}^{\times}$ (v_2) + $\prod_{i=1}^{n-2} supp\ \gamma_{se}{}^{\times}(v_{3i+2})$ + supp $\gamma_{se}{}^{\times}$ (v_{3n-1}) = 2$\times$ 4^{n-2} $\times$2 = 4^{n-1}.

Theorem 2.15: Let G = $P_{3n+1}, n \in N$ Then

supp $\gamma_{se}{}^{+}$ (G) = 4n+ 1 and

supp $\gamma_{se}{}^{\times}$ (G) = 4^n.

Proof: Let G = $P_{3n+1}, n \in N$. Let V(G) =$\{v_i;\ 1 \le i \le 3n+1\}$. Then S_1=$\{v_1, v_3, v_{6,} \dots, v_{3n-3}, v_{3n}\}$ and S_2=$\{v_2, v_5, v_{8,} \dots, v_{3n-1}, v_{3n+1}\}$are two distinct γ_{se} - sets of G.deg(v_1) = deg(v_{3n+1}) = 1 and deg(v_i) = 2, 2 ≤ i ≤ 3n.

Consider S_1. (Proof is similar for S_2)

supp γ_{se}^{+} (v_1) = deg(v_2)= 2

For i = 3, 6,…, 3n – 3, supp γ_{se}^{+} (v_i) = deg(v_{i-1}) +deg(v_{i+1}) = 4

supp γ_{se}^{+} (v_{3n}) = deg(v_{3n-1}) + deg(v_{3n+1}) = 3

Hence supp γ_{se}^{+} (G) = supp γ_{se}^{+} (v_1) +$\sum_{i=1}^{n-1}$ supp $\gamma_{se}^{+}(v_{3i})$ + γ_{se}^{+} (v_{3n}) = 2+ (n – 1)4 + 3 = 4n+ 1

supp $\gamma_{se}^{\times}$ (v_1)= deg(v_2)= 2

For i = 3, 6,…, 3n – 3, supp γ_{se}^{+} (v_i) = deg(v_{i-1}) + deg(v_{i+1}) = 4

supp γ_{se}^{+} (v_{3n}) = deg(v_{3n-1}) × deg(v_{3n+1}) = 2 × 1 = 2

Hence supp $\gamma_{se}^{\times}$ (G) = supp $\gamma_{se}^{\times}$ (v_1) ×$\prod_{i=1}^{n-1} supp\ \gamma_{se}^{\times}(v_{3i})$ ×supp $\gamma_{se}^{\times}(v_{3n})$ = 2× 4^{n-1} ×2=4^n.

Hence supp γ_{se}^{+} (G) = 4n+ 1 and supp $\gamma_{se}^{\times}$ (G) = 4^n.

Theorem 2.16: Let G = $P_{3n+2}, n \in N$ Then

supp γ_{se}^{+} (G) = 4(n+ 1) and

supp $\gamma_{se}^{\times}$ (G) = 4^{n+1}.

Proof: Let G = $P_{3n+2}, n \in N$. Let V(G) =$\{v_i;\ 1 \le i \le 3n+2\}$.

Then S = $\{v_1, v_3, v_6, \dots, v_{3n}, v_{3n+2}\}$ is the unique γ_{se} - set of G.

deg(v_1) = deg(v_{3n+2}) = 1 and deg(v_i) = 2, 2 ≤ i ≤ 3n + 1.

supp γ_{se}^{+} (v_1) = deg(v_2)= 2

For i = 3, 6,…, 3n, supp γ_{se}^{+} (v_i)= deg(v_{i-1}) +deg(v_{i+1}) = 4

Supp γ_{se}^{+} (v_{3n+2})= deg(v_{3n+1}) = 2

Hence supp $\gamma_{se}{}^{+}$ (G) = supp $\gamma_{se}{}^{+}$ (v_1) + $\sum_{i=1}^{n}$ supp $\gamma_{se}{}^{+}(v_{3i})$ + supp $\gamma_{se}{}^{+}$ (v_{3n+2})= 2 + n(4) +2 = 4(n+ 1).

supp $\gamma_{se}{}^{\times}$ (v_1)= deg(v_2) = 2

For i = 3, 6,…, 3n, supp $\gamma_{se}{}^{\times}$ (v_i) = deg(v_{i-1}) $\times$deg(v_{i+1}) = 4

Supp $\gamma_{se}{}^{\times}$ (v_{3n+2}) = deg(v_{3n+1}) = 2

Hence supp $\gamma_{se}{}^{\times}$ (G) = supp $\gamma_{se}{}^{\times}$ $(v_1)\times\prod_{i=1}^{n} supp\ \gamma_{se}{}^{\times}(v_{3i})$ $\times$ supp $\gamma_{se}{}^{\times}$ (v_{3n+2})= 2$\times$ 4^n $\times$2 = 4^{n+1}.

Theorem 2.17: Let G = $C_{3n}, n \in N$ Then

supp $\gamma_{se}{}^{+}$ (G) = 4nand

supp $\gamma_{se}{}^{\times}$ (G) = 4^n.

Proof: Let G = $C_{3n}, n \in N$ Let V(G) =$\{v_i;\ 1 \leq i \leq 3n\}$

Then S_1=$\{v_1, v_4, v_7, \dots, v_{3n-2}\}$, S_2 = $\{v_2, v_5, v_8, \dots, v_{3n-1}\}$, S_3 = $v_3, v_6, v_9, \dots, v_{3n}\}$ are three distinct γ_{se} - sets of G.deg(v_i) = 2, 1 $\leq$ i $\leq$ 3n.

Consider S_1= $\{v_1, v_4, v_7, \dots, v_{3n-2}$. (Proof is similar for S_2 and S_3)

For i=1, 4, 7,,,…,3n – 2, supp $\gamma_{se}{}^{+}$ (v_i)= deg(v_{i-1})+ deg(v_{i+1})=2+2=4

Hence supp $\gamma_{se}{}^{+}$ (G) =$\sum_{i=1}^{n}$ supp γ_{se}^{+} (v_{3i-2}) = 4n

For i=1, 4, 7,…3n – 2, supp $\gamma_{se}{}^{\times}$ (v_i)=deg(v_{i-1})×deg(v_{i+1})=2$\times$ 2=4

Hence supp $\gamma_{se}{}^{\times}$ (G) =$\prod_{i=1}^{n} supp\ \gamma_{se}{}^{\times}(v_{3i-2})$ = 4^n

Hence supp $\gamma_{se}{}^{+}$ (G) = 4n and supp $\gamma_{se}{}^{\times}$ (G) = 4^n

Theorem 2.18: Let G = $K_{1,n}, n \in N$ Then

supp $\gamma_{se}{}^{+}$ (G) = n and

supp $\gamma_{se}{}^{\times}$ (G) = 1

Proof: Let G = $K_{1,n}, n \in N$. Let V(G) ={ v, v_i; 1≤ $i \leq n$ } where v is the central point. Then S = {v} is the unique γ_{se} - set of G.deg(v)=n and deg(v_i)= 1, 1≤ $i \leq n$

supp γ_{se}^{+} (v) = $\sum_{i=1}^{n} \deg(v_i)$= n(1) = n

Hence supp γ_{se}^{+} (G) = supp γ_{se}^{+} (v) = n.

supp $\gamma_{se}^{\times}$ (v) =$\prod_{i=1}^{n} \deg(v_i)$= 1^n = 1

Hence supp $\gamma_{se}^{\times}$ (G) =supp $\gamma_{se}^{\times}$ (v) = 1

Theorem 2.19: Let G = K_n, $n \in N$. Then

supp γ_{se}^{+} (G) = $(n-1)^2$ and

supp $\gamma_{se}^{\times}$ (G) = $(n-1)^{n-1}$

Proof: Let G = $K_n, n \in N$. Let v_i; 1≤ $i \leq n$ be the points of G. Then S_i = {v_i}, 1≤ $i \leq n$ are n distinct γ_{se} - sets of G.deg (v_i)= n – 1, 1≤ $i \leq n$.

Consider the set S_i (Proof is similar for the other sets)

supp γ_{se}^{+} (v_i) = $\sum_{j=1, j \neq i}^{n} \deg(v_j)$ = (n–1)(n–1) =$(n-1)^2$,1≤ $i \leq n$

Hence supp γ_{se}^{+} (G) =supp γ_{se}^{+} (v_i) = $(n-1)^2$.

supp $\gamma_{se}^{\times}$ (v_i) = $\prod_{j=1, j\neq i}^{n} \deg(v_j)$ = $(n-1)^{n-1}$,1≤ $i \leq n$

Hence supp $\gamma_{se}^{\times}$ (G) =supp $\gamma_{se}^{\times}$ (v_i) = $(n-1)^{n-1}$

Hence supp γ_{se}^{+} (G) = $(n-1)^2$ and supp $\gamma_{se}^{\times}$ (G) = $(n-1)^{n-1}$

Remark 2.20: Let G be a nontrivial connected strong efficient graph on n ≥ 2 points. Then 1≤ supp γ_{se}^{+} (G) ≤ $(n-1)^2$ and 1≤supp $\gamma_{se}^{\times}$ (G) ≤ $(n-1)^{n-1}$.

Theorem 2.21: Let G = $D_{m,n}$, m, n ∈ N. Then

supp γ_{se}^{+}(G) = $m+(n+1)^2$, if m ≥ n

supp $\gamma_{se}^{\times}$ (G) = $(n+1)^{n+1}$, if m ≥ n

Proof: Let G = $D_{m,n}$. m $\geq$ n. Let V(G) ={ u, u_i, v, v_j; 1$\leq$ i $\leq$ n}

Let E(G) = {uv, uu_i, vv_j; 1$\leq$ i $\leq$ m, 1$\leq$ i $\leq$ n}.

Then S = { u, v_j ; $1 \leq j \leq n$ } is the unique γ_{se} - set of G.deg (u) = m + 1, deg (v) = n + 1, deg (u_i) = deg (v_j) = 1, 1$\leq$ $i \leq m$, 1$\leq$ $j \leq n$

supp $\gamma_{se}{}^{+}$ (u) = $\sum_{i=1}^{m} \deg(u_i) + \deg(v)$ = m +n+1

For j = 1, 2,…, n, supp $\gamma_{se}{}^{+}$ (v_j) = deg(v) = n +1

Hence supp $\gamma_{se}{}^{+}$ (G) = supp γ_{se}^{+} (u) +$\sum_{j=1}^{n}$ supp $\gamma_{se}{}^{+}(v_j)$ = m +n +1+n(n+1) = $m + (n+1)^2$

supp $\gamma_{se}{}^{\times}$ (u) = $\prod_{i=1}^{m} \deg(u_i) \times \deg(v)$ = 1(n+1) = n+1

For j = 1, 2,…, n, supp$\gamma_{se}{}^{\times}$ (v_j) = deg(v) = n +1

Hence supp $\gamma_{se}{}^{\times}$ (G) = supp $\gamma_{se}{}^{\times}$(u) $\times \prod_{i=1}^{n}$ supp $\gamma_{se}{}^{\times}(v_j)$ =(n+1) $\times$ $(n+1)^n$=$(n+1)^{n+1}$

3. Conclusion

In this paper, open support strong efficient domination number of some standard graphs under addition and multiplication is studied. Similar studies can be done for the other domination parameters.

References

[1] S. Balamurugan, M. Anitha, C. Karnan and P. Palani Kumar, Open Support of a Graph Under Multiplication, International Journal of Mathematics Trends and Technology, Vol. 65, Issue 5, (May 2019), 134-138.

[2] S. Balamurugan, M. Anitha, P. Aristotle and C. Karnan, A Note on Open Support of a Graph Under Addition I, International Journal of Mathematics Trends and Technology, Vol. 65, Issue 5, (May 2019), 110-114.

[3] Bange. D. W, Barkauskas. A. E. and Slater. P. J., Efficient dominating sets in graphs, Application of Discrete Mathematics,189-199, SIAM, Philadelphia(1988).

[4] Harary. F., Graph Theory, Addison – Wesley(1969).

[5] F. Harary, T. W. Haynes and P. J. Slater, Efficient and excess domination in graphs, J. Combin. Math. Combin .Comput., 26(1998),83-95.

[6] Haynes. T W., Stephen T. Hedetmiemi, Peter J. Slater. Fundamentals of domination in graphs, Advanced Topics, Marce; Dekker, Inc, New York (1998).

[7] Meena. N, Further Results on Sum of Strong Efficient Domination Number and Chromatic Number, International Journal of Innovative Science, Engineering & Technology, Vol. 3, Issue 11, (November 2016), 300-304.

[8] Meena. N, Strong Efficient Domination Number of Inflated Graphs of Some Standard Graphs, International Journal of Scientific and Innovative Mathematical Research, Vol. 2, Issue 5, (May 2014), 435-440.

[9] Meena. N, Subramanian. A, Swaminathan. V, Strong efficient domination and strong independent saturation number of graphs, International Journal of Mathematics and Soft Computing, Vol. 3, No. 2 (2013), 41-48

[10] Meena. N, Subramanian. A, Swaminathan. V, Strong Efficient Domination in Graphs, International Journal of Innovative Science, Engineering & Technology, Vol. 1, Issue 4, (June 2014), 172-177.

[11] Sampath Kumar. E and Pushpa Latha. L, Strong Weak domination and domination balance in a graph, Discrete Math., 161: (1996), 235 -242.