

Josna James and Shiny Jose Effective Edge Domination in iterated Jump Graph Mapana – Journal of Sciences 2024, Vol. 23, No. 2, 151-162 ISSN 0975-3303|https://doi.org/10.12723/mjs.69.9

# **Effective Edge Domination in iterated Jump** Graph of Intuitionistic Fuzzy Graph

Josna James\*, Shiny Jose†

# Abstract

Intuitionistic Fuzzy Graphs (InFGs) serve as a sophisticated framework for modeling complex and uncertain phenomena across diverse domains, such as decision-making, economics, medicine, computer science, and engineering. In this research, we develop and analyse the properties of jump graphs in the context of InFGs. The vertex set of the jump graph  $J(G)$  of a graph G is defined as the edge set of  $G$ , with adjacency between vertices in  $J(G)$ established if and only if the corresponding edges in G are non-incident. We systematically construct sequences of jump graphs for InFGs through iterative processes and investigate the structural characteristics of these sequences. Moreover, we introduce the concept of an effective edge dominating set for jump graphs of InFGs and rigorously determine the effective edge domination number for certain classes of graphs. These contributions enhance the theoretical foundation of InFGs and extend their applicability to solving real-world problems characterized by uncertainty and complexity.

Keywords: Intuitionistic Fuzzy Jump graph, effective edge dominating set, evanesce number

# AMS subject classifications (2010)

03E72, 05C72

Mahatma Gandhi University Kottayam, josnakottaram@gmail.com

<sup>†</sup> St. George's College Aruvithura, shinyjosedavis@gmail.com

# 1 Introduction

In 1965, L.A. Zadeh created fuzzy set theory (FST), which enhanced classical set theory by incorporating a degree of uncertainty. Due to the inherent imprecision of real-world problems, FST has had a substantial impact on graph theory. Fuzzy graph theory can be used to simulate complicated real-world scenarios such as decision making, communication networks, social network analysis, pattern recognition, and medical diagnosis in specific ways.

Building on this foundation, Atanassov [1], [2] refined FST by developing Intuitionistic Fuzzy Set (InFS) theory, which introduces a degree of non-belongingness alongside the degree of belongingness for elements in the set. This advancement has spurred extensive research into the properties and applications of InFS, particularly in decision making, pattern recognition, and medical diagnosis. Then in 1994, Rosenfeld [3] introduced Intuitionistic Fuzzy Graph Theory, by subsequent representation and extension of fuzzy graph theory.

The line graph  $L(G)$  of a finite, simple and undirected graph  $G$ , has its vertices as the edges of G and two vertices of L(G) are neighbours if and only if the associated edges are incident in G. Jump graph is the complement of the line graph. That is the vertices of jump graphs are neighbours if and only if, the associated edges are not incident in G. In 2012, M. Akram, B. Davvaz [4] introduced intuitionistic fuzzy line graphs and detailed its properties.

In 1962, Ore and Berge [5] introduced dominating sets in graphs, Arumugam and velammal introduced edge domination for graphs [6], which has many application in problems such as network routing, resource allocation etc. Then Parvathi and Thamizhendhi [7] extended dominating set and domination number to InFGs. In 2019, James Josna and Shiny jose [8] introduced third type of intuitionistic fuzzy graph. [9] James Josna and Shiny Jose put forward a new covering for intuitionistic fuzzy graphs named as path induced vertex covering for intuitionitic fuzzy graph. In 2015,[10], Anupama,S.B and Y.B Maralabhavi defined connected domination number of a jump graph and in 2017, [11] they extended their studies to domination parameters on jump graph.

In this study, we extend the concept of jump graphs to InFGs and conduct a comprehensive investigation of their properties. Through iterative construction of consecutive jump graphs, we investigate the mathematical and structural properties of these graphs. In the framework of intuitionistic fuzzy graphs, we also provide and investigate the effective edge dominance number and effective edge domination set for jump graphs. We offer a greater comprehension of these ideas through thorough analysis and illustrative examples, which advances the theoretical development and wider applicability of intuitionistic fuzzy graphs.

The study introduces and investigates a new graph feature for jump graphs of InFGs: the evanesce number. This invariant sheds new perspective on the fundamental behavior of InFGs and characterizes the iteration phase where the jump graph disappears. An important theoretical outcome for the understanding of graph invariant in the context of InFGs is the proof that isomorphic graphs have the same evanescence number. This discovery improves the capacity to categorize and contrast various InFGs according to their fundamental features.

# 2 Prerequisites

Definition 2.1 [11] The Jump graph of a graph  $G$  is the graph whose vertices are edges of G and two vertices of jump graph are neighbours if and only if the corresponding edges in G do not share a common end point.

**Definition 2.2** An **Intuitionistic Fuzzy Graph (InFG)** [12] is a pair  $\hat{G} = (\hat{V}, \hat{E})$ , where  $\hat{V} = {\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n}$  denote the vertex set of  $\hat{G}$  and  $\hat{E} \subseteq \hat{V} \times$  $\hat{V}$ , denote the edge set of  $\hat{G}$  such that

- 1.  $\tau_V: \hat{V} \rightarrow [0,1]$  and  $\xi_V: \hat{V} \rightarrow [0,1]$  denote the degree of belongingness and degree of non belongingness of nodes  $\hat{v}_i \in \hat{V}$ , satisfying  $0 \leq \tau_{\hat{v}} + \xi_{\hat{v}} \leq 1$ ,  $\forall \hat{v}_i \in \hat{v}, i = 1, 2, \dots, n.$
- 2. For each  $(\hat{v}_i, \hat{v}_j) \in \hat{E}$ ,  $\tau_{\hat{E}} : \hat{E} \rightarrow [0, 1]$  and  $\xi_{\hat{E}} : \hat{E} \rightarrow [0, 1]$  such that

 $\hat{E}(\hat{v}_i, \hat{v}_j) \le \Lambda[\tau_{\hat{V}}(\hat{v}_i), \tau_{\hat{V}}(\hat{v}_j)],$  $E_{\hat{E}}(\hat{v}_i, \hat{v}_j) \leq V[\xi_{\hat{V}}(\hat{v}_i), \xi_{\hat{V}}(\hat{v}_j)]$  and  $0 \leq \tau_{\hat{E}}(\hat{v}_i, \hat{v}_j) + \xi_{\hat{E}}(\hat{v}_i, \hat{v}_j) \leq 1$ ,  $(\hat{v}_i, \hat{v}_j) \in \hat{E}, i =1, 2, \dots, n.$ 

Mapana - Journal of Sciences, Vol. 23, No.1 ISSN 0975-3303

[13] An InFG,  $\hat G$ = ( $\hat V$ ,  $\hat E$ ,  $\tau$ ,  $\xi$ ) is said to be complete if,  $\tau_{\hat E}(v_i)$ **Definition 2.3** [13] An InFG,  $\hat{G} = (\hat{V}, \hat{E}, \tau, \xi)$  is said to be complete if,  $\tau_{\hat{E}}(v_i, v_j) = \Lambda(\tau_{\hat{V}}(v_i), \tau_{\hat{V}}(v_j))$  and  $\xi_{\hat{E}}(v_i, v_j) = \nu(\xi_{\hat{V}}(v_i), \xi_{\hat{V}}(v_j))$ ,  $\forall v_i, v_j \in \hat{V}$ . .

**Definition 2.4** [13] Two InFGs  $\hat{G}_1 = (\hat{V}_1, \hat{E}_1)$  and  $\hat{G}_2 = (\hat{V}_2, \hat{E}_2)$  are said to be isomorphic, depicted by  $\hat{G}_1 \cong \hat{G}_2$ , if there exist a bijective map  $i : \hat{V}_1 \rightarrow$  $\hat{V}$  which satisfies

$$
i(v_i) = v_i, \ v_i \in \hat{V}_1, v_i \in \hat{V}_2
$$
  

$$
\tau_{\hat{V}_1}(v_i) = \tau_{\hat{V}_2}(i(v_i)) \text{ and } \xi_{\hat{V}_1}(v_i) = \xi_{\hat{V}_2}(i(v_i)), \ \forall v_i \in \hat{V}_1
$$
  

$$
\tau_{\hat{E}_1}(v_i, v_j) = \tau_{\hat{E}_2}(i(v_i, v_j)) \text{ and } \xi_{\hat{E}_1}(v_i, v_j) = \xi'_{\hat{E}_2}(i(v_i, v_j)), \ \forall (v_i, v_j) \in \hat{E}_1
$$

[15] The vertex cardinality of an InFG,  $\hat G$  =  $(\hat V,\hat E)$  is defined by the contract of the contrac

$$
|\hat{V}| = \sum_{\hat{u} \in \hat{V}} \frac{1 + \tau_{\hat{V}}(\hat{u}) - \xi_{\hat{V}}(\hat{u})}{2} \tag{1}
$$

and it is called the order of  $\hat G$  =  $(\hat V,\hat E)$  and is delineated by  $o(\hat G)$  $\left(\right)$ 

[15] The cardinality of the InFG,  $\hat{G} = (\hat{V}, \hat{E})$  is defined by

$$
|\hat{G}| = |\hat{V}| + |\hat{E}| \tag{2}
$$

**Definition 2.7** [16] Let Ĝbe an InFG graph, let û and v̂ be two vertices of Ĝ,<br>if the edge (û, v̂) is an effective edge then say that û dominates v̂.

[16] A subset D of  $\hat{V}$  is called a dominating set (DS) of InFG, if for every  $\hat{v} \in \hat{V} - D$ , there exists  $\hat{u} \in D$  such that  $\hat{u}$  dominates  $\hat{v}$ .

[17] The edge cardinality of an InFG ,  $\hat{G} = (\hat{V}, \hat{E})$  is defined by the contract of the contrac

$$
|\hat{E}| = \sum_{(\hat{u},\hat{v}) \in \hat{E}} \frac{1 + \tau_{\hat{E}}(\hat{u},\hat{v}) - \xi_{\hat{E}}(\hat{u},\hat{v})}{2}
$$
(3)

and it is called the order of InFG  $\hat G$  =  $(\hat V, \hat E)$  and is delineated by q $(\hat G)$  $\left\{\right\}$ 

**Definition 2.10** [16] An edge  $(\hat{u}, \hat{v})$  of the InFG,  $\hat{G}$  is said to be an effective edge if,  $\tau_{\hat{F}}(\hat{u}, \hat{v}) = \Lambda \{\tau_{\hat{V}}(\hat{u}), \tau_{\hat{V}}(\hat{v})\}$  and  $\xi_{\hat{F}}(\hat{u}, \hat{v}) = \nu \{\xi_{\hat{V}}(\hat{u}), \xi_{\hat{V}}(\hat{v})\}$  such that  $0 \leq \frac{1}{\hat{v}}$  $\tau_{\hat{F}}(\hat{u}, \hat{v}) + \xi_{\hat{F}}(\hat{u}, \hat{v}) \leq 1$ 

# 3 Main Result

**Definition 3.1** Let  $G^* = (V, E)$  be a simple graph, let  $V_1 = (\tau_{V_1}, \xi_{V_1})$  and  $E_1$  $= (\tau_{E_1}, \xi_{E_1})$  be the InF sets of V and E respectively. Let  $J(G^*) = (P, Q)$  be a jump graph of G<sup>\*</sup> and  $V_2 = (\tau_{V_2}, \xi_{V_2})$  and  $E_2 = (\tau_{E_2}, \xi_{E_2})$  be the InF sets of jump (G<sup>\*</sup>), then jump graph for InFG J<sub>IF</sub> (G) = (P, Q, V<sub>2</sub>, E<sub>2</sub>) of the InFG G  $= (V_1, E_1)$  is defined as

(i)  $\tau_{V_2}(Z_x) = \tau_{E_1}(x) = \tau_{E_1}(u'_x, v'_x)$ ,  $v'_x$ ) ) and the set of  $\overline{a}$ (ii)  $\xi_{V_2}(Z_x) = \xi_{E_1}(x) = \xi_{E_1(u'_x, v'_x)}$  $(v'_x)$ ) and the contract of  $\mathcal{L}$  $(iii)$   $\tau_{E_2}(Z_x, Z_y) = \Lambda(\tau_{V_2}(Z_x), \tau_{V_2}(Z_y)) = \Lambda(\tau_{E_1}(x), \tau_{E_1}(y))$  $(iv) \xi_{E_2}(Z_x, Z_y) = \vee(\xi_{V_2}(Z_x), \xi_{V_2}(Z_y)) = \vee(\xi_{E_1}(x), \xi_{E_1}(y))$  $\forall Z_x, Z_y \in P$ ,  $Z_xZ_y \in Q$ .

Illustration 3.1 Consider the InFG, G shown in the figure 1, with vertex set  $\{u_1, u_2, u_3, u_4, u_5\}$ 

and edge set  $\{u_1u_4, u_2u_4, u_2u_5, u_3u_4, u_3u_5\}$ . Jump graph of figure 1 is shown in figure 2.

Proposition 3.1 Jump graph of InFG is a strong InFG.



Figure 1: Intuitionistic Fuzzy G



Figure 2: Jump graph of G  $J_{IF}^{1}$   $^{(G)}$ 

**Proof:** Let  $u_1$  and  $u_2$  be any two vertices of the jump graph of the InFG, G, then by the definition of jump graph, there exist edges  $e_1$ and  $e_2$  in G corresponding to the vertices  $u_1$  and  $u_2$  in  $J_{IF}(G)$ , iff  $e_1$ and  $e_2$  share a common end point. Then the edge  $(u_1, u_2)$  will get a belongingness degree  $\tau_{E_2}(u_1, u_2) = \Lambda (\tau_{E_1}(e_1), \tau_{E_1}(e_2))$  and  $\xi_{E_2}(u_1, u_2)$ =  $V(\xi_{E_1}(e_1), \xi_{E_1}(e_2))$ , hence  $(u_1, u_2)$  become an effective edge. Hence the proof.

**Lemma 3.1** Let S be a subgraph of the InFG, G, the  $J_{IF}(S)$  is an induced subgraph of  $J_{IF}(G)$ 

**Proof:** Since, S is an InF subgraph of  $G = (V, E)$ ,  $V(S) \subseteq V(G)$ ,  $E(S)$  $E(G)$ ,  $\tau_V(S)(u) \leq \tau_V(G)(u)$  and  $\zeta_V(S)(u) \geq \zeta_V(G)(u)$ ,  $\forall u \in V(S)$ ,  $E(S)(u, v) \ge \tau E(G)(u, v)$  and  $\mathcal{E}(S)(u, v) \ge \mathcal{E}(G)(u, v)$ ,  $\forall (u, v) \in E(S)$ 

 $E(S) \subseteq E(G) \rightarrow V(JIF(S)) \subseteq V(JIF(G))$ 

Let  $e$  and  $e'$  are non incident edges in S, then  $e$  and  $e'$  are non incident in G also. Hence  $E(JIF(S)) \subseteq E(JIF(G))$ 

Also, if  $\tau_{E(S)}(e) \leq \tau_{E(G)}(e)$  and  $\zeta_{E(S)}(e) \geq \zeta_{E(G)}(e)$ ,  $\tau_{E(S)}(e') \leq$  $E(G)(e')$  and  $\mathcal{E}_E(S)(e') \geq \mathcal{E}_E(G)(e')$  then  $\tau_E(f_{iF}(S))(e,e') \leq \tau_E(f_{iF}(S))$  $IF$ (G))( $(e^r)$  and  $\mathcal{E}[(f_{iF}(S))(e,e^r) \geq \mathcal{E}[(f_{iF}(G))(e,e^r)]$ . Hence we get  $f'_{iF}(S)$  $IF$  $(S)^{'} \subseteq J_{rr}(G)$  $IF^{\sim}$  $(G)$  and  $\mathbb{R}$  and  $\mathbb$ IF

Also, if  $e_1$  and  $e_2$  are non incident edges in G then if  $e_1$ ,  $e_2 \in S$ , then they are non incident in S. Hence  $J_{IF}(S)$  is an induced subgraph of  $J_{IF}(G)$ 

 $\binom{0}{F}$  = G. For  $n \geqslant 1$ , the n<sup>th</sup> Jump graph of G is the Jump graph of  $\int_{I_{\rm F}}^{n-1}(\ddot{G})$ , denoted by  $J^n_{I_{\rm F}}(G)$ .

The smallest number  $n \geqslant 0$ , such that  $J^n_{\text{IF}}(G) = \phi$  is called the evanesce number of the InFG, G, denoted by  $\Omega(G)$ . That is the graph G evanesce for a finite  $\Omega(G)$ , then G is called  $\Omega$ -finite InFG. If no such  $\Omega(G)$ exist then  $\Omega(G) = \infty$ , then G is called  $\Omega$ - infinite InFG.

**Proposition 3.2** For a complete InFG, G,  $\Omega(G) = 2$ 

#### Proof:

Since for a complete InFG,  $\tau_E(v_i, v_j) = \Lambda \{\tau_V(v_i), \tau_V(v_j)\}$  and  $\xi_E(v_i, v_j)$  $V_{V}(v_i)$ ,  $\zeta_{V}(v_j)$ },  $\forall (v_i, v_j) \in E(\hat{G})$ . Hence  $E(J^1_{\phantom{1}I^c}(G)) = \varphi$ , therefore  $J$  $2_{IF}(G) = \phi$ 

#### Example 3.1

The evanesce number of the InFG in Figure 1 is 5. That is  $\Omega(G) = 5$ 

**Proposition 3.3** If  $n \geq 1$  is the evanesce number of the InFG G, then  $E(J_{IF}^{n-1}(G)) = \Phi$ 

#### Proof:

Proof is obvious since, if  $E(J_{IF}^{n-1}(G)) \neq \emptyset$  then  $J_{IF}^{n} \neq \emptyset$ 

**Theorem 3.1** If  $G_1 = (\hat{V}_1, \hat{E}_1)$  and  $G_2 = (\hat{V}_2, \hat{E}_2)$  are two isomorphic graphs then  $\Omega(G_1) = \Omega(G_2)$ 

### Proof:

Since  $G_1$  and  $G_2$  be two isomorphic graphs, the there exist a bijection  $i: \hat{V}_1 \to \hat{V}_2$  such that  $i(v_i) = v'$ , for  $v_i \in \hat{V}_1$ , satisfying  $\tau_{\hat{V}_1}(v_i) = \tau_{\hat{V}_2}(i(v_i))$ and  $\xi_{\hat{v}_1}(v_i) = \xi_{\hat{v}_2}(i(v_i))$ ,  $\forall v_i \in \hat{V}_1 \tau_{\hat{E}_1}(v_i, v_j) = \tau_{\hat{E}_2}(i(v_i, v_j))$  and  $\xi_{\hat{E}_1}(v_i, v_j)$  $E_{\hat{E}_2}(i(v_i,v_j))$ ,  $\forall (v_i,v_j) \in \hat{E}_1$ 

Hence size of  $G_1$  is equal to size of  $G_2$ , Also corresponding to every non incident edges in  $G_1$ , there exist non incident edges in  $G_2$ . Hence there exist an isomorphism between  $J_{IF}(G_1)$  and  $J_{IF}(G_2)$ .

Then continuing the iteration on jump graphs, we get the jump graph of both the graphs vanishes on same iteration, hence  $\Omega(G1) = \Omega(G2)$ 

**Definition 3.4** Let  $I_{IF}$  (G)(P, Q, V<sub>2</sub>, E<sub>2</sub>) be the InF jump graph of the InFG G =  $(V_1, E_1)$ . A subset  $K \subseteq V_2(J_{IF}(G))$  is said to be an effective edge dominating set of  $J_{IF}(G)$ , if every vertex  $v \in V_2(J_{IF}(G)) - K$ , there exist  $u \in$  $E$  such that u effectively dominates  $v$ 

Theorem 3.2 Every edge of InF jump graph is an effective edge.

**Proof:** Let  $\hat{G} = (V, E)$  be an InFG,  $J_{IF}(\hat{G}) = (V_1, E_1)$  be the InF jump graph of  $\hat{G}$ . Let  $e_1$  and  $e_2$  are two non incident vertices of  $\hat{G}$ , then  $e_1$  and  $e_2$  are neighbours vertices of  $J_{IF}(\hat{G})$ . By the definition of InF jump graph,  $\tau_{E_1}(e_1, e_2) = \Lambda(\tau_{V_1}(e_1), \tau_{V_1}(e_2))$   $\xi_{E_1}(e_1, e_2) = \Lambda(\xi_{V_1}(e_1), \xi_{V_1}(e_2))$ . Hence it is clear that  $(e_1, e_2)$  is an effective edge.

Also, if G have at least one pair of non neighbours edges, then  $I_{IF}(\tilde{G})$  $\left(\right)$ has atleast one effective edge.

**Definition 3.5** An effective edge DS K of the jump graph  $I_{IF}(G)$  is called a minimal effective edge DS if, for every vertex  $v \in K$ , K-v is not an effective edge DS.

Definition 3.6 The minimum of InF vertex cardinality taken over all minimal effective edge DS of  $J_{IF}(G)$  is called the effective edge dominating number of  $J_{IF}(G)$  and is delineated by  $\mathcal{P}_{ED}(J_{IF}(G))$  and the corresponding DS is called the minimum effective edge DS delineated by  $M_{ED}(J_{IF}(G))$ , and the number of vertices in  $M_{ED}(J_{IF}(G))$  is delineated by  $n(M_{ED}(J_{IF}(G)))$ .

**Example 3.2:** The vertex subsets  $K_1 = \{v_1, v_3\}$ ,  $K_2 = \{v_1, v_2, v_5\}$ ,  $K_3 = \{v_2, v_4, v_5\}$  are effective edge DSs for the InF Jump graph G shown in Figure: 2.  $\mathcal{P}_{ED}(J_{IF}(G)) = \Lambda / |K_1|, |K_2|, |K_3|$ , therefore  $\mathcal{P}_{ED}(J_{IF}(G)) = \Lambda / 0.9$ ,  $1.275, 1.425 \}= 0.9$ 

**Proposition 3.4** The effective edge DS for jump graph of complete InF graph  $\hat{G}$  = ( $\hat{V}$ ,  $\hat{E}$ ) is  $\hat{E}(\hat{G})$ . Hence its effective edge domination number  $\wp_{ED}$  $(J_{IF}(\hat{G}) = |E|)$ 

The proof of the theorem is obvious

**Theorem 3.3** For an InFG  $\hat{G} = (\hat{V}, \hat{E})$ ,  $M_{ED}(J_{IF}(\hat{G})) \neq \hat{E}$  iff  $\Omega(\hat{G}) \neq 2$ 

#### Proof:

 $\hat{G}$  = 2,  $J_{IF}^{1}(\hat{G})$  is a null graph, that is having only isolated vertices. then the minimum effective edge DS  $M_{ED}(J_{IF}(\hat{G}))$  $\eta$ )) is the edge set of  $\tilde{G}$ . .

 $\hat{G}$ ) = 1, then  $\hat{G}$  itself is a null graph, hence  $M_{ED}(J_{IF}(\hat{G}))$  $\eta$ ))  $=$   $\phi$ 

 $\hat{G}$  > 2, then there exist at least one effective edge in  $J_{IF}(\hat{G})$ .  $\int$ ). Hence  $M_{ED}(J_{IF}(\hat{G})) \subset \hat{E}$ 

Let G be an InFG,  $\{J^n{}_{IF}(G)\}$  be the sequence of jump graphs of G, and  $\wp_{ED}^*$  be the effective domination number corresponding to the jump graph J<sup>n</sup> (G). Then the sequence { $\mathcal{S}^n{}_{ED}$ },  $n = 1,2, \cdots$  is called the effective

**Example 3.3** Consider the figure 1, since the evanesce number of  $G = 5$ ,  $J_{IF}^5(G) = \phi$ . The effective domination number sequence corresponding to sequence  $\{J_{IF}^n(G)\}\$ , n=1,2,3,4 is found to be  ${0.9,0.825,0.8,0.8}$ 

**Theorem 3.4** For an  $\Omega$  -finite InFG G,  $\{\varnothing_{ED}^n\}$ ,  $n = 1,2, \cdots$  is a monotonic decreasing sequence

The proof of the theorem is obvious

#### **Observations**

- (i)  $n(M_{ED} U_{IF}(C3))) = 3$
- (ii)  $\Omega(C_2) = 2$
- (iii)  $n(M_{_{ED}}(J_{_{IF}}(P_3))) = 2$
- (iv)  $\Omega(P_3) = 2$

**Proposition 3.5** If  $\Omega(G) = 2$  for an InFG G then its jump graph  $J_{IF}(G)$  is a null graph

#### Proof:

Suppose  $J_{IF}^1(G)$  is not a null graph, that means there exist atleast one edge in  $J_{IF}(G)$ , hence  $J_{IF}^1(G) \neq \Phi$ .

# **Conflict of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

# Funding

This research received no external funding.

# Acknowledgement

We would like to thank St. Thomas College, Palai for their support and resources that were instrumental in completion of this research.

#### Author Contributions

All authors contributed to the conceptualization, methodology, investigation, data curation, writing, review, and editing of the manuscript.

# 4 Conclusion

In this study, we introduced the concept of the Jump graph for Intuitionistic Fuzzy Graphs (InFGs), defined as the complement of the Intuitionistic Fuzzy line graph. Through successive iterations of the Jump graph, we observed that for certain InFGs, the Jump graph eventually vanishes after a finite number of iterations. We defined the evanescence number of a graph G as the smallest non-negative integer  $J_{IF}^{n}(G)$ , is empty. We demonstrated that isomorphic graphs share the same evanescence number. Ad- ditionally, we proposed the concept of the effective edge dominating set (DS) for Intuitionistic Fuzzy Jump graphs. We established that the minimum edge dominating set of a Jump graph is equivalent to the edge set of its corresponding InFG if and only if the evanescence number is two.

The introduction of the jump graph for InFGs and the concept of the evanescence number provide new insights into the structural properties and behaviors of Intuitionistic Fuzzy Graphs. The finding that isomorphic graphs share the same evanescence number contributes to the un- derstanding of graph invariant in the context of InFGs. Furthermore, the study of effective edge dominating sets in InF jump graphs opens up new avenues for optimizing network design

and resource allocation in systems modeled by Intuitionistic Fuzzy Graphs. These contribu- tions enhance the theoretical foundation of Intuitionistic Fuzzy Graph theory and extend its applicability to practical problems involving uncertainty and complexity.

#### References

- [1] Atanassov, Krassimir T. "New operations defined over the Intuitionistic Fuzzy sets." Fuzzy sets and Systems 61.2 (1994): 137-142.
- [2] Atanassov, Krassimir. "Review and new results on Intuitionistic Fuzzy sets." preprint Im-MFAIS-1-88, Sofia 5.1 (1988).
- [3] Rosenfeld, Azriel. "Fuzzy graphs." Fuzzy sets and their applications to cognitive and decision processes. Academic press, 1975. 77-95.
- [4] Akram, Muhammad, and R. Parvathi. "Properties of Intuitionistic Fuzzy line graphs." Notes on Intuitionistic Fuzzy sets 18.3 (2012): 52-60.
- [5] Gupta, Preeti. "Domination in graph with application." Indian J. Res 2.3 (2013): 115-117.
- [6] Arumugam, S., and S. Velammal. "Edge domination in graphs." Taiwanese journal of Mathematics (1998): 173-179.
- [7] Parvathi, R., and G. Thamizhendhi. "Domination in Intuitionistic Fuzzy graphs." Notes on Intuitionistic Fuzzy Sets 16.2 (2010): 39-49.
- [8] James, Josna, and Shiny Jose. "Intuitionistic fuzzy graph of third type." International journal of Scientific Research in Mathematical and Statistical Sciences 6.1 (2019): 221-224.
- [9] James, Josna, and Shiny Jose. "Path Induced Vertex Covering for Intuitionistic Fuzzy Graph and its Application in Disaster Management." Mapana Journal of Sciences 21.3 (2022).
- [10] Anupama, S. B., Y. B. Maralabhavi, and Venkanagouda M. Goudar. "Connected domination number of a jump graph." Journal of Computer and Mathematical Sciences 6.10 (2015): 538-545.
- [11] Anupama, S. B., Y. B. Maralabhavi, and V. M. Goudar. "Some Domination Parameters on Jump graph." (2017): 47-55.
- [12] Parvathi, R., and M. G. Karunambigai. "Intuitionistic Fuzzy graphs." Computational intelligence, theory and applications. Springer, Berlin, Heidelberg, 2006. 139-150.
- [13] Karunambigai, M. G., R. Parvathi, and O. K. Kalaivani. "A study on atanassov's Intuitionistic Fuzzy graphs." 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011). IEEE, 2011.
- [14] Rao, N. Pratap Babu. "On Non Bondage Number of a Jump Graph." International Journal of Mathematics Trends and Technology 57.4 (2018): 292-295.
- [15] Karunambigai, M. G., S. Sivasankar, and K. Palanivel. "Some properties of a regular Intuitionistic Fuzzy graph." International Journal of Mathematics and Computation 26.4 (2015): 53-61.
- 16] Senthilkumar, V. "Types of domination in Intuitionistic Fuzzy graph by strong arc and effective ARC." Bulletin of Pure & Applied Sciences-Mathematics and Statistics 37.2 (2018): 490- 498.