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Path Induced Vertex Covering for Intuitionistic Fuzzy Graph and its Application in Disaster Management

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Abstract

Graph theory is one of the popular tools for modelling real-world problems. Due to the fuzziness and uncertainty surrounding these problems, intuitionistic fuzzy graphs have taken the lead over both graph theory and fuzzy graph theory. Due to their increased use, conditional covering problems and set covering problems are now receiving a lot of attention. Here, we added a new vertex covering called the Intuitionistic Fuzzy Path Induced Vertex Covering (IFPVC) to the Intuitionistic Fuzzy Graph (IFG). An algorithm is created to find the IFPVC set and IFPV covering number for IFG. Additionally, an application in disaster management is given to examine the viability of the suggested covering set.

Keywords: Intuitionistic Fuzzy Graph, Intuitionistic Fuzzy Path Induced Vertex Covering, covering number, disaster management

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1. Introduction

Covering problems are a prominent area of research in graph theory due to its increasing usage in practical problems. Facility location problems can be resolved using covering problems. Quite a lot of work has been done in solving covering and facility location problems in a crisp sense. But, due to the uncertainty and ambiguity in real-life situations, the parametric values cannot remain crisp. Consequently, fuzzy graph theory can be a key tool in dealing with such circumstances. In a fuzzy context, a number of researchers have investigated vertex-covering problems, edge-covering problems, clique-covering problems, etc. Anushree Bhattacharya and Madhumangal Pal studied Fuzzy covering problems of fuzzy graphs and its application to investigate the Indian economy in the new normal [6], they also introduced vertex covering problems for fuzzy graphs and applied it in the installation of CCTV [5], Sonia Mandal and Nupur Patra analysed the application of fuzzy covering problems in disaster management [12].

Intuitionistic Fuzzy Sets (IFS) by K. T. Attannasov [4] emerged after fuzzy sets, and IFS took a step above them since its elements are represented by giving them marks for belongingness and nonbelongingness. The usage of intuitionistic fuzzy graphs to model real-world problems is therefore essential. Intuitionistic fuzzy relation was introduced by K.T Attannasov, in 2006 Parvathi and Karunambigai discussed the concept of Intuitionistic Fuzzy Graphs [15]. Akram and Akmal defined Intuitionistic fuzzy graph structures [3]. SankarSahoo and Madhumangal put forward covering and paired domination for Intuitionistic fuzzy graphs using strong arcs and specified its properties[16].

In this study, we introduced a new vertex covering for intuitionistic fuzzy graphs called Intuitionistic Fuzzy Path Induced Vertex Covering (IFPVC), which was inspired by the numerous applications of covering problems and made feasible by the literature's relative scarcity of intuitionistic fuzzy covering problems. An algorithm is developed to determine the IFPVC sets and covering number to the IFGs. We also utilised it to find the closest relief centre for a region where a disaster had struck.

2. Prerequisites

Definition 2.1[4] Let X be a universe of discourse, an Intuitionistic Fuzzy Set (IFS) X* on X is an object having the form X* = { < x, $\tau_{X^*}(x), \xi_{X^*}(x)$ > | x ∈ X }, where the function $\tau_{X^*}(x) : X \to [0,1]$ is the degree of membership and, $\xi_{X^*}(x) : X \to [0,1]$ is the degree of non-membership of the elements in the set X satisfying $0 \leq \tau_{X^*}(x) + \xi_{X^*}(x) \leq 1$.

Definition 2.2 [15] An Intuitionistic Fuzzy Graph (IFG) is a pair $\tilde{G} = (\tilde{V}, \tilde{E})$ where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_n\}$ denote the vertex set of G and, $\tilde{E} \subseteq \tilde{V} \times \tilde{V}$ denote the edge set of G˜ such that $\tau_{\tilde{V}} : \tilde{V} \to [0,1]$

And $\xi_{\tilde{V}} : \tilde{V} \to [0,1]$ denote the degree of belongingness and degree of non belongingness of nodes $\tilde{v}_i \in \tilde{V}$, satisfying $0 \leq \tau_{\tilde{V}} + \xi_{\tilde{V}} \leq 1, \forall \tilde{v}_i \in \tilde{V}, i = 1,2,\cdots,n$.

For each $(\tilde{v}_i, \tilde{v}_j) \in \tilde{E}$, $\tau_{\tilde{E}}: \tilde{E} \to [0,1]$ and $\xi_{\tilde{E}}: \tilde{E} \to [0,1]$ such that

$\tau_{\tilde{E}}(\tilde{v}_i, \tilde{v}_j) \leq \min[\tau_{\tilde{V}}(\tilde{v}_i), \tau_{\tilde{V}}(\tilde{v}_j)]$,

$\xi_{\tilde{E}}(\tilde{v}_i, \tilde{v}_j) \leq \max[\xi_{\tilde{V}}(\tilde{v}_i), \xi_{\tilde{V}}(\tilde{v}_j)]$

and $0 \leq \tau_{\tilde{E}}(\tilde{v}_i, \tilde{v}_j) + \xi_{\tilde{E}}(\tilde{v}_i, \tilde{v}_j) \leq 1, \forall (\tilde{v}_i, \tilde{v}_j) \in \tilde{E}$, $i = 1,2,\ldots,n$.

Definition 2.3 [2], [10] A sequence of distinct vertices $\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_n$ in an IFG $\tilde{G} = (\tau_{\tilde{V}}, \xi_{\tilde{V}}, \tau_{\tilde{E}}, \xi_{\tilde{E}})$ is said to be a Path P, such that for all $(i, j = 1,2,\cdots, n)$ either one of the following conditions

are satisfied

$\tau_{\tilde{E}_{i,j}} > 0$ and $\xi_{\tilde{E}_{i,j}} = 0$ for some i and j

$\tau_{\tilde{E}_{i,j}} = 0$ and $\xi_{\tilde{E}_{i,j}} > 0$ for some i and j

(iii) $\tau_{\tilde{E}_{i,j}} > 0$ and $\xi_{\tilde{E}_{i,j}} > 0$ for some i and *j*

Definition 2.4 [10] Any two vertices connected by a path in an intuitionistic fuzzy graph are construed to be connected.

Definition 2.5 [11] A Generalized Triangular Intuitionistic Fuzzy Number (GTIFN) is defined as the number $\tilde{\alpha} = (k, l, m; M_{\tau_{\tilde{\alpha}}}, M_{\xi_{\tilde{\alpha}}})$

Whose membership and non-membership are given by

$$\tau(\tilde{\alpha})(x) = \begin{cases} \frac{(x-k)M_{\tau_{\tilde{\alpha}}}}{l-k}, k \leqslant x \leqslant l \\ \frac{(m-x)M_{\xi_{\tilde{\alpha}}}}{m-l}, l \leqslant x < m \\ 0, k > x \quad or \quad x > m \end{cases} \quad \xi(\tilde{\alpha})(x) = \begin{cases} \frac{l-x+(x-a)M_{\tau_{\tilde{\alpha}}}}{l-k}, \quad if k \leq x \leq l \\ \frac{x-l+(m-x)M_{\xi_{\tilde{\alpha}}}}{m-a}, \quad if \quad l \leq x \leq m \\ 1 if x < k or x > m \end{cases} \tag{1}$$

Where $M_{\tau_{\tilde{\alpha}}}$ denote the maximum degree of membership and $M_{\xi_{\tilde{\alpha}}}$ denote the minimum degree of membership, such that $M_{\tau_{\tilde{\alpha}}} \in [0,1], M_{\xi_{\tilde{\alpha}}} \in [0,1]$ and $0 \leq M_{\tau_{\tilde{\alpha}}} + M_{\xi_{\tilde{\alpha}}} \leq 1$,

Definition 2.6 [13] Let $\tilde{\alpha}_i = (k_i, l_i, m_i; M_{\tau_{\tilde{\alpha}_i}}, M_{\xi_{\tilde{\alpha}_i}}) i = 1,2$ be two GTIFNs then the operations laws for GTIFNs is

(i) $\tilde{\alpha}_1 + \tilde{\alpha}_2 = ((k_1 + k_2, l_1 + l_2, m_1 + m_2); min(M_{\tau_{\tilde{\alpha}_1}}, M_{\tau_{\tilde{\alpha}_2}}), max(M_{\xi_{\tilde{\alpha}_1}}, M_{\xi_{\tilde{\alpha}_2}}))$

(ii) $\tilde{\alpha}_1 \tilde{\alpha}_2 = ((k_1 k_2, l_1 l_2, m_1 m_2); min(M_{\tau_{\tilde{\alpha}_1}}, M_{\tau_{\tilde{\alpha}_2}}), max(M_{\xi_{\tilde{\alpha}_1}}, M_{\xi_{\tilde{\alpha}_2}}))$

(iii) $\lambda \tilde{\alpha}_1 = ((\lambda k_1, \lambda l_1, \lambda m_1), M_{\tau_{\tilde{\alpha}_1}}, M_{\xi_{\tilde{\alpha}_1}})$

Definition 2.7 [1], [19] Let $\tilde{\alpha} = (k, l, m; M_{\tau_{\tilde{\alpha}}}, M_{\xi_{\tilde{\alpha}}})$ be a generalized triangular intuitionistic fuzzy number then the score function $\tilde{S}(\tilde{\alpha})$ and accuracy function $\tilde{H}(\tilde{\alpha})$ of $\tilde{\alpha}$ are defined by

$$\tilde{S}(\tilde{\alpha}) = \frac{(k + 2l + m)M_{\tau_{\tilde{\alpha}}}}{4} \quad (2)$$

$$\tilde{H}(\tilde{\alpha}) = \frac{(k + 2l + m)(1 - M_{\xi_{\tilde{\alpha}}})}{4} \quad (3)$$

Definition 2.8 [1],[19] If $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be two GTIFNs, $\tilde{S}(\tilde{\alpha}_i)$ and $\tilde{H}(\tilde{\alpha}_i)$ be the score function and accuracy function of $\tilde{\alpha}_i$ s, then (i) If $\tilde{S}(\tilde{\alpha}_1)<\tilde{S}(\tilde{\alpha}_2)$ then $\tilde{\alpha}_1 < \tilde{\alpha}_2$

ii, If $\tilde{S}(\tilde{\alpha}_1)=\tilde{S}(\tilde{\alpha}_2)$and $\tilde{H}(\tilde{\alpha}_1)=\tilde{H}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1=\tilde{\alpha}_2$

iii, If $\tilde{S}(\tilde{\alpha}_1)=\tilde{S}(\tilde{\alpha}_2)$ and $\tilde{H}(\tilde{\alpha}_1)=\tilde{H}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 < \tilde{\alpha}_2$.

3. Path induced vertex covering for IFG

Definition 3.1 Consider an IFG $\tilde{G} = (\tilde{V}, \tilde{E})$, the membership and non-membership values to the edges and vertices are represented by a generalized triangular intuitionistic fuzzy number. Let x and y be two vertices in the vertex set of G, and let P be a path from x to y, that is there exist a sequence of vertices $x_1, x_2, \cdots, x_n \in \tilde{V}(\tilde{G})$ such that $(x, x_1), (x_1, x_2), \cdots (x_n, y)$ are edges of $\tilde{G}$. The length of a path is the sum of the weights of edges in that path. Suppose, $((k_1, l_1, m_1); M_{\tau_{\tilde{\alpha}_1}}, M_{\xi_{\tilde{\alpha}_1}})$, $((k_2, l_2, m_2); M_{\tau_{\tilde{\alpha}_2}}, M_{\xi_{\tilde{\alpha}_2}})$, $((k_3, l_3, m_3); M_{\tau_{\tilde{\alpha}_3}}, M_{\xi_{\tilde{\alpha}_3}})$, …, $(k_{n+1}, l_{n+1}, m_{n+1}); M_{\tau_{\tilde{\alpha}_{n+1}}}, M_{\xi_{\tilde{\alpha}_{n+1}}})$ be the edge weights of the corresponding edges of the path P. Then the length of the path P is defined as

$$l_p(x,y) = ((\sum_{i=1}^{n+1} k_i, \sum_{i=1}^{n+1} l_i, \sum_{i=1}^{n+1} m_i), \min_{i=1}^{n+1} M_{\tau_{\tilde{\alpha}_i}}, \min_{i=1}^{n+1} M_{\xi_{\tilde{\alpha}_i}}) \tag{4}$$

Therefore corresponding to every path from x to y, we can find $l_p(x,y)$.

Definition 3.2 Let $\tilde{G} = (\tilde{V}, \tilde{E})$ be an intuitionistic fuzzy graph, then the shortest path between x and y is the path with minimum distance.

Definition 3.3 Let P_1, P_2, ⋯, $P_n$be the paths from x to y, then the path P_i such that $l_p(x,y)$.is the minimum is called the Shortest path from x to y. The minimum can be calculated by definition 2.8. Let P = P_i be the shortest path from x to y, then define $l_d(x,y)$.= $l_p(x,y)$ is the distance from x to y.

Theorem 3.1 For every connected IFG G, between any two vertices x and y, $l_d(x,y)$ exists and it is unique.

Proof: In an intuitionistic fuzzy graph, any two vertices connected by a path are assumed to be connected. Therefore there exists at least one path between the vertices. Let P_1,P_2, ⋯$P_n$be the distinct paths connecting vertices x and y. Suppose P_i be the shortest path among them. Then, $l_d(x,y)$. = $l_{P_i}(x,y)$, that is $l_d(x,y)$ unique.

Theorem 3.2 For every two vertices x and y, in a connected IFG G, $l_d(x,y) = l_d(y,x)$.

Proof: Proof is obvious from the definition itself.

Definition 3.4 Let $\tilde{\alpha}_i = \left((k_i, l_i, m_i); M_{\tau_{\tilde{\alpha}_i}}, M_{\xi_{\tilde{\alpha}_i}}\right)$ are GTFNs for $i = 1,2,\cdots,n$. Then the membership average index and non-membership average index for $\tilde{\alpha}_i$ s are defined as

$$AI_\tau(\tilde{\alpha}_i) = \frac{(k+l+m)M_{\tau_{\tilde{\alpha}_i}}}{3} \tag{5}$$

$$AI_\xi(\tilde{\alpha}_i) = \frac{(k+l+m)M_{\xi_{\tilde{\alpha}_i}}}{3} \tag{6}$$

For every vertex $x \in \tilde{V}(\tilde{G})$, assign a number called covering radius denoted by $\Re(x)$. In this paper we consider that number as a generalized triangular intuitionistic fuzzy number. The covering radius can be fixed for every vertex of the IFG or it can depend on the vertex.

Definition 3.5 Let $\tilde{G} = (\tilde{V}, \tilde{E})$ be an IFG, and $l_d(x,y)$ denote the distance or length between the vertices x and y. If

$$AI_\tau(l_d(x,y)) \leq AI_\tau(\Re(x)), AI_\xi(l_d(x,y)) \geq AI_\xi(\Re(x))$$

Then, the vertex y is said to be strongly covered by the vertex x. If

$$AI_\tau(l_d(x,y)) \leq AI_\tau(\Re(x)), AI_\xi(l_d(x,y)) < AI_\xi(\Re(x))$$

Then, the vertex y is said to be weakly covered by the vertex x.

Definition 3.6

$$C^S(x) = \{y \in \tilde{V}(\tilde{G}) | AI_\tau(l_d(x,y)) \leq AI_\tau(\Re(x)), AI_\xi(l_d(x,y)) \geq AI_\xi(\Re(x))\}$$

is defined as the Strong Intuitionistic Fuzzy Path induced covering set of x.

Definition 3.7

$$C^S(x) = \{y \in \tilde{V}(\tilde{G}) | AI_\tau(l_d(x,y)) \le AI_\tau(\Re(x)), AI_\xi(l_d(x,y)) < AI_\xi(\Re(x))\}$$

is defined as the Weak covering set of x.

Definition 3.8 An Intuitionistic fuzzy Path induced vertex covering set (IFPVCS) for x, $C(x) = C^S(x) \cup C^W(x)$

A IFPVC covering set C(x) of x is said to be maximal covering set if there exists no vertex u such that $C(u) \supseteq C(x)$

Definition 3.9 Let $\tilde{G}$ be an IFG, Let $K \subseteq \tilde{V}(\tilde{G})$, Then the set

$$C(k) = \cup_i C(x_i), x_i \in K \tag{11}$$

is the IFPV covering set for K. If $C(K) = \tilde{V}(\tilde{G})$, then this covering set is called the IF path induced vertex covering set (IFPVC) of $\tilde{G}$.

Definition 3.10 Let $K \subseteq \tilde{V}(\tilde{G})$ be IFPVC of $\tilde{G}$. Then K is said to be minimal IFPVC set of $\tilde{G}$, if there exist no IFPVC set $K^0 \subseteq \tilde{V}(\tilde{G})$ such that $K^0 \subseteq K$.

The minimum IFPV covering set of $\tilde{G}$ is the covering set of $\tilde{G}$ with least cardinality. And this cardinality is called the IFPVC number of the $\tilde{G}$. And is denoted by α_0

Theorem 3.3 Let G be an IFG G with weight of each vertex is given by its covering radius, then $x \in C^S(x)$ and $x \notin C^W(x)$.

Proof: Consider a vertex x covering radius $\Re(x)$, then

$AI_\tau(l_d(x,y)) = AI_\tau(\Re(x)), AI_\xi(l_d(x,y)) = AI_\xi(\Re(x))$. Then from equation (9) and equation (10), $x \in C^S(x)$ and $x \notin C^W(x)$.

Theorem 3.4 Let G be an IFG G with weight of each vertex is given by its covering radius, then the IFPVCS for the vertex $C(x) \neq \varphi$.

Proof: Suppose, $C(x) = \varphi$, since, $C(x) = C^S(x) \cup C^W(x)$, both $C^S(x)$ and $C^W(x)$ equals φ. This contradicts theorem 3.3.

4. Illustration

Consider the Intuitionistic Fuzzy Graph $\tilde{G}$ given in the Figure 1. Let the edge weight of $(u_1,u_2) = ((6,8,11);0.6,0.4)$, $(u_1,u_5) = ((5,6,9);0.3,0.6)$,

$(u_2,u_4) = ((4,8,10);0.7,0.2)$, $(u_2,u_6) = ((4,7,9);0.6,0.3)$, $(u_3,u_4) = ((12,16,19);0.4,0.5)$, $(u_3,u_5) = ((2,3,4);0.5,0.3)$, $(u_5,u_6) = ((5,8,12);0.3,0.5)$. And let the covering radius for each vertex is given as $\Re(u_i) = ((8,11,12);0.45,0.5)$, $i = 1,2,\cdots,n$.

Then, $l_d(u_i,u_j)$ for $i,j = 1,2,\cdots n$ is

$l_d(u_1,u_2) = ((6,8,11);0.6,0.4)$,

$l_d(u_1,u_3) = ((7,9,13);0.3,0.6)$, $l_d(u_1,u_4) = ((10,16,21);0.6,0.4)$,

$l_d(u_1,u_5) = ((5,6,9);0.3,0.6)$, $l_d(u_1,u_6) = ((10,14,21);0.3,0.6)$,

$l_d(u_2,u_3) = ((11,18,25);0.3,0.5)$, $l_d(u_2,u_4) = ((4,8,10);0.7,0.2)$,

$l_d(u_2, u_5) = ((11,14,20); 0.3,0.6)$
,
$l_d(u_2, u_6) = ((4,7,9); 0.6,0.3)$,

$l_d(u_3, u_4) = ((12,16,19); 0.4,0.5)$, $l_d(u_3, u_5) = ((2,3,4); 0.5,0.3)$,

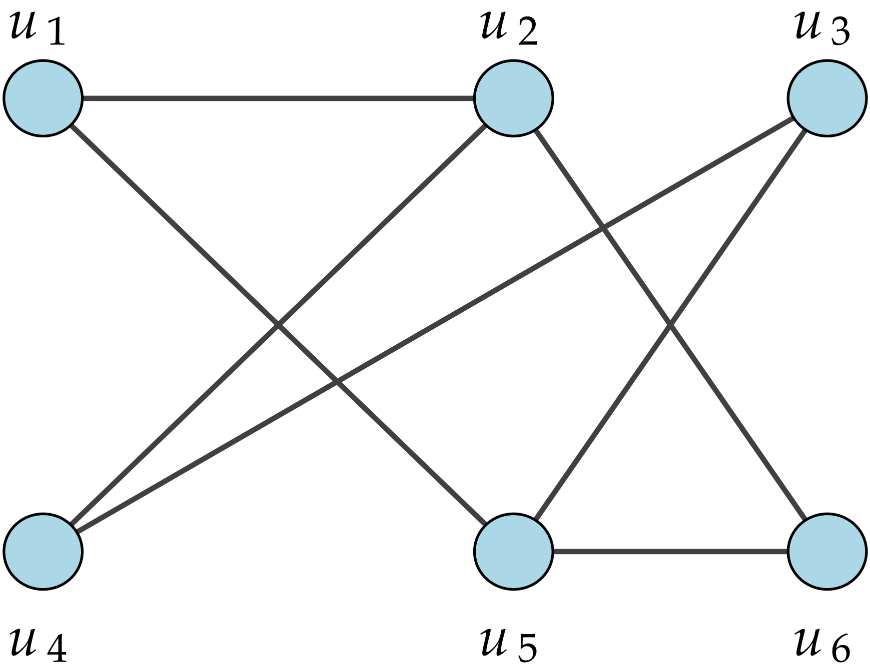

Figure 1: Intuitionistic Fuzzy graph $\tilde{G}$

$l_d(u_3, u_6) = ((7,11,16); 0.3,0.5)$, $l_d(u_4, u_5) = ((11,14,20); 0.3,0.6)$
,
$l_d(u_4, u_6) = ((19,27,35); 0.3,0.5)$, $l_d(u_5, u_6) = ((5,8,12); 0.3,0.5)$

Given the covering radius $\Re(u_i) = ((8,11,12); 0.45,0.5), i = 1,2,\cdots, n$

Hence, the IFPV covering set of $C(u_i)$, i = 1,2, ⋯ , n is

$C(u_1) = \{u_3, u_5, u_6\}, C(u_2) = \{u_5, u_6\}$,

$C(u_3) = \{u_1, u_5, u_6\}, C(u_4) = u_5,$

$C(u_5) = \{u_1, u_2, u_3, u_4, u_6\}, C(u_6) = \{u_1, u_2, u_3, u_5\}$

From this it is clear that $\{u_5\}$ can cover all vertices of $\tilde{G}$. Hence, $\{u_5\}$ is a IFPVC set for $\tilde{G}$. Also, it is the minimum IFPVC covering set for $\tilde{G}$. Therefore IFPVC number $\alpha_0 = 1$.

5. Algorithm for finding path induced vertex covering in IFG

Input: Intuitionistic Fuzzy Graph $\tilde{G} = (\tilde{V}, \tilde{E})$, generalized triangular intuitionistic fuzzy number as edge weight, covering radius $\Re(u)$ for each vertex

Output: IFPV Covering set of $\tilde{G}$ and IFPV covering number.

Step 1: Assign i → i = 1

Take $u_i \in \tilde{V}$

Step 2: For all path P from u_i to $u_j, j = 1,2,\cdots,n,$

Find $l_P(u_i, u_j) = ((\sum_P k, \sum_P l, \sum_P l); \min M_\tau, M_\xi), \forall\, i, j = 1,2, \ldots n.$

Step 3: Denote $l_d(u_i, u_j) = \min\, (l_P(u_i, u_j)$

$\forall j = 1,2,\cdots,n,$ (Find minimum using definition 2.8)

Step 4: Find Average index $AI_\tau(l_d(u_i,u_j)) = \frac{(k+l+m)M_{\tau_{l_d}(u_i,u_j)}}{3}$ and $AI_\xi(l_d(u_i,u_j)) = \frac{(k+l+m)M_{\xi_{l_d}(u_i,u_j)}}{3}$

Step 5: If If $AI_\tau(l_d(x,y)) \leq AI_\tau(\Re(x))$ and $AI_\xi(l_d(x,y)) \geq AI_\xi(\Re(x))$ then $u_j \rightarrow C^S$,

If $AI_\tau(l_d(x,y)) \leq AI_\tau(\Re(x))$ and $AI_\xi(l_d(x,y)) < AI_\xi(\Re(x))$ then $u_j \rightarrow C^W$

Step 6: $C(u_i) = C^S \cup C^W$

Step 7: If $\tilde{V} = \cup\, C(u_i)$, Stop IFPV Covering set of $\tilde{G} = \bigcup_i u_i$ and IFPV covering number = $|\bigcup_i u_i|$ otherwise $i \rightarrow i+1$; and go to step 2.

6. Application for finding a Relief Centre during a Disaster

Disasters are occurred either by natural or by human made, both case cause heavy devastation to nature and human. Natural disasters are mainly floods, earthquakes, volcanoes, tsunamis, storm sage, etc. During such a tense period, various organizations, relief centres, and people are very eager to help the victims. But it is necessary to identify which team can reach the affected zone immediately and provide their urgent needs. During such situations, the factors such as estimated travelling time, cost, distance, and other natural and human made circumstances, favour the evacuation team to reach the disaster point. Such uncertainties can be modelled by an Intuitionistic Fuzzy graph. In a country, there are places which are relief centres, non relief centres, calamity areas, non-calamity areas, model these places as vertices of IFG.

Due to the topographic structure and climatic change, Kerala is highly imperiled to natural disasters. The western Ghats of Kerala, that is the Wayanad, Idukki, Kottayam, and Pathanamthitta districts are the major victims of landslides. In 2018, due to heavy-monsoons in the month of August, September, and October, frequent mudflows and landslides occurred, and which cause a heavy loss in the life and property of many in the state.

In 2019, a massive landslide occurred at Puthumala in Wayanad district, an entire village was completely uprooted. Consider the places of Wayanad district as the vertices of a Graph and connect these vertices with edges if any type of transportation is possible between these places. Consider the following 9 places in the Wayanad district of Kerala State (Figure 2), since it is a forest area, the major transportation between these places is by road. During heavy monsoon, the chance for heavy mudflow and landslides are high in this area, hence the estimated travelling time from one place to another change, also other cost distance and other natural and human made circumstances also vary, these factors can be modelled as membership and non-membership degrees to the edges, which is depicted in Table: 1, that values are just approximate values, given to study the working of our algorithm.

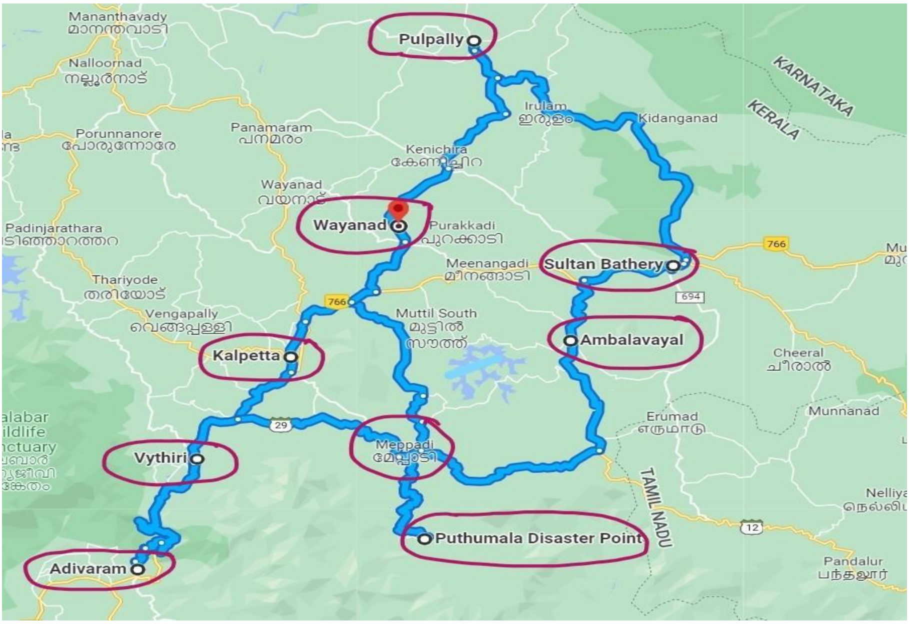

Figure 2: A route map connecting 9 places in Wayanad district

Since, in 2020 a heavy landslide occurred at Puthumala, We are going to find the IFPV covering set for Puthumala. Suppose the covering radius $\Re$ (Puthumala) = $((25,26,27);\ 0.4,0.5)$.

$l_d($

$Puthumala, Adivaram) = ((38,47,55);0.2,0.6), l_d(Puthumala, Kalpetta) =$

$((7,9,10);0.5,0.3), l_d(Puthumala, Meppadi) =$

$((5,7,8);0.55,0.4), l_d\ (Puthumala, Vythiri) = ((21,27,33);0.2,0.6)$

,

$l_d(Puthumala, Wayanad) = ((38,47,52);0.2,0.7), l_d(Puthumala, Ambalavayal) =$

$((17,21,23);0.55,0.4),$

$l_d(Puthumala, SultanBathery) = ((39,46,50);0.55,0.4), l_d(Puthumala, Pulpally) =$

$((46,58,67);0.2,0.7)$

Hence, the IFPVC covering set for Puthumala within the covering radius $\Re(Puthumala) = ((25,26,27);\ 0.4,0.5)$

C^S(Puthumala) = {Adivaram, Wayanad}, C^W(Puthumala)={Kalpetta, Vythiri, Meppadi}

C(Puthumala) = {*Kalpetta,Adivaram,V ythiri,Meppadi,Wayanad*}

During a landslide or mud flow at Puthumala, they can seek an immediate help from Kalpetta, Adivaram, Vythiri, Meppadi, Wayanad, if any one of it is a relief centre. Otherwise, increase the covering radius and can locate another relief centre.

Name of places	Intuitionistic fuzzy value representing estimated time,cost, distance and other variables
Wayanad - Pulpally	((8,11,15); 0.3,0.6)

Wayanad - kalpetta	((13,14,19); 0.5,0.3)
Wayanad - Vythiri	((20,23,25); 0.2,0.7)
Kalpetta - Vythiri	((11,15,17); 0.4,0.3)
kalpetta - Puthumala	((7,9,10); 0.5,0.3)
Sultan Bathery - Ambalavayal	((22,25,27); 0.8,0.16)
Ambalavayal - Meppadi	((12,14,15); 0.6,0.35)
Meppadi - Puthumala	((5,7,8); 0.55,0.4)
Vythiri - Meppadi	((16,20,25); 0.2,0.6)
Pulpally - Sultan Bathery	((30,34,39); 0.2,0.6)
Adivaram - Vythiri	((17,20,22); 0.35,0.4)

Table 1: Intuitionistic fuzzy value representing estimated time, cost, distance and other variables

Suppose a relief centre is located at Kalpetta and the coverage radius $\Re$ (Kalpetta) = {((24.25.26);0.5,0.3)}.

$l_d(Kalpetta, V\ ythiri) = ((11,15,17); 0.4,0.3), l_d(Kalpetta, Adivaram) = ((28,35,39); 0.35,0.4)$

$l_d(Kalpetta, Wayanad) = ((13,14,19); 0.5,0.3), l_d(Kalpetta, Puthumala) = ((7,9,10); 0.5,0.3), l_d(Kalpetta, Meppadi) = ((17,16,18); 0.5,0.4), l_d(Kalpetta, Ambalavayal) = ((29,30,33); 0.5,0.4), l_d(Kalpetta, SultanBatheri) = ((51,59,73); 0.2,0.6), l_d(Kalpetta, Pulpally) = ((21,25,24); 0.3,0.6)$

Therefore the IFPV Covering set for Kalpetta, $C(Kalpetta)$ =

$\{Vythiri, Adivaram, Wayanad, Puthumala, Meppadi, Ambalavayal\}$

Hence, if any kind of evacuation is needed in any of these places, the relief centre at Kalpetta can handle it.

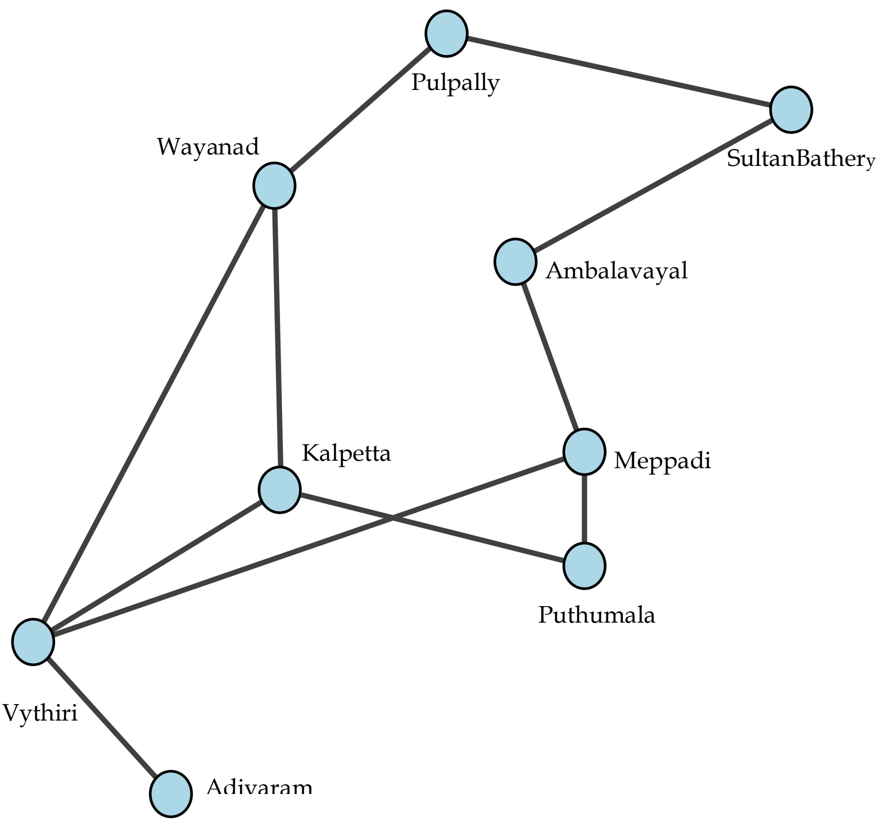

Figure 3: Intuitionistic Fuzzy graph model of Figure 2

7. Sensitivity Analysis

Due to the uncertainty, the covering radius of each vertex may change, hence sensitivity analysis can be done for varying covering radius. If we change the covering radius for the vertex kalpetta, then its IFPV covering set also changes. For, $\Re(Kalpetta) = ((20,21,22); 0.3,0.5)$, then $C^W = \{Vythiri, Puthumala\}$ and $C^S = \varphi$

For, $\Re(Kalpetta) = ((21,23,28); 0.4,0.35)$, C^W = {Vythiri, Wayanad, Puthumala, Meppadi}, C^S= {Meppadi}. Hence, C(Pulpally) = { Vythiri, Wayanad, Puthumala, Meppadi, Pulpally}

Relief centre	Covering radius	IFPV covering set
Kalpetta	((24,25,26);0.5,0.3)	{Vythiri, Wayanad, Puthumala, Meppadi, Adivaram, Pulpally, Sultan Bathery}
Kalpetta	((20,21,22);0.3,0.5)	{Vythiri, Puthumala}
Kalpetta	((21,23,28);0.4,0.35)	{Vythiri, Wayanad, Puthumala, Meppadi, Pulpally}

Observed that, ((20,21,22);0.3,0.5) ≤ ((21,23,28);0.4,0.35) ≤ ((24,25,26);0.5,0.3),

Implies

$C_{((20,21,22);0.3,0.5)}$(Kapetta) ⊆ $C_{((21,23,28);0.4,0.35)}$ (Kalpetta)

⊆ $C_{((24,25,26);0.5,0.3)}$ (Kalpetta)

8. Conclusion

In this paper, a new vertex covering to Intuitionistic Fuzzy Graph named Intuitionistic fuzzy path induced vertex covering (IFPVC) is defined. An illustrative example is provided to understand the definition. An algorithm is designed to find the IFPVC set and IFPVC number to IFGs. Finally, we applied the algorithm to find the nearest relief centre for the landslide occurred region in Wayanad district of Kerala. The covering places of a relief centre are also found out. In addition, we are interested in expanding on this result by employing a Trapezoidal IF number, Pentagonal IF number, etc. instead of a Triangular IF number in order to solve various facility location difficulties in the future.

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