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M modulo *N* Graceful Labeling on Arbitrary Super Subdivision of Ladder Graph

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Abstract

In this paper we show that arbitrary supersubdivision of ladder graph and supersubdivision of ladder graph are M modulo N graceful Labeling. Furthermore, on the given graph, a C++ program is used to trace the M modulo N graceful labelling.

Keywords: Graceful Labeling, Supersubdivision of ladder graph, Arbitrary supersubdivision of ladder graph, M modulo N graceful Labeling, Algorithm of labelling.

1. Introduction

A Ladder graph L_n is assumed an arbitrary supersubdivision of Ladder graph $ASS(L_n)$ if every edge of L_n is arbitrarily replaced by $K_{2,m}$, ($m \geq 1$) . If m is not arbitrary and fixed, the resulting graph is known as supersubdivision of Ladder graph $SS(L_n)$, and if $m = 1$, the resulting graph is referred to subdivision of Ladder graph $S(L_n)$. Numerous graphs that belong to this family have been developed and validated to use a M modulo N graceful labelling, where $\mathrm{M} =$

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1 to N and N is a positive integer [5]. The ladder and the graph that goes with it, satisfied the several labelling technologies [2], [3]. Reviewed and discussed the graph labelling of several graphs were conducted [1]. Arbitrary supersubdivision of ladder graph any it's types are referred to be graceful graphs [4].

2. M modulo N graceful labelling is produced by the ladder and the accompanying graph.

Theorem: 2.1 For every N (Positive integer) and $\mathrm{M} = 1 \text{ to } N$, the arbitrary supersubdivision of Ladder graph is M modulo N graceful graph.

Proof: In $ASS(L_n)$ every edge of L_n is replaced by $K_{2,m}$, $m \geq 1$ (m is arbitrary). The vertices of L_n is denoted by $u_1, u_2, \ldots, u_{2n}$ and the vertices in K_{2,m_i}, $i = 1$ to $3n - 2$ is denoted by v_j, $j = 1$ to $\sum_{i=1}^{3n-2} m_i$ in $ASS(L_n)$. Let $ASS(L_n)$ have $2n + \left[\sum_{i=1}^{3n-2} m_i\right]$ vertices and $2\left[\sum_{i=1}^{3n-2} m_i\right]$ edges.

Construction on M modulo N graceful labelling of Vertices in $ASS(L_n)$.

Consider the vertices $u_1, u_2, \ldots, u_{2n}$ in L_n:

$$f(u_j) = \left[2\left(\sum_{i=1}^{3n-2} m_i\right) - j\right]N + \mathrm{M},\ j = 1 \text{ to } n. \quad (1)$$

$$f(u_{2n-j+1}) = \left[2\left(\sum_{i=1}^{3n-2} m_i\right) - \sum_{i=1}^{n} m_{n-1+i} - j\right]N + \mathrm{M},\ j = 1 \text{ to } n. \quad (2)$$

Take the vertices v_t, $t = 1$ to $\sum_{i=1}^{n-1} m_i$ in K_{2,m_j}, $j = 1 \text{ to } n-1$:

$$f\left(v_{k+1+\sum_{i=0}^{j-1} m_i}\right) = \left[2\left(\sum_{i=0}^{j-1} m_i\right) - j + 1 + 2k\right]N,\ k = 0 \text{ to } m_j - 1,\ j = 1 \text{ to } n - 1,\ m_0 = 0. \text{ ----(3)}$$

Consider the vertices v_t, in $t = \sum_{i=1}^{n-1} m_i + s$, $s = 1$ to $\sum_{i=1}^{n} m_{(n-1)+i}$ in $K_{2,m_{(n-1)+j}}$, $j = 1 \text{ to } n$:

$$f\left(v_{\sum_{i=1}^{(n-1)} m_i+\sum_{i=1}^{(n-(j-1))} m_{n-1+i}-k}\right) = \left[\left(\sum_{i=1}^{2n-1} m_i\right)+\left(\sum_{i=1}^{n-1} m_i\right)-\left(\sum_{i=1}^{j-1} m_{2n-i}\right)-j-k\right]N,\ k=0 \text{ to } m_{(2n-j)}-1,\ j=1 \text{ to } n. \quad (4)$$

For the vertices $\boldsymbol{v_t}$, in $\boldsymbol{t}=\sum_{i=1}^{2n-1} \boldsymbol{m_i}+$ s, $\boldsymbol{s}=\mathbf{1}$ to $\sum_{i=1}^{n-1} \boldsymbol{m}_{(2n-1)+i}$ in $\boldsymbol{K}_{2, m_{2n-1+j}}$, $\boldsymbol{j}=\mathbf{1}$ **to** $\boldsymbol{n}-\mathbf{1}$.

$$f\left(v_{\sum_{i=1}^{(2n-1)} m_i+\sum_{i=1}^{(j-1)} m_{2n-1+i}+1+k}\right)$$

$$=\left[\left(\sum_{i=1}^{2n-1} m_i\right)+\left(\sum_{i=1}^{n-1} m_i\right)+2\left(\sum_{i=1}^{j-1} m_{2n-1+i}\right)-(j-1)+2k\right]N,$$

$$k=0 \text{ to } m_{2n+j-1}-1,\ j=1 \text{ to } n-1. \quad (5)$$

Hence, the vertices are mapping with distinct labelling as

$$\{[2\sum_{i=1}^{3n-2} m_i-1]N+M, [2\sum_{i=1}^{3n-2} m_i-2]N+M, \ldots, [2\sum_{i=1}^{3n-2} m_i-n]N+M\} \cup \{[2\sum_{i=1}^{3n-2} m_i-1-\sum_{i=1}^{n} m_{n-1+i}]N+M, [2\sum_{i=1}^{3n-2} m_i-2-\sum_{i=1}^{n} m_{n-1+i}]N+M, \ldots, [2\sum_{i=1}^{3n-2} m_i--\sum_{i=1}^{n} m_{n-1+i}]N+M\} \cup \{0, 2N, \ldots, [2(m_1-1)]N, [2m_1-1]N, [2m_1+1]N, [2m_1+3]N, \ldots, [2m_1+2m_2-3]N, \ldots, 2\left[\sum_{i=0}^{n-2} m_i-n+2\right]N, \left[2\sum_{i=0}^{n-2} m_i-n+4\right]N, \ldots, [2\sum_{i=0}^{n-2} m_i-n+2m_{n-1}]N\} \cup$$

$$\{[\sum_{i=1}^{2n-1} m_i+\sum_{i=1}^{n-1} m_i-1]N, \left[\sum_{i=1}^{2n-1} m_i+\sum_{i=1}^{n-1} m_i-2\right]N, \ldots, \left[\sum_{i=1}^{2n-1} m_i+\sum_{i=1}^{n-1} m_i-m_{2n-1}\right]N,$$

$$[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i - m_{2n-1} - 2]N, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i - m_{2n-1} - 3]N, \ldots, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i - m_{2n-1} - m_{2n-2} - 1]N, \ldots, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i - \sum_{i=1}^{n-1} m_{2n-i} - m_n - n + 1]N\} \cup \{[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i]N, \left[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2\right]N, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 4]N, \ldots, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2m_{2n} - 2]N, \left[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2m_{2n} - 1\right]N, \left[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2m_{2n} + 1\right]N, \ldots,$$

$$[\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2\sum_{i=1}^{n-2} m_{2n-1+i} - n + 2m_{3n-2}]N\}$$

$$= \{0, 2N, \ldots, 2(m_1 - 1)N, \ldots, [\sum_{i=1}^{2n-1} m_i + \sum_{i=1}^{n-1} m_i + 2\sum_{i=1}^{n-2} m_{2n-1+i} - n + 2m_{3n-2}]N, \ [2\sum_{i=1}^{3n-2} m_i - n - \sum_{i=1}^{n} m_{n-1+i}]N + M, \ldots,$$

$$[2\sum_{i=1}^{3n-2} m_i - 2]N + M, [2\sum_{i=1}^{3n-2} m_i - 1]N + M\}.$$

Take the edges incident with vertices between $(\boldsymbol{u_j}, \boldsymbol{m_j})$ **and** $(\boldsymbol{u_{j+1}}, \boldsymbol{m_j})$, $\boldsymbol{j} = \mathbf{1}$ **to** $\boldsymbol{n} - \mathbf{1}$.

$f^*(e_i) = \left[2\left(\sum_{k=1}^{3n-2} m_k\right) - i\right]N + \mathrm{M},\ i = 1\ to\ 2\sum_{k=1}^{n-1} m_k.$ (6)

Consider the edges incident with vertices between $(\boldsymbol{u_{n-(j-1)}}, \boldsymbol{m_{(n-1)+j}}), \boldsymbol{j} = \mathbf{1}\ \boldsymbol{to}\ \boldsymbol{n} - \mathbf{1}$.

$f^*(e_{t+i}) = \left[2\left(\sum_{k=1}^{3n-2} m_k\right) - t - i\right]N + \mathrm{M},\ \mathrm{i} = 1\ \text{to}\ \sum_{j=1}^{n} m_{(n-1)+j},\ t = 2\sum_{k=1}^{n-1} m_k.$ (7)

Assume the edges incident with vertices between $(\boldsymbol{u_{n+j}}, \boldsymbol{m_{(n-1)+j}}),\ \boldsymbol{j} = \mathbf{1}\ \boldsymbol{to}\ \boldsymbol{n} - \mathbf{1}$.

$f^*(e_{t+i}) = \left[2\left(\sum_{k=1}^{3n-2} m_k\right) - t - i\right]N + \mathrm{M},\ \mathrm{i} = 1\ \text{to}\ \sum_{j=1}^{n} m_{(n-1)+j},\ t = \sum_{k=1}^{2n-1} m_k + \sum_{k=1}^{n-1} m_k.$ (8)

Let the edges incident with vertices between $(\boldsymbol{u_{2n-j+1}}, \boldsymbol{m_{2n+j-1}})$ **and** $(\boldsymbol{u_{2n-j}}, \boldsymbol{m_{2n+j-1}}),\ \boldsymbol{j} = \mathbf{1}$ **to** $\boldsymbol{n} - \mathbf{1}$.

$f^*(e_{t+i}) = \left[2\left(\sum_{k=1}^{3n-2} m_k\right) - t - i\right]N + \mathrm{M},\ \mathrm{i} = 1\ \text{to}\ 2\sum_{j=1}^{n-1} m_{(2n+j-1)},\ t = 2\sum_{k=1}^{2n-1} m_k.$ (9)

Therefore, the edge mapping labelling are 1-1 and hence

$$\{[2\sum_{k=1}^{3n-2} m_k - 1]N + \mathrm{M}, [2\sum_{k=1}^{3n-2} m_k - 2]N + \mathrm{M}, \dots, [2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{n-1} m_k]N + \mathrm{M}\} \cup$$

$$\{[2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{n-1} m_k - 1]N + \mathrm{M}, [2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{n-1} m_k - 2]N + \mathrm{M}, \dots, [2\sum_{k=1}^{3n-2} m_k$$

$$-2\sum_{k=1}^{n-1} m_k - \sum_{j=1}^{n} m_{(n-1)+j}]N + \mathrm{M}\} \cup \{[2\sum_{k=1}^{3n-2} m_k - \sum_{k=1}^{2n-1} m_k$$
$$- \sum_{k=1}^{n-1} m_k - 1]N + \mathrm{M},$$

$$[2\sum_{k=1}^{3n-2} m_k - \sum_{k=1}^{2n-1} m_k - \sum_{k=1}^{n-1} m_k - 2]N + M, \dots, [2\sum_{k=1}^{3n-2} m_k - \sum_{k=1}^{2n-1} m_k$$
$$- \sum_{k=1}^{n-1} m_k -$$

$$\sum_{j=1}^{n} m_{(n-1)+j}]N + \mathrm{M}\} \cup \{ [2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{2n-1} m_k - 1]N$$
$$+ \mathrm{M}, [2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{2n-1} m_k$$

$$-2]N + \mathrm{M}, \dots, [2\sum_{k=1}^{3n-2} m_k - 2\sum_{k=1}^{2n-1} m_k - 2\sum_{j=1}^{n-1} m_{2n+j-1}]N + \mathrm{M}$$
$$= \{\mathrm{M}, N + \mathrm{M}, 2N + \mathrm{M},$$

$$3N + \mathrm{M}, \dots, [2\sum_{i=1}^{3n-2} m_i - 2]N + M, [2\sum_{i=1}^{3n-2} m_i - 1]N + M \}.$$

From the definition of f and f *, it's clear that arbitrary supersubdivision of Ladder graph $ASS(L_n)$ is M modulo N graceful labelling for all N and $\mathrm{M} = 1$ to N.

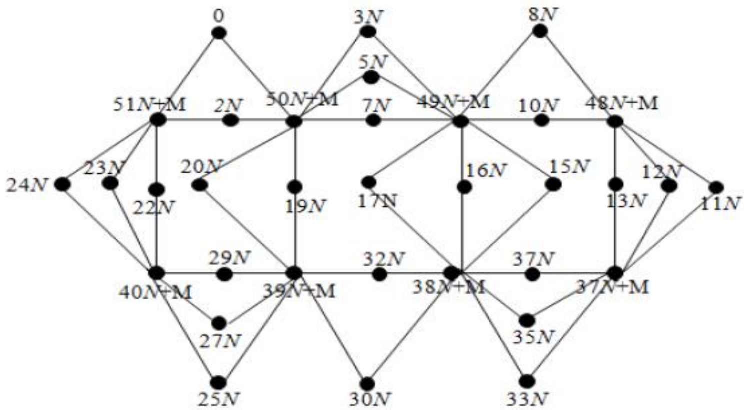

Figure 1: M modulo N graceful labelling of $ASS(L_4)$.

Example:1 3 modulo 5 graceful labelling of $ASS(L_4)$.

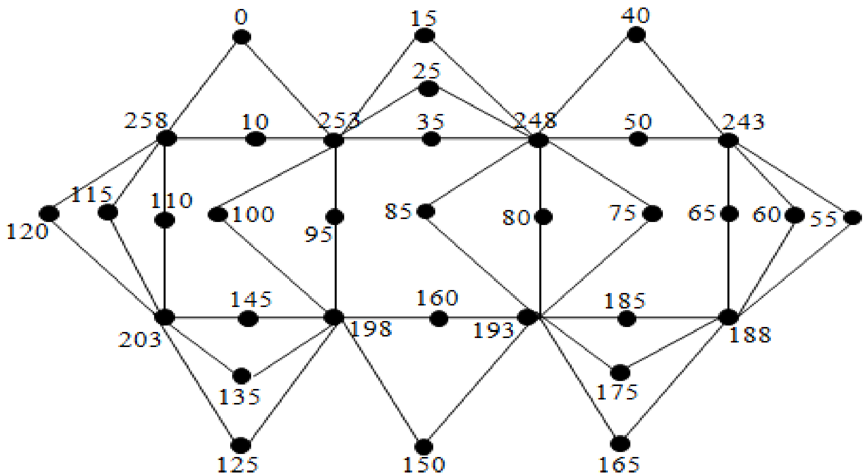

Figure 2: 3 modulo 5 graceful labelling of $ASS(L_4)$.

Theorem: 2.2 *A* super subdivision of Ladder graph $SS(L_n)$ is M modulo N graceful labelling for all positive integer N and $M = 1$ to N.

Proof: In $SS(L_n)$, every edge of L_n is replaced by $K_{2,m}$. The vertices of L_n is denoted by $u_1, u_2, \ldots, u_{2n}$ and the vertices of $K^i{}_{2,m}, i = 1$ to $3n-2$ is denoted by v_j, $j = 1$ to $m(3n-2)$ in $SS(L_n)$. Let $SS(L_n)$ has $2n + m(3n-2)$ vertices and $2m(3n-2)$ edges.

Description of M modulo *N* Graceful labelling on $SS(L_n)$.

Consider the vertices $u_1, u_2, \ldots, u_{2n}$ in L_n:

$$f(u_j) = [6mn - 4m - j]N + \mathrm{M}, j = 1 \text{ to } n. \quad (10)$$

$$f(u_{2n-j+1}) = [5mn - 4m - j]N + \mathrm{M}, j = 1 \text{ to } n. \quad (11)$$

Let the vertices v_t, $\boldsymbol{in\ t = 1}$ **to** $\boldsymbol{m(n-1)}$ in $\boldsymbol{K^j_{2,m}}$, j = 1 to n – 1.

$$f(v_{m(j-1)+k+1}) = [(2m-1)(j-1) + 2k]N, k = 0 \text{ to } m - 1, j = 1 \text{ to } n-1. \quad (12)$$

Take the vertices v_t, $\boldsymbol{in\ t = 1}$ **to** $\boldsymbol{m(n-1) + s, s = 1}$ **to** $\boldsymbol{mn}$ in $\boldsymbol{K^{(n-1)+j}_{2,m}}$, j = 1 to n.

$$f(v_{2nm-jm-k}) = [(3mn - 2m) - (j-1)m - j - k]N, k = 0 \text{ to } m-1, j = 1 \text{ to } n. \quad (13)$$

Assume the vertices $\boldsymbol{v_t}$, $\boldsymbol{in\ t = 1\ to\ m(2n-1) + s,\ s = 1\ to\ m(n-1)}$ in $\boldsymbol{K^{(2n-1)+j}_{2,m}}$, j = 1 to n – 1.

$$f(v_{m(2n-2+j)+1+k}) = [(3mn - 2m) + (j-1)(2m-1) + 2k]N, k = 0 \text{ to } m-1, j = 1 \text{ to } n-1. \quad (14)$$

Hence, $\{[6mn-4m-1]N + \mathrm{M}, [6mn-4m-2]N + \mathrm{M}, \ldots, [6mn-4m-n]N + \mathrm{M}\} \cup$

$\{[5mn-4m-1]N + \mathrm{M}, [5mn-4m-2]N + \mathrm{M}, \ldots, [5mn-4m-n]N + \mathrm{M}\} \cup \{0, 2N, 4N,$

$\dots, [2m-2]N, [2m-1]N, \ [2m+1]N, \dots, [(2m-1)(n-2)]N, \dots, [2mn-2m-n]N\} \cup$

$[3mn-2m-1]N, \ [3mn-2m-2]N, \ \dots, \ [3m(n-1)]N, \ [3m(n-1)-2]N, \ [3m(n-1)-3]N, \dots, [(2m-1)(n-1)]N\} \cup \{[3mn-2m]N, \ [3mn-2m+2]N, \ [3mn-2m+4]N, [3mn-2m+6]N, \dots, [3mn-2]N, [3mn-1]N, \dots, \ [5mn-4m-n]N = \{0, 2N, \dots,$

$[2mn-2m-n]N, \dots, [6mn-4m-2]N + \mathrm{M}, [6mn-4m-1]N + \mathrm{M}$ are confirmed that vertex mapping is 1-1.

Assume the edges incident with vertices between $(\boldsymbol{u_j}, \boldsymbol{m_j}) \boldsymbol{and} (\boldsymbol{u_{j+1}}, \boldsymbol{m_j}), \ \boldsymbol{j = 1 \ to \ n-1}$.

$f^*(e_i) = [(6n-4)m - i]N + \mathrm{M}, i = 1 \ to \ 2(n-1)m.$ ---- (15)

Let the edges incident with vertices between $(\boldsymbol{u_{n-(j-1)}}, \boldsymbol{m_{(n-1)+j}})$, $\boldsymbol{j = 1 \text{ to } n-1}$.

$f^*(e_{t+i}) = [(6n-4)m - t - i]N + \mathrm{M}, i = 1 \text{ to } mn, t = 2(n-1)m.$ (16)

Take the edges incident with vertices between $(\boldsymbol{u_{n+j}}, \boldsymbol{m_{(n-1)+j}}), \ \boldsymbol{j = 1 \text{ to } n-1}$.

$f^*(e_{t+i}) = [(6n-4)m - t - i]N + \mathrm{M}, i = 1 \text{ to } nm, t = (3n-2)m.$ (17)

Consider the edges incident with vertices between (u_{2n-j+1}, m_{2n+j-1}) and (u_{2n-j}, m_{2n+j-1}), $j = 1 \text{ to } n-1$.

$f^*(e_{t+i}) = [(6n-4)m - t - i]N + \mathrm{M}, i = 1 \text{ to } 2(n-1)m, \ t = (4n-2)m.$ --- (18)

Hence,

$\{[6mn-4m-1]N + \mathrm{M}, [6mn-4m-2]N + \mathrm{M}, \dots, [4mn-2m]N + \mathrm{M}\} \cup \{[4mn$

$- 2m - 1]N + \mathrm{M}, [4mn - 2m - 2]N + \mathrm{M}, \ldots, [3mn - 2m]N + \mathrm{M}\}$
$\cup \{ [3mn - 2m - 1]$

$N + \mathrm{M}, [3mn - 2m - 2]N + \mathrm{M}, \ldots, [2m(n-1)]N + \mathrm{M} \}$
$\cup \{[2m(n-1) - 1]N + M,$

$[2m(n-1) - 2]N + \mathrm{M}, \ldots, \mathrm{M}\} = \{\mathrm{M}, N + \mathrm{M}, 2N + \mathrm{M}, \ldots,$
$[6mn - 4m - 2]N + \mathrm{M}, [6mn - 4m - 1]N + \mathrm{M}\}$ confirms that the edge mapping is 1-1.

From the definition of f and f^*, it's clear that arbitrary supersubdivision of Ladder graph $ASS(L_n)$ is M modulo N graceful labelling for all N and $\mathrm{M} = 1$ to N.

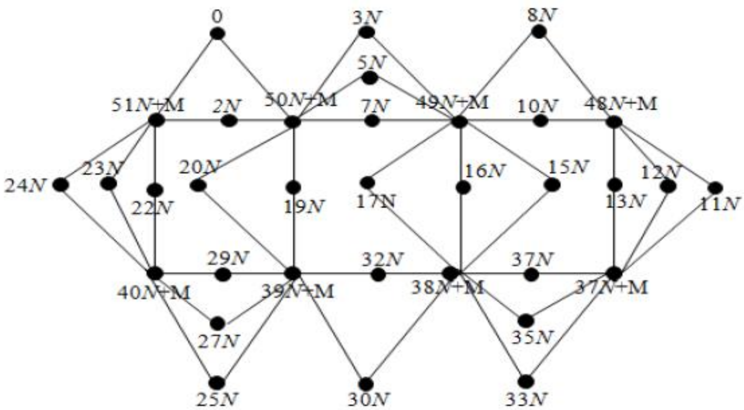

Figure 1: M modulo N graceful labelling of $ASS(L_4)$.

Example:1 3 modulo 5 graceful labelling of $ASS(L_4)$.

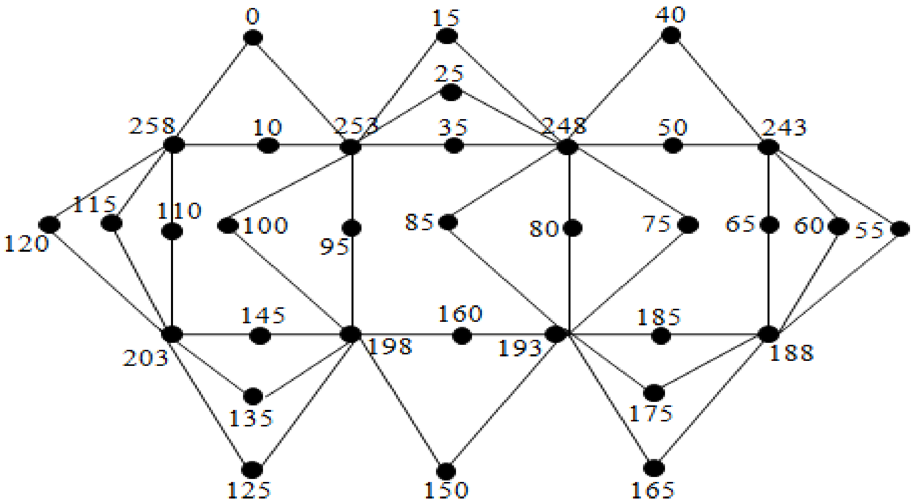

Figure 2: 3 modulo 5 graceful labelling of $ASS(L_4)$.

***Theorem:* 2.2** *A* Supersubdivision of Ladder graph $SS(L_n)$ is M modulo N graceful labelling for all positive integer N and $M = 1$ to N.

***Proof*:** In $SS(L_n)$, every edge of L_n is replaced by $K_{2,m}$. The vertices of L_n is denoted by $u_1, u_2, \ldots, u_{2n}$ and the vertices of $K^i{}_{2,m}, i = 1$ to $3n-2$ is denoted by v_j , $j = 1$ to $m(3n-2)$ in $SS(L_n)$. Let $SS(L_n)$ has $2n + m(3n-2)$ vertices and $2m(3n-2)$ edges.

Description of M modulo *N* Graceful labelling on $SS(L_n)$.

Consider the vertices $u_1, u_2, \ldots, u_{2n}$ in L_n:

$f(u_j) = [6mn - 4m - j]N + M, j = 1$ to $n. - - - - (10)$

$f(u_{2n-j+1}) = [5mn - 4m - j]N + M, j = 1$ to $n. - - - - (11)$

Let the vertices v_t, *in* $t = 1$ to $m(n - 1)$ in $K^j_{2,m}$, j = 1 to n – 1.

$f(v_{m(j-1)+k+1}) = [(2m-1)(j-1)+2k]N, k = 0$ to $m-1, j = 1$ to $n-1$. ----(12)

Take the vertices v_t, $in\ t = 1$ to $m(n-1)+s, s = 1$ to mn in $K_{2,m}^{(n-1)+j}$, j = 1 to n.

$f(v_{2nm-jm-k}) = [(3mn-2m)-(j-1)m-j-k]N, k = 0$ to $m-1, j = 1$ to n. ----(13)

Assume the vertices v_t, $in\ t = 1\ to\ m(2n-1)+s,\ s = 1\ to\ m(n-1)$ in $K_{2,m}^{(2n-1)+j}$, j = 1 to n – 1.

$f(v_{m(2n-2+j)+1+k}) = [(3mn-2m)+(j-1)(2m-1)+2k]N, k = 0$ to $m-1$, $j = 1$ to $n-1$. -------(14)

Hence, $\{[6mn-4m-1]N+M, [6mn-4m-2]N+M, \dots, [6mn-4m-n]N+M\} \cup$

$\{[5mn-4m-1]N+M, [5mn-4m-2]N+M, \dots, [5mn-4m-n]N+M\} \cup \{0, 2N, 4N,$

$$\dots, [2m-2]N, [2m-1]N, [2m+1]N, \dots, [(2m-1)(n-2)]N, \dots, [2mn-2m-n]N\} \cup$$

$[3mn-2m-1]N$, $[3mn-2m-2]N$, ..., $[3m(n-1)]N$, $[3m(n-1)-2]N$, $[3m(n-1)-3]N, \dots, [(2m-1)(n-1)]N\} \cup \{[3mn-2m]N$, $[3mn-2m+2]N$, $[3mn-2m+4]N$, $[3mn-2m+6]N, \dots, [3mn-2]N, [3mn-1]N, \dots$, $[5mn-4m-n]N = \{0, 2N, \dots,$

$[2mn-2m-n]N, \dots, [6mn-4m-2]N+M, [6mn-4m-1]N+M$ are confirmed that vertex mapping is 1-1.

Assume the edges incident with vertices between (u_j, m_j) and (u_{j+1}, m_j), $j = 1\ to\ n-1$.

$f^*(e_i) = [(6n-4)m-i]N+M, i = 1\ to\ 2(n-1)m$. ---- (15)

Let the edges incident with vertices between $(\boldsymbol{u_{n-(j-1)}}, \boldsymbol{m_{(n-1)+j}}),\ \boldsymbol{j} = \mathbf{1}$ **to** $\boldsymbol{n}-\mathbf{1}$**.**

$f^*(e_{t+i}) = [(6n - 4)m - t - i]N + \mathrm{M}, i = 1$ to $mn, t = 2(n - 1)m$. -----(16)

Take the edges incident with vertices between $(\boldsymbol{u_{n+j}}, \boldsymbol{m_{(n-1)+j}}),\ \boldsymbol{j} = \mathbf{1}$ **to** $\boldsymbol{n}-\mathbf{1}$.

$f^*(e_{t+i}) = [(6n - 4)m - t - i]N + \mathrm{M}, i = 1$ to $nm, t = (3n - 2)m$. -----(17)

Consider the edges incident with vertices between (u_{2n-j+1}, m_{2n+j-1}) and (u_{2n-j}, m_{2n+j-1}), $j = 1$ to $n-1$.

$f^*(e_{t+i}) = [(6n - 4)m - t - i]N + \mathrm{M}, i = 1$ to $2(n - 1)m,\ t = (4n - 2)m$. --- (18)

Hence,

$$\{[6mn - 4m - 1]N + \mathrm{M}, [6mn - 4m - 2]N + \mathrm{M}, \dots, [4mn - 2m]N + \mathrm{M}\} \cup \{[4mn$$

$$- 2m - 1]N + \mathrm{M}, [4mn - 2m - 2]N + \mathrm{M}, \dots, [3mn - 2m]N + \mathrm{M}\} \cup \{ [3mn - 2m - 1]$$

$$N + \mathrm{M}, [3mn - 2m - 2]N + \mathrm{M}, \dots, [2m(n-1)]N + \mathrm{M}\} \cup \{[2m(n-1) - 1]N + M,$$

$[2m(n-1) - 2]N + \mathrm{M}, \dots, \mathrm{M}\} = \{\mathrm{M}, N + \mathrm{M}, 2N + \mathrm{M}, \dots, [6mn - 4m - 2]N + \mathrm{M}, [6mn - 4m - 1]N + \mathrm{M}\}$ confirms that the edge mapping is 1-1.

Hence supersubdivision of Ladder graph $SS(L_n)$ is M modulo N graceful labelling for all N and $M = 1\ to\ N$.

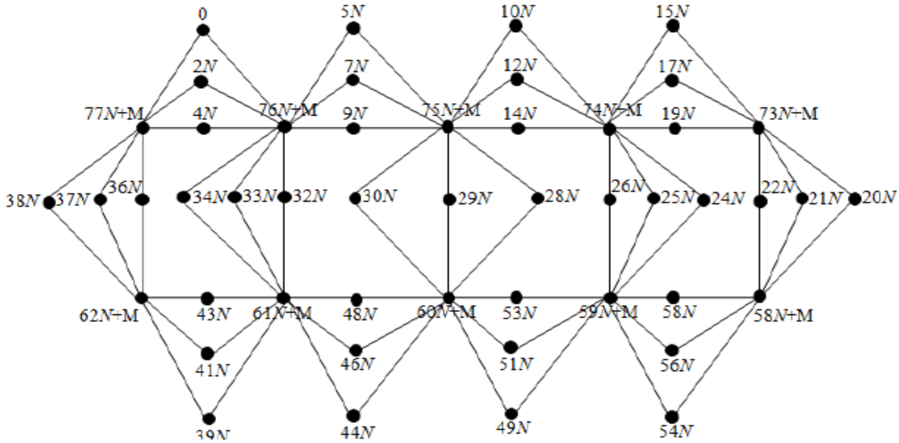

Figure 3: M modulo N graceful labelling of super subdivision of Ladder graph $SS(L_5)$.

Example 2: 5 modulo 10 graceful labelling of super subdivision of Ladder graph $SS(L_5)$.

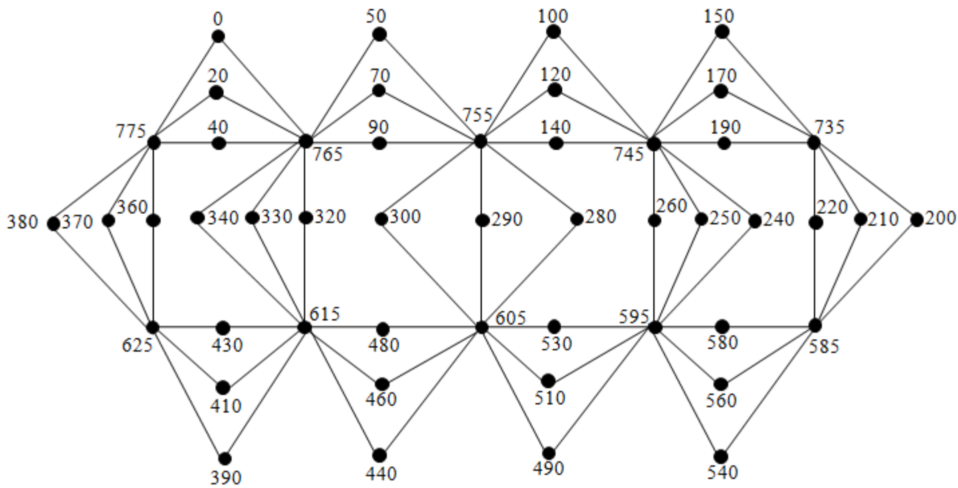

Figure 4: 5 modulo 10 graceful labelling of supersubdivision of Ladder graph $SS(L_5)$.

3. Algorithm for evaluating M modulo N graceful labelling.

3.1 C++ Programming Algorithms.

Algorithm of M modulo N graceful labelling on arbitrary supersubdivision of Ladder graph, supersubdivision of Ladder graph, Subdivision of Ladder graph for all positive integer N, M $=$ 1 to N.

In this algorithm we able to find

i. M modulo N graceful labelling of $ASS(L_n)$, when m is arbitrary in$K_{2,m}$.

ii. M modulo N graceful labelling of $SS(L_n)$, when m is not arbitrary in$K_{2,m}$.

iii. M modulo N graceful labelling of $S(L_n)$, when $m = 1$ in $K_{2,m}$.

```
#include<iostream.h>
#include<conio.h>
void main()
{
clrscr();
int i,j,t,n,M,N,Y,p,m[100],Z,q,q1,q2,q3,q4,q5,q6,q7,q8,q9,k;
q=0,q1=0,q3=0,q4=0;
cout<<endl<<"Enter the N value: N = ";
cin>>N;
cout<<endl<<"Enter the n value: n = ";
cin>>n;
cout<<endl<<"Enter the mi value: ";
cout<<endl<<"Is The graph is ASS(Ln) if yes type 1:Z=";
cin>>Z;
if(Z==1)
{
cout<<"M modulo N graceful labelling of Arbitrary
supersubdivision of Ladder graph";
for(i=1;i<=(3*n)-2;i++)
```

```
{
cout<<" m["<<i<<"] = ";
cin>>m[i];
q=q+(2*m[i]);
}
}
else
{
cout<<"M modulo N graceful labelling of supersubdivision of
Ladder graph";
cout<<" enter m Value : m= ";
cin>>p;
for(i=1;i<=(3*n)-2;i++)
{
m[i]=p;
q=q+(2*m[i]);
}
}
cout<<endl<<"q="<<q<<endl;
for(i=1;i<=(2*n)-1;i++)
{
q3=q3+m[i];
if(i<=n)
{
q1=q1+m[n-1+i];
}
if(i<=n-1)
{
q4=q4+m[i];
}
}
cout<<"Want to find particular M if yes enter 1 :";
cin>>Y;
if(Y==1)
{
cout<<endl<<"Enter the M value: M = ";
cin>>M;
goto X;
}
```

```
for(M=1;M<=N;M++)
{
X:
cout<< M<<" modulo"<<N<< " graceful labelling for vertices:";
for(j=1;j<=n;j++)
{
cout<<" f(u"<<j<<")="<<(q-j)*N+M;
}
for(j=1;j<=n;j++)
{
cout<<" f(u"<<(2*n)-j+1<<")="<<(q-q1-j)*N+M;
}
for(j=1;j<=n-1;j++)
{
q2=0;
for(i=1;i<=j-1;i++)
{
 q2=q2+m[i];
 }
for(k=0;k<=m[j]-1;k++)
{
cout<<" f(v"<<1+q2+k<<")="<<(2*q2-j+1+2*k)*N;
 }
}
for(j=1;j<=n;j++)
{
 q5=0; q6=0;
 for(i=1;i<=n-(j-1);i++)
{
 q5=q5+m[n-1+i];
 }
 for(i=1;i<=j-1;i++)
{
 q6=q6+m[2*n-i];
 }
for(k=0;k<=m[2*n-j]-1;k++)
{
cout<<" f(v"<<q4+q5-k<<")="<<(q3+q4-q6-j-k)*N;
}
```

```
}
for(j=1;j<=n-1;j++)
{
 q7=0; q8=0 ;
 for(i=1;i<=j-1;i++)
{
 q7=q7+m[2*n-1+i];
 q8=q7-1;
 }
 for(k=0;k<=m[2*n-1+j]-1;k++)
 {
cout<<" f(v"<<q3+q7+1+k<<")="<<(q3+q4+2*q7-(j-1)+2*k)*N;
 } }
cout<<endl<<M<<" modulo "<<N<< " graceful labelling for edge:";
for(i=1;i<=2*q4;i++)
{
cout<<" f*(e"<<i<<")="<<(q-i)*N+M;
}
for(i=1;i<=q1;i++)
{
cout<<" f*(e"<<2*q4+i<<")="<<(q-(2*q4)-i)*N+M;
 }
 for(i=1;i<=q1;i++)
{
cout<<" f*(e"<<q3+q4+i<<")="<<(q-(q3+q4)-i)*N+M;
 }
 q9=0;
 for(j=1;j<=n-1;j++)
{
 q9+=m[2*n+j-1] ;
}
for(i=1;i<=2*q9;i++)
{
cout<<" f*(e"<<2*q3+i<<")="<<(q-(2*q3)-i)*N+M;
}
if(Y==1)
{
goto R;
}
```

```
}
R:
if(Z==1)
{
if(Y==1)
{
cout<<endl<<M<<" modulo "<<N<<" graceful labelling of Arbitrary supersubdivision of Ladder graph";
}
else
{
cout<<endl<<"M modulo N graceful labelling of Arbitrary supersubdivision of Ladder graph";
}}
else
{
if(p==1)
{
if(Y==1)
{
cout<<endl<<M<<" modulo "<<N<<" graceful labelling of subdivision of Ladder graph";
}
else
{
cout<<endl<<"M modulo N graceful labelling of subdivision of Ladder graph";
}
}
else
{
if(Y==1)
{
cout<<endl<<M<<" modulo "<<N<<" graceful labelling of supersubdivision of Ladder graph";
}
else
{
```

```
cout<<endl<<"M modulo N graceful labelling of supersubdivision of Ladder graph";
}
}
}
getch();
}.
```

4. Conclusions

In this work we proved that arbitrary supersubdivision of ladder graph and supersubdivision of ladder graph are M modulo N graceful labelling. The developed C ++ algorithm reduces the manual work for finding M modulo N graceful labelling when the size of the graph is large. In C++ Programming algorithm, when m is arbitrary, then its known as M modulo N graceful labelling on arbitrary supersubdivision of ladder graph, when m is fixed, then its known as M modulo N graceful labelling on supersubdivision of ladder graph and when m is 1, then it is known as M modulo N graceful Labeling on subdivision of ladder graph. In future, we apply our labelling techniques in various different communication network problems which one need on high security.

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