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Fibonacci Vertex Prime Labelings of Some Graphs

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Abstract

A graph $G(|V(G)| = p, |E(G)| = q)$ has an FVP labeling if a bijective function f exists from $E(G)$ to the set $\{F_2, F_3, \dots, F_{q+1}\}$ having the property that the labels of the edges incident at any vertex of degree atleast two are relatively prime. An FVP graph is one that accepts an FVP labeling. In this work we realise that several graphs such as Path, Cycle, Wheel, Gear, Helm, Shell, Bistar and Umbrella graphs are FVP graph.

Keywords: Fibonacci number, Vertex prime labeling, Fibonacci vertex prime labelling.

1. Introduction

A thorough analysis of graph labelings is provided by Gallian [4]. Roger Entringer first proposed the idea of prime labeling, which was then addressed by Tout et al. [9]. Eventually many developments were brought in this field. The cycle C_n on n vertices was shown to be a prime graph by Deresky et al. [3]. Prime labeling of various path and cycle-related graphs was illustrated in [5],[7].

Chandrakala and Sekar [2] introduced the concept of Fibonacci prime labeling. They showed the Fibonacci prime nature of a few cycle-related graphs. Furthermore, they provided the Fibonacci prime labeling of the Uddukkai and octopus graph in [1]. T.Deretsky

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et al. [3] proposed a new concept called Vertex Prime labeling (VP labeling). M. Bapat has demonstrated various VP graphs in [8]. Kavitha et al. [6] established VP labeling for several Helm-related graphs. In this article, we consider finite simple graphs with no directed edges.

2. Main Results

Definition 2.1. A graph $G(|V(G)| = p, |E(G)| = q)$ has a Fibanocci Vertex Prime (FVP) labeling if a bijective function f exists from $E(G)$ to the set $\{F_2, F_3, \dots, F_{q+1}\}$ having the property that the labels of the edges incident at any vertex of degree atleast two are relatively prime. An FVP graph is the one that accepts an FVP labelling.

Theorem 2.2. Path is an FVP graph.

Proof. Assume P_n to be a path with $V(P_n) = \{v_i\}_{i=1}^{n}$ and $E(P_n) = \{v_i v_{i+1}\}_{i=1}^{n-1}$.

Define $f: E(P_n) \to \{F_2, F_3, \dots, F_n\}$ such that $f(v_i v_{i+1}) = F_{i+1}$, whenever $1 \le i \le n-1$.

Let $f'(v) = \gcd\{f(e): e\ is\ incident\ at\ v, \deg(v) \ge 2\}$ $.Then,$
$f'(v_i) = gcd\big(f(v_{i-1}v_i), f(v_i v_{i+1})\big) = \gcd(F_i, F_{i+1}) = 1$, whenever $(2 \le i \le n-1)$. Hence, a path -P_n is an FVP graph.

Theorem 2.3. Cycle C_n is an FVP graph.

Proof. Assume C_n be a cycle graph with $V(C_n) = \{v_i\}_{i=1}^{n}$ and $E(C_n) = \{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_n v_1\}$. Define $f: E(C_n) \to \{F_2, F_3, \dots, F_{n+1}\}$ as follows.

$f(v_i v_{i+1}) = F_{i+1}$, whenever $1 \le i \le n-1$,

$f(v_n v_1) = F_{n+1}$.

Let $f'(v) = \gcd\{f(e): e\ is\ incident\ at\ v, \deg(v) \ge 2\}$.

Then, $f'(v_i) = gcd\big(f(v_{i-1}v_i), f(v_i v_{i+1})\big) = \gcd(F_i, F_{i+1}) = 1$, whenever $(2 \le i \le n)$. For $f'(v_1) = \gcd(F_2, F_{n+1}) = 1$. Hence, cycle C_n is an FVP graph.

Theorem 2.4. Wheel graph W_n is an FVP graph.

Proof. Assume W_n to be the wheel graph with $n + 1$vertices and $2n$ edges.

Let $V(W_n) = \{v_i\}_{i=0}^{n}$ where v_0 be the apex vertex and

$E(W_n) = \{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_n v_1\} \cup \{v_0 v_i\}_{i=1}^{n}$. Define $f: E(W_n) \to \{F_2, F_3, \dots, F_{2n+1}\}$ as follows.

$f(v_i v_{i+1}) = F_{2i+1}$, whenever $1 \le i \le n-1$,

$f(v_n v_1) = F_{2n+1}$,

$f(v_0 v_i) = F_{2i}$, whenever $1 \le i \le n$.

Let $f'(v) = \gcd\{f(e)$: $e\ is\ incident\ at\ v, \deg(v) \ge 2\}$. Then, $f'(v_i) = gcd(f(v_{i-1}v_i), f(v_i v_{i+1}), f(v_0 v_i) = \gcd(F_{2i-1}, F_{2i+1}, F_{2i}) = 1$, whenever $(2 \le i \le n)$.

$f'(v_1) = \gcd(F_2, F_{3,}F_{n+1}) = 1$

$f'(v_0) = \gcd(f(v_0 v_i)) = \gcd\{F_{2i},\ \ for\ 1 \le i \le n\} = 1$.

Hence, Wheel graph W_n is an FVP graph.

Theorem 2.5. Gear graph G_n is an FVP graph.

Proof. Assume G_n to be the Gear graph with $2n + 1$vertices and $3n$ edges.

Let $V(G_n) = \{v_i\}_{i=0}^{n} \cup \{u_i\}_{i=1}^{n}$, where v_0 be the apex vertex and

$E(G_n) = \{v_i u_i\}_{i=1}^{n} \cup \{u_i v_{i+1}\}_{i=1}^{n-1} \cup u_n v_1 \cup \{v_0 v_i\}_{i=1}^{n}$.

Define $f: E(G_n) \to \{F_2, F_3, \dots, F_{3n+1}\}$ as follows.

$f(v_i u_i) = F_{3i}$, whenever $1 \le i \le n$,

$f(u_i v_{i+1}) = F_{3i+1}$, whenever $1 \le i \le n-1$,

$f(u_n v_1) = F_{3n+1}$

$f(v_0 v_i) = F_{3i-1}$, whenever $1 \le i \le n$.

Let $f'(v) = \gcd\{f(e)$: $e\ is\ incident\ at\ v,\ \deg(v) \ge 2\}$. Then, $f'(v_i) = gcd(f(u_{i-1}v_i), f(v_i u_i), f(v_0 v_i)) = \gcd(F_{3i-2}, F_{3i}, F_{3i-1}) = 1$, for $(2 \le i \le n)$.

$f'(v_1) = \gcd(F_2, F_{3,}F_{3n+1}) = 1$.

$f'(u_i) = \gcd(f(v_i u_i), f(u_i v_{i+1})) = \gcd\{(F_{3i}, F_{3i+1}) = 1, 1 \le i \le n\}$.

For $f'(v_0) = \gcd(f(v_0 v_i)) = \gcd\{F_{3i-1},\ \ for\ 1 \le i \le n\} = 1$.

Hence, Gear graph G_n is an FVP graph.

Theorem 2.6. Shell graph $(n, n-3)$ is an FVP graph.

Proof: Assume $C(n,n-3)$ be the shell graph with $\left|V\left(C(n,n-3)\right)\right| = n$ and $\left|E\left(C(n,n-3)\right)\right| = 2n-3$. Then, $V\left(C(n,n-3)\right) = \{v_i\}_{i=1}^{n}$ where, v_1 be the apex vertex and $\left|E\left(C(n,n-3)\right)\right| = \{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_n v_1\} \cup \{v_1 v_m\}_{m=3}^{n-1}$. Define $f: E\left(C(n,n-3)\right) \to \{F_2, F_3, \dots, F_{2n-2}\}$ as follows.

$f(v_i v_{i+1}) = F_{2i-1}$, whenever $2 \le i \le n-1$,

$f(v_1 v_i) = F_{2i-2}$, whenever $2 \le i \le n-1$,

Let $f'(v) = \gcd\{f(e)\colon e \; is \; incident \; at \; v, \deg(v) \ge 2\}$. Then, $f'(v_i) = gcd\left(f\left(v_{i-1,}v_i\right), f(v_i v_{i+1}), f(v_1 v_i)\right) = \gcd(F_{2i-3}, F_{2i-1}, F_{2i-2}) = 1$, for $(3 \le i \le n-1)$.

For $f'(v_1) = \gcd(f(v_1 v_i),\ 1 \le i \le n) = \gcd(F_2, F_4, F_6, \dots\dots F_{2n-2}) = 1$.

For $f'(v_2) = \gcd\left(f(v_1 v_2), f(v_2 v_3)\right) = \gcd(F_2, F_3) = 1$.

$f'(v_n) = \gcd\left(f(v_{n-1} v_n), f(v_n v_1)\right) = \gcd(F_{2n-3}, F_{2n-2}) = 1$.

Hence, shell graph $C(n, n-3)$ is an FVP graph.

Theorem 2.7. Helm graph H_n is an FVP graph.

Proof: Assume H_n to be the Helm graph with $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$.

Let $V(H_n) = \{v_i\}_{i=0}^{n} \cup \{v_i{}^*\}_{i=1}^{n}$, where, v_0 be the apex vertex and $E(H_n) = \{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_n v_1\} \cup \{v_0 v_i\}_{i=1}^{n} \cup \{v_i v_i^*\}_{i=1}^{n}$.

Define $f: E(H_n)) \to \{F_2, F_3, \dots, F_{3n+1}\}$ as follows.

$f(v_i v_{i+1}) = F_{3i}$, whenever $1 \le i \le n-1$,

$f(v_n v_1) = F_{3n}$,

$f(v_0 v_i) = F_{3i-1}$, whenever $1 \le i \le n$,

$f(v_i v_i^*) = F_{3i+1}$, whenever $1 \le i \le n$.

Let $f'(v) = \gcd\{f(e)\colon e \; is \; incident \; at \; v, \deg(v) \ge 2\}$. Then,

$f'(v_i) = gcd(f(v_{i-1} v_i), f(v_i v_{i+1}), f(v_0 v_i), f(v_i v_i^*)) = \gcd(F_{3i-3}, F_{3i}, F_{3i-1}, F_{3i+1}) = 1$, for $(2 \le i \le n)$ and $f'(v_1) = \gcd\left(F_2, F_{3,} F_4, F_{3n}\right) = 1$.

For $f'(v_0) = \gcd\left(f(v_0 v_i)\right) = \gcd\{F_{3i-1},\ for\ 1 \le i \le n\} = 1$.

Hence, Helm graph H_n is an FVP graph.

Theorem 2.8. Bistar $B_{n,n}$ is an FVP graph.

Proof: Assume $B_{n.n}$ to be the Bistar graph with $|V(B_{n,n})| = 2n+2$ and $|E(B_{n,n})| = 2n+1$.

Let $V(B_{n.n}) = \{u_i\}_{i=0}^{n} \cup \{v_i\}_{i=1,}^{n}$ and $(B_{n,n}) = u_0v_0 \cup \{u_0u_i\}_{i=1}^{n} \cup \{v_0v_i\}_{i=1}^{n}$.

Define $f: E(B_{n.n}) \rightarrow \{F_2, F_3, \dots, F_{2n+2}\}$ as follows.

$f(u_0u_i) = F_{i+2}$, whenever $1 \leq i \leq n$,

$f(v_0v_i) = F_{n+2+i}$, whenever $1 \leq i \leq n$,

$f(u_0v_0) = F_2$.

Let $f'(v) = \gcd\{f(e)\colon e \text{ is incident at } v, \deg(v) \geq 2\}$. The vertices u_0 *and* v_0 in $B_{n.n}$ have $d(v) \geq 2$.

Then, $f'(u_0) = \gcd(f(u_0u_i), f(u_0v_0)) = \gcd(\{F_{i+2}, 1 \leq i \leq n\}, F_2)=1$.

$f'(v_0) = \gcd(f(v_0v_i), f(u_0v_0)) = \gcd(\{F_{n+2+i}, 1 \leq i \leq n\}, F_2)=1$.

Hence, $B_{n,n}$ is an FVP graph.

Theorem 2.9. Umbrella graph $U(m,n)$ is an FVP graph.

Proof: Assume $U(m,n)$ to be the umbrella graph with $|V(U(m,n)| = m+n$ and $|E(U(m,n)| = 2m+n-2$.

The vertex set $V(U(m,n)) = \{v_i\}_{i=1}^{m} \cup \{u_i\}_{i=1,}^{n}$ and edge set $E(U(m,n)) = \{v_iv_{i+1}\}_{i=1}^{m-1} \cup \{u_iu_{i+1}\}_{i=1}^{n-1} \cup \{u_1v_i\}_{i=1}^{m}$.

Define $f: E(U(m,n)) \rightarrow \{F_2, F_3, \dots, F_{2m+n-1}\}$ as follows.

$f(v_iv_{i+1}) = F_{n+2i}$, whenever$1 \leq i \leq m-1$,

$f(u_1v_i) = F_{n+2i-1}$, whenever $1 \leq i \leq m$,

$f(u_iu_{i+1}) = F_{i+1}$, whenever $1 \leq i \leq n-1$,

Let $f'(v) = \gcd\{f(e)\colon e \text{ is incident at } v, \deg(v) \geq 2\}$. Then,

Then, $f'(v_i) = gcd(f(v_{i-1}v_i), f(v_iv_{i+1}), f(u_1v_i)) = \gcd(F_{n+2i-2}, F_{n+2i}, F_{n+2i-1}) = 1$, for $(2 \leq i \leq m-1)$.

For $f'(v_1) = \gcd(f(u_1v_1), f(v_1v_2)) = \gcd(F_5, F_6) = 1$.

$f'(v_n) = \gcd(f(v_{m-1}v_m), f(u_1v_m)) = \gcd(F_{n+2m-2}, F_{n+2m-1}) = 1$.

$f'(u_1) = \gcd\big(f(u_1v_i)\ 1 \le i \le m), f(u_1u_2)\big) = \gcd\ ((F_{n+2i-1}, 1 \le i \le n), F_2) = 1.$

$f'(u_i) = \gcd\big(f(u_{i-1}u_i), f(u_iu_{i+1})\big) = \gcd\ ((F_i, F_{i+1}) = 1.\ \ for\ 1\ \le i\ \le n).$

Hence, $U(m, n)$ is an FVP graph.

Conclusion

From the above paper we examined several types of graphs such as Path, Cycle, Wheel, Gear, Helm, shell, Bistar and Umbrella graphs are FVP graph.

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