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$b\mu_{ij}$ - Open Sets in Bigeneralized Topological Spaces

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Abstract

The purpose of this article is to discuss in detail about the $b\mu_{ij}$-open sets in bigeneralized topological space. Also, we investigated some characterizations and properties.

Keywords: Bigeneralized Topological Space, $b\mu_{ij}$-open set, $b\mu_{ij}$-closed set, $b\mu_{ij}$-interior, $b\mu_{ij}$-closure.

1. Introduction

On 2002, A. Csaszar [5] was the first to investigate the generalized concepts of topological spaces. Later, C. Boonpak [3] extended the notion of generalized topological space into bigeneralized topological space (briefly BGTS) and defined some open sets. The characteristics of semi and pre-open sets in BGTS were studied by Anees Fathima and Jamuna Rani in [1] and [8]. This article discusses $b\mu_{ij}$-open set in BGTS. We also examine its properties and provide some characterizations.

2. Preliminaries

Consider a non-empty set X and $\gamma \in \Gamma$. If $M \subset \gamma(M)$, then the subset M of X is said to be γ-open[4] and for any subset M of X, the function

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$\gamma^*: \wp(X) \rightarrow \wp(X)$[4] can be defined as $\gamma^*(M) = X - \gamma(X - M)$. For any non empty set X and $\eta \epsilon \wp(X)$, η is said to be GT [5] if $\phi \epsilon \eta$ and η is closed under arbitrary union. A function $\gamma \epsilon \Gamma$ is said to be μ-friendly [6] if $\gamma(M) \cap L \subset \gamma(M \cap L)$ for any subset M of X and $L \in \mu$. If $\gamma \epsilon \Gamma$ and $\mu = \{M \subset X / M \subset \gamma(M)\}$ is the collection of all γ-open sets, then μ is called GT [5] and (X, μ) is a GTS. In [11], Γ_4 is denoted the collection of all μ-friendly functions and (X, μ) is referred ad the γ-space [7]. Also, by [7] the results established in [11] are valid for quasi-topological spaces.

For a non-empty set X, a triple $(X, \mu_{1,} \mu_2)$ is said to be BGTS where $\mu_{1,}$ μ_2 are generalized topologies on X. For any non-empty subset M of X, $i_{\mu_n}(M)$ and $c_{\mu_n}(M)$ denotes the interior and closure of M with respect to μ_n respectively, for $n = 1{,}2$[3]. For the BGTS, a subset M is said to be μ_{pq}-semi open [1](resp. μ_{pq}-preopen [8]) if $M \subset c_{\mu_p} i_{\mu_q}(M)$ (resp. $M \subset i_{\mu_p} c_{\mu_q}(M)$) where $p, q = 1{,}2$ and $p \neq q$.

Theorem 2.1. [10] Consider a GTS (X, μ). The following properties apply to the subsets M and N of X.

1. $i_\mu(X - M) = X - c_\mu(M)$ and $c_\mu(X - M) = X - i_\mu(M)$.
2. $i_\mu(M) = M$ for $M \in \mu$ and $c_\mu(M) = M$ for $X - M \in \mu$.
3. $i_\mu(M) \subseteq i_\mu(N)$ and $c_\mu(M) \subseteq c_\mu(N)$ for $M \subseteq N$.
4. $i_\mu(M) \subseteq M$ and $M \subseteq c_\mu(M)$.
5. $i_\mu(i_\mu(M)) = i_\mu(M)$ and $c_\mu(c_\mu(M)) = c_\mu(M)$.

Theorem 2.2 [9] For any γ-space (X, γ), $L \cap c_\gamma(M) \subset c_\gamma(L \cap M)$, for any γ-open set L and for any subset M of X.

Theorem 2.3. For any quasi-topological space (X, μ), the following apply to any subset M, N of X.

(1) $M \cap N$ is μ- open where M and N are μ- open [11].

(2) For all subsets M and N of X, $i_\mu(M \cap N) = i_\mu(M) \cap i_\mu(N)$[7].

(3) For all subsets M and N of X, $c_\mu(M \cup N) = c_\mu(M) \cup c_\mu(N)$[11].

Theorem 2.4. [2] For BGTS $(X, \mu_{1,} \mu_2)$, $\mu_i \epsilon \Gamma_4$ where $i = 1{,}2$. Then the following properties hold for any subset M of X.
(1) $i_{\pi_{ij}}(M) = M \cap i_{\mu_i} c_{\mu_j}(M)$.

(2) $i_{\sigma_{ij}}(M) = M \cap c_{\mu_i} i_{\mu_j}(M)$.

(3)$i_{\mu_i}\left(c_{\sigma_{ij}}(M)\right) = i_{\mu_i} c_{\mu_j}(M)$.

(4) $i_{\mu_i}\left(c_{\pi_{ji}}(M)\right) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(M)$.

(5) $c_{\sigma_{ij}}\left(i_{\pi_{ji}}(M)\right) = i_{\pi_{ji}}\left(c_{\sigma_{ij}}(M)\right) = i_{\mu_i} c_{\mu_j}(M)$.

(6) $i_{\pi_{ji}}\left(i_{\sigma_{ij}}(M)\right) = i_{\alpha_{ji}}(M)$.

(7) $i_{\sigma_{ij}}\left(i_{\beta_{ij}}(M)\right) = i_{\sigma_{ij}}(M)$.

(8) $i_{\pi_{ij}}\left(i_{\beta_{ji}}(M)\right) = i_{\pi_{ij}}(M)$.

3. $\boldsymbol{b\mu_{ij}}$- open sets

Definition 3.1. Any subset A of BGTS (X, μ_1, μ_2) is said to be $b\mu_{ij}$-open if $A \subset i_{\mu_i} c_{\mu_j}(A) \cup c_{\mu_j} i_{\mu_i}(A)$ where i is not equal to j and $i, j = 1,2$. Also $b_{ij}(\mu)$ denotes the collection of $b\mu_{ij}$- open sets. By $i_{b_{ij}}(A)$, we mean the $b\mu_{ij}$- interior of a subset A of X and it can be as the union of all $b\mu_{ij}$- open sets contained in A.

Also, every μ_i-open set is $b\mu_{ij}$-open. But the illustration below demonstrates that the converse need not be true. For let $X = \{1,2,3,4\}$, $\mu_1 = \{\emptyset, \{1\}, \{3,4\}, \{1,3,4\}\}$, $\mu_2 = \{\emptyset, \{2\}, \{1,4\}, \{2,4\}, \{1,2,4\}\}$. Then the collection of all $b\mu_{ij}$ -open sets is $\{\emptyset, \{1\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,3,4\}\}$. Here $\{3\}$ is $b\mu_{12}$-open but not μ_1-open.

Theorem 3.2. For a BGTS (X, μ_1, μ_2) and $\mu_i \in \Gamma_4$ for $i = 1,2$, the statements below are equivalent.

(1) A is $b\mu_{ij}$-open.

(2)$A = i_{\pi_{ij}}(A) \cup i_{\sigma_{ji}}(A)$.

(3) $A \subset c_{\pi_{ji}}(i_{\pi_{ij}}(A))$.

Proof. $(i) \Rightarrow (ii)$ If A is $b\mu_{ij}$-open, then $A \subset i_{\mu_i}c_{\mu_j}(A) \cup c_{\mu_j}i_{\mu_i}(A)$. Now by Theorem 2.4, $i_{\pi_{ij}}(A) \cup i_{\sigma_{ji}}(A) = \left(A \cap i_{\mu_i}c_{\mu_j}(A)\right) \cup \left(A \cap c_{\mu_j}i_{\mu_i}(A)\right) = A \cap \left(i_{\mu_i}c_{\mu_j}(A) \cup c_{\mu_j}i_{\mu_i}(A)\right) = A$.

$(ii) \Rightarrow (iii)$ If $A = i_{\pi_{ij}}(A) \cup i_{\sigma_{ji}}(A)$, then $A = i_{\pi_{ij}}(A) \cup \left(A \cap c_{\mu_j}i_{\mu_i}(A)\right) \subset i_{\pi_{ij}}(A) \cup c_{\mu_j}i_{\mu_i}(A) = c_{\pi_{ji}}i_{\pi_{ij}}(A)$ by Theorem 2.4 which proves (c).

$(iii) \Rightarrow (i)$ If $\subset c_{\pi_{ji}}(i_{\pi_{ij}}(A)) \Rightarrow A \subset i_{\pi_{ij}}(A) \cup c_{\mu_j}i_{\mu_i}(A) = \left(A \cap i_{\mu_i}c_{\mu_j}(A)\right) \cup c_{\mu_j}i_{\mu_i}(A) = \left(A \cup c_{\mu_j}i_{\mu_i}(A)\right) \cap \left(i_{\mu_i}c_{\mu_j}(A) \cup c_{\mu_j}i_{\mu_i}(A)\right) \Rightarrow A \subset i_{\mu_i}c_{\mu_j}(A) \cup c_{\mu_j}i_{\mu_i}(A)$. Hence A is $b\mu_{ij}$-open.

Theorem 3.3. Let A_l be $b\mu_{ij}$-open for all $l \in \Delta$ and $\mu_i \epsilon \Gamma_4$ in a bigeneralized topological space (X, μ_1, μ_2). Then $\cup\{A_l/l \in \Delta\}$ is $b\mu_{ij}$-open.

Proof. Let $A = \cup\{A_l/l \in \Delta\}$. Then $A \subset \cup\, c_{\pi_{ji}}i_{\pi_{ij}}(A_l) \subset c_{\pi_{ji}}(\cup\, (i_{\pi_{ij}}(A_l)) \subset c_{\pi_{ji}}(i_{\pi_{ij}}(\cup A_l) = c_{\pi_{ji}}i_{\pi_{ij}}(A)$. Therefore, $\cup\{A_l/l \in \Delta\}$ is $b\mu_{ij}$-open.

Remark 3.4. The following example demonstrates that the intersection of any two $b\mu_{ij}$-open sets does not have to be a $b\mu_{ij}$-open.

Let $X = \{1,2,3,4\}$, $\mu_1 = \left\{\phi, \{3\}, \{1,4\}, \{1,3\}, \{3,4\}, \{1,3,4\}\right\}$, $\mu_2 = \left\{\phi, \{2,4\}, \{2,3,4\}\right\}$.

$b\mu_{12} - O(X) = \{\phi, \{1\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, X\}$. Here $\{1,2,4\}$ and $\{2,3,4\}$ are $b\mu_{12}$-o (X) but $\{1,2,4\} \cap \{2,3,4\} = \{2,4\}$ which is not $b\mu_{12}$-open.

Theorem 3.5. For BGTS (X, μ_1, μ_2), then the statements below are true for any subset A of X.

(i) $i_{b_{ij}}(A)$ is the largest $b\mu_{ij}$-open set contained in A.

(ii) A is $b\mu_{ij}$-open if and only if $A = i_{b_{ij}}(A)$.

(iii) $x \in i_{b_{ij}}(A)$ if and only if there is a $b\mu_{ij}$-open set M containing x such that $M \subset A$.

(iv) $i_{b_{ij}} \in \Gamma_{012-}$.

Definition 3.6. In a BGTS (X, μ_1, μ_2), a subset A is $b\mu_{ij}$-closed if its complement is $b\mu_{ij}$-open. Clearly, A is $b\mu_{ij}$-closed if and only if $c_{\mu_i} i_{\mu_j}(A) \cap i_{\mu_j} c_{\mu_i}(A) \subset A$. By $c_{b_{ij}}(A)$, we mean the $b\mu_{ij}$-closure of a subset A of X and it can be defined as the intersection of all $b\mu_{ij}$-closed sets containing A.

Theorem 3.7. For BGTS (X, μ_1, μ_2), then the statements below are true for any subset A of X.

(i) $c_{b_{ij}}(A)$ is the smallest $b\mu_{ij}$-closed set containing A.

(ii) A is $b\mu_{ij}$- closed if and only if $A = c_{b_{ij}}(A)$.

(iii) $x \in c_{b_{ij}}(A)$ if and only if for every $b\mu_{ij}$-closed set M containing x such that $M \cap A \neq \phi$.

(iv) $c_{b_{ij}} \in \Gamma_{012+}$.

Theorem 3.8. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A of X.

(i) $(i_{b_{ij}})^* = c_{b_{ij}}$.

(ii) $(c_{b_{ij}})^* = i_{b_{ij}}$.

(iii) $i_{b_{ij}}(X - A) = X - c_{b_{ij}}(A)$.

(iv) $c_{b_{ij}}(X - A) = X - i_{b_{ij}}(A)$.

Theorem 3.9. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A.

(i) $c_{b_{ij}}(A) = c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A)$.

(ii) $i_{b_{ij}}(A) = i_{\sigma_{ij}}(A) \cup i_{\pi_{ji}}(A)$.

Proof. (i) Since $c_{b_{ij}}(A) \subset c_{\sigma_{ij}}(A)$ and $c_{b_{ij}}(A) \subset c_{\pi_{ji}}(A)$, we have $c_{b_{ij}}(A) \subset c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A)$. Also, $c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A) = (A \cup i_{\mu_i} c_{\mu_j}(A)) \cap (A \cup c_{\mu_j} i_{\mu_i}(A)) = A \cup \left(i_{\mu_i} c_{\mu_j}(A) \cap c_{\mu_j} i_{\mu_i}(A) \right) \subset A \cup$

$\left(i_{\mu_i} c_{\mu_j} \left(c_{b_{ij}} (A) \right) \cap c_{\mu_j} i_{\mu_i} \left(c_{b_{ij}} (A) \right) \right) \subset A \cup c_{b_{ij}}(A) = c_{b_{ij}}(A)$,
Since $c_{b_{ij}}(A)$ is $b\mu_{ij}$- closed. Therefore, $c_{b_{ij}}(A) = c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A)$.

(ii) can be proved in the same way that (i).

Theorem 3.10. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A.

(i) A is $b\mu_{ij}$- open if and only if $c_{\sigma_{ij}}(A) \cup c_{\pi_{ji}}(A) = i_{\mu_i} c_{\mu_j}(A)) \cup c_{\mu_j} i_{\mu_i}(A))$.

(ii) A is $b\mu_{ij}$- closed if and only if $i_{\sigma_{ij}}(A) \cap i_{\pi_{ji}}(A) = c_{\mu_i} i_{\mu_j}(A)) \cap i_{\mu_j} c_{\mu_i}(A))$.

Proof. (i) By definition, $A \in b\mu_{ij}(A)$ if and only if $A \subset \left(i_{\mu_i} c_{\mu_j}(A) \cap c_{\mu_j} i_{\mu_i}(A) \right)$. Now, $c_{\sigma_{ij}}(A) \cup c_{\pi_{ji}}(A) = \left(A \cup i_{\mu_i} c_{\mu_j}(A) \right) \cup \left(A \cup c_{\mu_j} i_{\mu_i}(A) \right) = A \cup \left(i_{\mu_i} c_{\mu_j}(A) \cup c_{\mu_j} i_{\mu_i}(A) \right)$. Since $A \subset \left(i_{\mu_i} c_{\mu_j}(A) \cap c_{\mu_j} i_{\mu_i}(A) \right)$, we have $c_{\sigma_{ij}}(A) \cup c_{\pi_{ji}}(A) = i_{\mu_i} c_{\mu_j}(A)) \cup c_{\mu_j} i_{\mu_i}(A))$.

(ii) From (i), the proof follows.

The relationship between the $b\mu_{ij}$- closure and $b\mu_{ij}$- interior operators are proved by Theorem 3.11.

Theorem 3.11. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A.

(i) $i_{\mu_i} \left(c_{b_{ij}}(A) \right) = c_{b_{ij}} \left(i_{\mu_i}(A) \right) = i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(ii) $c_{\mu_i} \left(i_{b_{ij}}(A) \right) = i_{b_{ij}} \left(c_{\mu_i}(A) \right) = c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(iii) $i_{b_{ij}}(c_{\sigma_{ji}}(A)) = i_{\sigma_{ij}}(c_{\sigma_{ji}}(A))$.

(iv) $c_{b_{ij}}(i_{\sigma_{ji}}(A)) = c_{\sigma_{ij}}(i_{\sigma_{ji}}(A))$.

(v) $i_{\sigma_{ji}}(c_{b_{ij}}(A)) = c_{\sigma_{ij}}(A) \cap c_{\mu_j} i_{\mu_i}(A)$.

(vi) $c_{\sigma_{ji}}(i_{b_{ij}}(A)) = i_{\sigma_{ij}}(A) \cap i_{\mu_j} c_{\mu_i}(A)$.

Proof. (i)$c_{b_{ij}}\left(i_{\mu_i}(A)\right) = c_{\sigma_{ij}}\left(i_{\mu_i}(A)\right) \cap c_{\pi_{ji}}\left(i_{\mu_i}(A)\right) = \left(i_{\mu_i}(A) \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)\right) \cap \left(i_{\mu_i}(A) \cup c_{\mu_j}i_{\mu_i}(A)\right) = \left(i_{\mu_i}(A) \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)\right) \cap c_{\mu_j}i_{\mu_i}(A) = i_{\mu_i}(A) \cup i_{\mu_i}c_{\mu_j}i_{\mu_i}(A) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.

Also by theorem 2.4(3) and 2.4(4), $i_{\mu_i}\left(c_{b_{ij}}(A)\right) = i_{\mu_i}(c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A)) = i_{\mu_i}(c_{\sigma_{ij}}(A)) \cap i_{\mu_i}(c_{\pi_{ji}}(A)) = i_{\mu_i}c_{\mu_j}(A) \cap i_{\mu_i}c_{\mu_j}i_{\mu_i}(A) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.

(ii) From (i), the proof follows.

(iii) $i_{b_{ij}}(c_{\sigma_{ji}}(A)) = i_{\sigma_{ij}}(c_{\sigma_{ji}}(A)) \cup (i_{\pi_{ji}}(c_{\sigma_{ji}}(A))$. Now, $i_{\pi_{ji}}(c_{\sigma_{ji}}(A) = c_{\sigma_{ji}}(A) \cap i_{\mu_j}c_{\mu_i}(c_{\sigma_{ji}}(A)) = c_{\sigma_{ji}}(A) \cap i_{\mu_j}c_{\mu_i}(A) \subset c_{\sigma_{ji}}(A) \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A) = (A \cup i_{\mu_j}c_{\mu_i}(A)) \cap c_{\mu_i}i_{\mu_j}c_{\mu_i}(A) = i_{\sigma_{ij}}(c_{\sigma_{ji}}(A))$. Also from the definition $i_{b_{ij}}(c_{\sigma_{ji}}(A)) \subset i_{\sigma_{ij}}(c_{\sigma_{ji}}(A))$. Hence the result follows.

(iv) From (iii), the proof follows.

(v) $i_{\sigma_{ji}}c_{b_{ij}}(A) = c_{b_{ij}}(A) \cap c_{\mu_j}i_{\mu_i}\left(c_{b_{ij}}(A)\right) = c_{b_{ij}}(A) \cap c_{\mu_j}i_{\mu_i}\left(c_{\mu_j}i_{\mu_i}(A)\right)$ by (i), $c_{b_{ij}}(A) \cap c_{\mu_j}i_{\mu_i}(A) = (c_{\sigma_{ij}}(A) \cap c_{\pi_{ji}}(A)) \cap c_{\mu_j}i_{\mu_i}(A) = c_{\sigma_{ij}}(A) \cap c_{\mu_j}i_{\mu_i}(A)$.

(vi) From (v), the proof follows.

Theorem 3.12. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A.

(i) $i_{\mu_j}i_{b_{ij}}(A) = i_{b_{ij}}i_{\mu_j}(A) = i_{\mu_j}(A)$.

(ii) $c_{\mu_j}c_{b_{ij}}(A) = c_{b_{ij}}c_{\mu_j}(A) = c_{\mu_j}(A)$.

(iii) $i_{b_{ij}}i_{\beta_{ij}}(A) = i_{\beta_{ij}}i_{b_{ij}}(A) = i_{b_{ij}}(A)$.

(iv) $c_{b_{ij}}c_{\beta_{ij}}(A) = c_{\beta_{ij}}c_{b_{ij}}(A) = c_{b_{ij}}(A)$.

(v) $i_{b_{ij}}c_{\alpha_{ij}}(A) = c_{\mu_i}i_{\mu_j}c_{\mu_i}(A)$.

(vi) $c_{b_{ij}}i_{\alpha_{ij}}(A) = i_{\mu_i}c_{\mu_j}i_{\mu_i}(A)$.

(vii) $c_{\alpha_{ij}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cup c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(viii) $i_{\alpha_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cap i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(ix) $c_{\alpha_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i} c_{\mu_j}(A)$.

(x) $i_{\alpha_{ji}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cap i_{\mu_j} c_{\mu_i} i_{\mu_j}(A)$.

(xi) $c_{\sigma_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j}(A)$.

(xii) $i_{\sigma_{ij}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cap c_{\mu_i} i_{\mu_j}(A)$.

(xiii) $c_{\pi_{ji}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i}(A)$.

(xiv) $i_{\pi_{ji}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cap i_{\mu_j} c_{\mu_i}(A)$.

(xv) $c_{\beta_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(xvi) $i_{\beta_{ij}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

Proof. (i) Clearly $i_{\mu_j} i_{b_{ij}}(A) \subset i_{\mu_j}(A)$. Also, $i_{\mu_j} i_{b_{ij}}(A) = i_{\mu_j}(i_{\sigma_{ij}}(A) \cup i_{\pi_{ji}}(A)) \supset i_{\mu_j}(i_{\sigma_{ij}}(A)) = i_{\mu_j}(A) \cup i_{\mu_j}(A) = i_{\mu_j}(A)$.

Also, $i_{b_{ij}}\left(i_{\mu_j}(A)\right) = i_{\sigma_{ij}}\left(i_{\mu_j}(A)\right) \cup i_{\pi_{ji}}\left(i_{\mu_j}(A)\right) = \left(i_{\mu_j}(A) \cap c_{\mu_i} i_{\mu_j} i_{\mu_j}(A)\right) \cup \left(i_{\mu_j}(A) \cap i_{\mu_j} c_{\mu_i} i_{\mu_j}(A)\right) = \left(i_{\mu_j}(A) \cap c_{\mu_i} i_{\mu_j}(A)\right) \cup \left(i_{\mu_j}(A) \cap i_{\mu_j} c_{\mu_i} i_{\mu_j}(A)\right) = i_{\mu_j}(A) \cup i_{\mu_j}(A) = i_{\mu_j}(A)$.

(ii) From (i), the proof follows.

(iii) By theorem 2.4(7) and (8), $i_{b_{ij}} i_{\beta_{ij}}(A) = i_{\sigma_{ij}}\left(i_{\beta_{ij}}(A)\right) \cup i_{\pi_{ji}}\left(i_{\beta_{ji}}(A)\right) = i_{\sigma_{ij}}(A) \cup i_{\pi_{ji}}(A) = i_{b_{ij}}(A)$.

Also, $i_{\beta_{ij}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}\left(i_{b_{ij}}(A)\right) = i_{b_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A) = i_{b_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A) \supset i_{b_{ij}}(A)$. Clearly, $i_{\beta_{ij}} i_{b_{ij}}(A) \subset i_{b_{ij}}(A)$. Hence proved.

(iv) From (iii), the proof follows.

(v) $i_{b_{ij}} c_{\alpha_{ij}}(A) = i_{\sigma_{ij}}\left(c_{\alpha_{ij}}(A)\right) \cup i_{\pi_{ji}}\left(c_{\alpha_{ij}}(A)\right) = \left(c_{\alpha_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)\right) \cup \left(c_{\alpha_{ij}}(A) \cap i_{\mu_j} c_{\mu_i}(A)\right) = c_{\alpha_{ij}}(A) \cap (c_{\mu_i} i_{\mu_j} c_{\mu_i}(A) \cup$

$i_{\mu_j} c_{\mu_i}(A) = c_{\alpha_{ij}}(A) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A) = \left(A \cup c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)\right) \cap c_{\mu_i} i_{\mu_j} c_{\mu_i}(A) = c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(vi) From (v), the proof follows.

(vii) $c_{\alpha_{ij}} i_{b_{ij}}(A) = i_{b_{ij}}(A) \cup c_{\mu_i} i_{\mu_j} c_{\mu_i}\left(i_{b_{ij}}(A)\right) = i_{b_{ij}}(A) \cup c_{\mu_i} i_{\mu_j}\left(c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)\right) = i_{b_{ij}}(A) \cup c_{\mu_i} i_{\mu_j} c_{\mu_i}(A)$.

(viii) From (vii), the proof follows.

(ix) $c_{\alpha_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i} c_{\mu_j}\left(c_{b_{ij}}(A)\right) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i} c_{\mu_j}(A)$ by (ii).

(x) From (ix), the proof follows.

(xi) $c_{\sigma_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j}\left(c_{b_{ij}}(A)\right) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j}(A)$ by (ii).

(xii) From (xi), the proof follows.

(xiii) $c_{\pi_{ji}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i}\left(c_{b_{ij}}(A)\right) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i} i_{\mu_i} c_{\mu_j} i_{\mu_i}(A) = c_{b_{ij}}(A) \cup c_{\mu_j} i_{\mu_i}(A)$.

(xiv) From (xiii), the proof follows.

(xv) $c_{\beta_{ij}} c_{b_{ij}}(A) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j} i_{\mu_i}\left(c_{b_{ij}}(A)\right) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j} i_{\mu_i} c_{\mu_j} i_{\mu_i}(A) = c_{b_{ij}}(A) \cup i_{\mu_i} c_{\mu_j} i_{\mu_i}(A)$.

(xvi) From (xv), the proof follows.

Theorem 3.12. In a BGTS (X, μ_1, μ_2), the following statements are true for any subset A.

(i) A is $b\mu_{ij}$- open if and only if $i_{b_{ij}} i_{\beta_{ij}}(A) = A$.

(ii) A is $b\mu_{ij}$- closed if and only if $c_{b_{ij}} c_{\beta_{ij}}(A) = A$.

Proof. (i) By theorem 3.12(vii), the result follows.

(ii) The proof of (ii) follows from (i).

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