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Skolem Difference Mean Graphs

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Abstract

Skolem difference mean labelings of some predefined graphs are studied.

Keywords: Skolem difference mean labeling, skolem difference mean graphs.

AMS Subject Classification (2010): 05C78

1. Introduction

Throughout this paper we consider only finite, undirected, simple graphs without loops or multiple edges. Let G be a graph with p vertices and q edges. For all terminologies and notations we follow [2]**.** A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey can be found in [4]. The concept of mean labeling was introduced by S. Somasundaram and R. Ponraj in [8] and the concept of skolem mean labeling was introduced by V. Balaji, D.S.T. Ramesh and A. Subramanian in [1]. Motivated by these definitions skolem difference mean labeling was introduced by K. Murugan and A. Subramanian in [7]and further results were proved in [5,6]. The following definitions are necessary for the present study. Definitions and concepts which are not specifically mentioned here are in the sense of Harary [2].

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Definition 1.1. A path is a walk if all the points and lines are distinct. A path on n vertices is denoted by P_n.

Definition 1.2.A bigraph (or bipartite graph) G is a graph whose point set can be partitioned into two subsets V_1 and V_2 such that every line of G joins V_1 with V_2.

Definition 1.3. A star is a complete bigraph $K_{1,n}$

Definition 1.4. If G has order n, the corona of G with H denoted by $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

Definition 1.5. A cycle denoted by C_n, consisting of n points, is a path with same initial and terminal point.

Definition 1.6. The graph $G^{(t)}$ denotes the one point union of t copies of G.

Definition 1.7. $G_1@G_2$ is nothing but one point union of G_1 and G_2. [3]

Definition 1.8. A graph G= (V, E) with p vertices and q edges is said to be a mean graph if it is possible to label the vertices $x \varepsilon V$ with distinct elements from the set 0,1,…q in such a way that the edge e=uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u)+f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u)+f(v) is odd and the resulting edges are distinct.

Definition 1.9. A graph $G(V,E)$ with p vertices and q edges is said to have skolem mean labeling if it is possible to label the vertices $x \varepsilon V$ with distinct elements $f(x)$ from $0,1,2,3 \ldots p$ in such a way that the edge $e=uv$ is labeled with $\frac{f(u)+f(v)}{2}$ if $|f(u)+f(v)|$ is even and $\frac{f(u)+f(v)+1}{2}$ if $|f(u)+f(v)|$ is odd and the resulting labels of the edges are distinct and from $1,2,3 \ldots p$. A graph that admits skolem mean labeling is called skolem mean graph.

Definition 1.10. A graph *G(V,E)* with *p* vertices and *q* edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \varepsilon V$ with distinct elements *f(x)* from *1,2,3…p+q* in such a way that the edge *e=uv* is labeled with $\frac{|f(u)-f(v)|}{2}$ if |*f(u)-f(v)*| is even and $\frac{|f(u)-f(v)|+1}{2}$ if |*f(u)-f(v)*| is odd and the resulting labels of the edges are distinct and from *1,2,3…q*. A graph that admits skolem difference mean labeling is called skolem difference mean graph.

The skolem difference mean labeling of C_3 is given in figure 1.

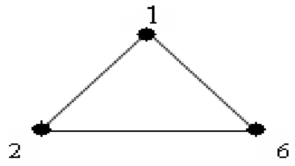

Figure 1

2. Main Results

Theorem 2.1 The path P_n is skolem difference mean for all values of *n*.

Proof: Let $V(P_n) = \{v_i; 1 \le i \le n\}$ and $E(P_n) = \{v_i v_{i+1}; 1 \le i \le n-1\}$

Define the function $f: V(P_n) \to \{1,2,3\ldots 2n-1\}$ as follows.

Case i. When *n* is odd.

$$f(v_{2i+1}) = 1+2i,\ 0 \le i < \frac{n+1}{2}$$

$$f(v_{2i}) = 2n+1-2i, 1 \le i < \frac{n+1}{2}$$

Case ii. When *n* is even.

$$f(v_{2i+1}) = 1+2i,\ 0 \le i < \frac{n}{2}$$

$$f(v_{2i}) = 2n+1-2i;\ 1 \le i \le \frac{n}{2}$$

In both the cases the induced edge labels are *1,2…n-1.*

Hence, the theorem.

The skolem difference mean labeling of the paths P_5 and P_6 are given below.

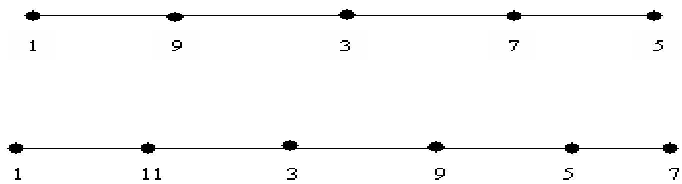

Figure 2

Theorem 2.2. If the path P_n is skolem difference mean, then the twig G obtained from the path P_n by attaching exactly two pendent edges to each internal vertex of the path is skolem difference mean.

Proof: Let the path P_n be skolem difference mean.

Let $V(P_n)=\{v_i;1\le i\le n\}$and $E(P_n)=\{v_iv_{i+1};1\le i\le n-1\}$.

Let $f: V(P_n) \rightarrow \{1,2\ldots 2n-1\}$ be the skolem difference mean labeling of the path.

Let f^* be the induced edge labeling of f.

Let G be the twig.

Let $V(G) =\{v_i,u_j,w_j;\ 1\le i\le n-1, 2\le j\le n-1\}$ and $E(G) = \{v_iv_{i+1},v_ju_j,v_jw_j;\ 1\le i\le n-1,\ 2\le j\le n-1\}$

Define $g: V(G)\rightarrow f(P_n)$ as follows.

Case i: When n is odd.

$$g(v_{2i+1})= f(v_{2i+1});0\le i< \frac{n+1}{2}$$

$$g(v_{2i})= f(v_{2i})+4n-8;1\leq i<\frac{n+1}{2}$$

$$g(u_{2i})= f(v_{2i})-4+4i;1\leq i<\frac{n+1}{2}$$

$$g(u_{2i+1})=f(v_{2i+1})+4n-6-4i;1\leq i<\frac{n-1}{2}$$

$$g(w_{2i})= f(v_{2i})+2n-8+4i;1\leq i<\frac{n+1}{2}$$

$$g(w_{2i+1})= f(v_{2i+1})+2n-2-4i\ ;1\leq i<\frac{n-1}{2}$$

Case ii: When *n* is even.

$$g(v_{2i+1})= f(v_{2i+1});0\leq i<\frac{n}{2}$$

$$g(v_{2i})= f(v_{2i})+4n-8;1\leq i\leq\frac{n}{2}$$

$$g(u_{2i})= f(v_{2i})-4+4i;1\leq i<\frac{n}{2}$$

$$g(u_{2i+1})=f(v_{2i+1})+4n-6-4i;1\leq i<\frac{n}{2}$$

$$g(w_{2i})= f(v_{2i})+2n-8+4i;1\leq i<\frac{n}{2}$$

$$g(w_{2i+1})= f(v_{2i+1})+2n-2-4i\ ;1\leq i<\frac{n}{2}$$

Let g^* be the induced edge labeling of g.

Then $g^*(v_iv_{i+1})= f^*(v_iv_{i+1})+2n-4;\ 1\leq i\leq n-1$

$$g^*(v_iu_i)=2n-2-i\ ;2\leq i\leq n-1$$

$$g^*(v_iw_i)=n-i\ ;2\leq i\leq n-1$$

In both the cases the induced edge labels are 1,2…3n-5.Hence the theorem.

The skolem difference mean labeling of the twigs obtained from P_5 and P_6 are given in figures 3 and 4.

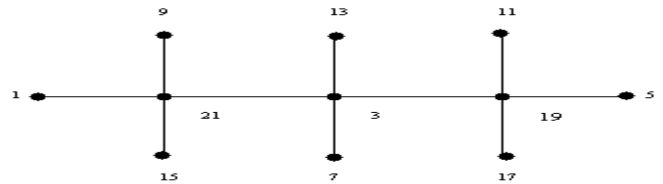

Figure 3

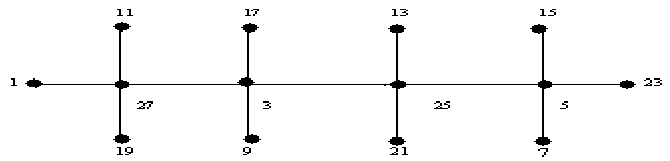

Figure 4

Theorem 2.3. If the path P_n is skolem difference mean, then the graph $P_n \odot S_2$ is skolem difference mean for all values of n.

Proof: Let $v_1, v_2 \ldots v_n$ be the vertices of the path P_n.

Let f be the skolem difference mean labeling of the given path as defined in theorem 2.1.

Let f^* be the induced edge labeling of f.

Let $V(P_n \odot S_2) = \{v_i, v_{i'}, v_{i''}; 1 \le i \le n\}$ and $E(P_n \odot S_2) = \{v_i v_{i+1}, v_j v_{j'}, v_j v_{j''}; 1 \le i \le n-1, 1 \le j \le n\}$

Define a labeling $g: (P_n \odot S_2) \to \{1, 2 \ldots 6n-1\}$ as follows.

Case i: n is odd.

$$g(v_{2i+1}) = f(v_{2i+1}); 0 \le i < \frac{n+1}{2}$$

$$g(v_{2i}) = f(v_{2i}) + 4n; 1 \le i < \frac{n+1}{2}$$

$$g(v_{2i+1'})=2+10i;\ 0\leq i<\frac{n+1}{2}$$

$$g(v_{2i'})=g(v_2)+4-10i;\ 1\leq i<\frac{n+1}{2}$$

$$g(v_{2i+1''})=4+10i;\ 0\leq i<\frac{n+1}{2}$$

$$g(v_{2i''})=g(v_2)+2-10i;\ 1\leq i<\frac{n+1}{2}$$

Case ii: *n* is even.

$$g(v_{2i+1})=f(v_{2i+1});0\leq i<\frac{n}{2}$$

$$g(v_{2i})=f(v_{2i})+4n;1\leq i\leq\frac{n}{2}$$

$$g(v_{2i+1'})=2+10i;0\leq i<\frac{n}{2}$$

$$g(v_{2i'})=g(v_2)+4-10i;1\leq i\leq\frac{n}{2}$$

$$g(v_{2i+1''})=4+10i;0\leq i<\frac{n}{2}$$

$$g(v_{2i''})=g(v_2)+2-10i;1\leq i\leq\frac{n}{2}$$

Let g^* be the induced edge labeling of g.

Then we have

$$g^*(v_iv_{i+1})=f^*(v_iv_{i+1})+2n$$

$$g^*(v_iv_{i'})=2i-1$$

$$g^*(v_iv_{i''})=2i$$

In both the case the induced edge labels are *1,2…3n-1*. Hence, the theorem.

The skolem difference mean labeling of the graph $P_5 \odot S_2$ is given in figure 5

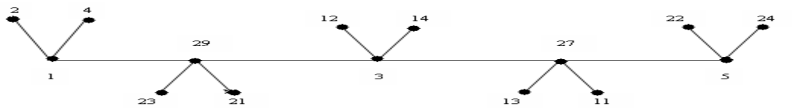

Fig 5

Theorem 2.4. The graph $C_5@K_{1,n}$ is skolem difference mean for all values of *n*.

Proof: Let V $(C_5@K_{1,n})=\{u_i,v_j;\ 1\le i\le 5, 1\le j\le n\}$ and

$$E(C_5@K_{1,n})=\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1, u_1v_j;\ 1\le j\le n\}$$

Define a function f: V $(C_5@K_{1,n}) \rightarrow \{1,2\ldots 2n+10\}$ by

$$f(u_1)=1$$

$$f(u_2)=9$$

$$f(u_3)=5$$

$$f(u_4)=4$$

$$f(u_5)=10 \text{ and}$$

$$f(v_j)=10+2j;\ 1\le j\le n$$

Then the induced edge labels are $1,2\ldots n+5$. Hence $C_5@K_{1,n}$ is skolem difference mean for all values of *n*.

The skolem difference mean labeling of $C_5@\ K_{1,3}$ is given in figure 6.

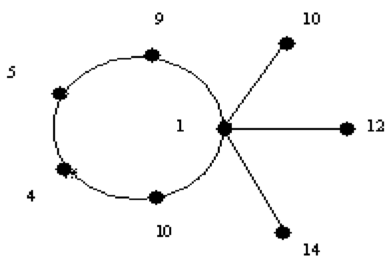

Fig 6

Theorem 2.5. The graph $K_1 \odot K_{1,n}$ is skolem difference mean for all values of n.

Proof: Let $V(K_1 \odot K_{1,n}) = \{u, u_i, v, v_i \,/\, 1 \le i \le n\}$ and $E(K_1 \odot K_{1,n}) = \{uv, uu_i, u_iv_i \,/\, 1 \le i \le n\}$

Define a function $f: V(K_1 \odot K_{1,n}) \to \{1, 2 \ldots 4n+3\}$ by

$$f(u)=1$$

$$f(u_i)=4n+2-2i,\ 1 \le i \le n$$

$$f(v) = 4n+3$$

$$f(v_i) = 4n+3-4i,\ 1 \le i \le n$$

Let f^* be the induced edge labeling of f. Then

$$f^*(uv)=2n+1$$

$$f^*(uu_i)=2n+1-i;\ 1 \le i \le n$$

$$f^*(u_iv_i)=i;\ 1 \le i \le n$$

The induced edge labels are $1, 2 \ldots 2n+1$. Hence the theorem.

The skolem difference mean labeling of $K_1 \odot K_{1,4}$ is given in figure 7.

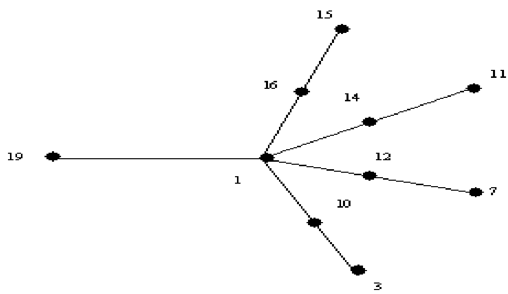

Fig7

Theorem 2.6. Let G be the graph $K_{1,n}^{(2)}$. Then G is skolem difference mean for all values of n.

Proof: Let $V(G)=\{u, u_i', u_i''; 1 \le i \le n\}$ and $E(G)=\{uu_i', uu_i''; 1 \le i \le n\}$

Define a function $f: V(G) \to \{1, 2 \ldots 4n+1\}$ by

$$f(u)=4n+1$$

$f(u_i')=2i-1;\ 1\leq i\leq n$

$f(u_i'')=2n-1+2i;\ 1\leq i\leq n$

Let f^* be the induced edge labeling of f.

Then $f^*(uu_i')=2n+1-i;\ 1\leq i\leq n$

$f^*(uu_i'')=n+1-i;\ 1\leq i\leq n$

Then the induced edge labels are *1,2..2n*. Hence, the theorem.

The skolem difference mean labeling of the graph $K_{1,3}^{(2)}$ is given in figure 8.

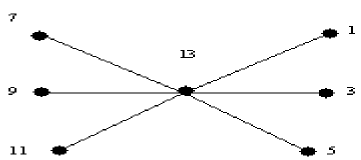

Fig 8

Theorem 2.7. Let G be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of S_2.Then G is skolem difference mean for all values of n.

Proof: Let $V(G)=\{u,u_i,u_i',u_i'';1\leq i\leq n\}$ and $E(G)=\{uu_i,\ u_iu_i',\ u_iu_i'';1\leq i\leq n\}$

Define a function $f:V(G)\rightarrow\{1,2\ldots 6n+1\}$ by

$f(u)=6n+1$

$f(u_i)=2i-1;1\leq i\leq n$

$f(u_i')=4n+3-2i;1\leq i\leq n$

$f(u_i'')=4n-2i;1\leq i\leq n$

Let f^* be the induced edge labeling of f. Then

$f^*(uu_i)=3n+1-i;\ 1\leq i\leq n$

$f^*(u_iu_i')=2n+2-2i;\ 1\leq i\leq n$

$f^*(u_iu_i'')=2n+1-2i;\ 1\leq i\leq n$

Then the induced edge labels are *1,2…3n*. Hence, the theorem.

The skolem difference mean labeling of the graph $K_{1,3} \odot S_2$ is given in figure 9.

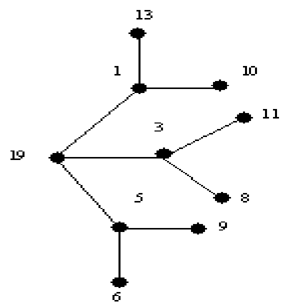

Fig 9

Theorem 2.8 Let *G* be a graph obtained by identifying a pendant vertex of P_3 with a leaf of $K_{1,n}$. Then *G* is skolem difference mean for all values of n.

Proof: Let $V(G)=\{v,v_i,u,w;\ 1\le i\le n\}$ and $E(G)=\{vv_i,v_nu,uw;\ 1\le i\le n\}$

Define $f:V(G)\rightarrow\{1,2\ldots 2n+5\}$ by

$f(v)=2n+5$

$f(v_i)=2i-1;\ 1\le i\le n$

$f(u)=2n+3$

$f(w)=2n+1$

Let f^* be the induced edge labeling of f.

Then

$f^*(vv_i)=n+3-i;\ 1\le i\le n$

$f^*(v_nu)=2$

$f^*(uw)=1$

The induced edge labels are $1,2\ldots n+2$. Hence, the theorem.

The skolem difference mean labeling of the graph obtained by identifying a pendent vertex of P_3 with a leaf of $K_{1,7}$ is given in figure 10.

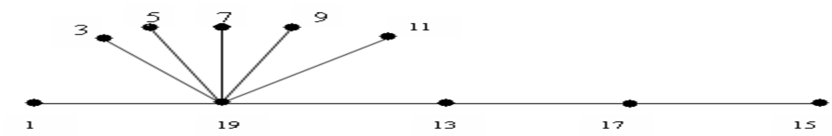

Fig 10

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