Mapana – Journal of Sciences
2023, Vol. 22, Special Issue 1, *151-161*
ISSN 0975-3303 | https://doi.org/10.12723/mjs.sp1.12

Optimization on Deteriorating Inventories during a Sudden Pandemic Situation

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Abstract

Today, the World Health Organization and many countries globally, have implemented a lockdown to restrict the spread of the novel coronavirus disease. For this concern, we have worked out a model on the deteriorating food supply chain with two different dealers during the pandemic situation. First, the authorized dealer sells the products directly to the customer. As the lock- down begins, he has two options to sell the products. (i) He owns the unsold products himself and awaits unlocking but this option gives more deterioration as the time goes. (ii) He passes on all those remaining unsold inventories to an unauthorized dealer for sale. The profit maximization is done through the genetic algorithm and compared with the results of both dealers.

Keywords: Deteriorating food products, Expiration date, Deterministic inventory model, Authorized and unauthorized dealer, Expiration rate, Selling price, Pandemic situation.

Introduction

A pandemic is characterized as an infectious disease that spreads to many people or attacks nearly all individuals from an area in several

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countries. As many of countries affected, the effect on supply chain operations is severe and long-lasting. The travel restrictions and lockdowns imposed by many countries worldwide have further influenced supply and demand. Due to these interruptions, short-term real- time projections (daily and weekly) about the pandemic and its impact on the supply chain have become a crucial management and policy-making requirement.

Due to the COVID-19 disease, the global situation has significantly affected the inventory control's ability to make the essential products available on time and in a reliable manner. Owing to its way, the national lockdown was declared in different countries due to spread and increase in the rate of infection for the welfare of people in the countries. Therefore, all the government banned the authorized dealers for selling the stocks but the unauthorized or small shops are permitted to meet the requirements of everyday life.

Deterioration may be generally considered due to different impacts of the stocks, including damage, contamination, environmental devastation, decay, losing utility, and several others. For example, in manufacturing sectors such as chemicals, medicines, food products, radioactive substances, the product deteriorates over a period of time. Uthayakumar and Tharani [6] formalized a remanufacturing model for deteriorated products that are serviceable and also, they have dealt a time deteriorating inventory model [7]. Koyuncu and Erol [2] presented a model to illustrate the effect of pandemic influenza with optimal resource distribution. Tayal et al., [4] proposed an inventory management system for deteriorating products under some preservation technology. Singh [3] offered an economic model that provided an inventory model for degrading products with allowable shortages and demand dependent on the season and supply. Here, demand changes according to the available stock and the season. In this model, the authorized dealer has two options based on the pandemic situation:

(i) At first, he himself holds the stock and waits for the unlock to sale those inventories,

(ii) Secondly, he moves those unsold inventories to an unauthorized dealer or a small vendor at a low price to distribute those inventories to the customer. The model is illustrated with the help of a numerical example. Graphical representations have also been used to demonstrate how different parameters vary.

Notations and Assumptions

Notations

Q	Ordered quantity
a, b	Parameters involving in demand
$\theta(t)$	Deteriorating rate
p_1	Selling price per unit during the interval $[0, T_1]$
p_2	Selling price per unit during the interval $[T_1, T_2]$ where $p_2 < p_1$
c	Acquisition cost per unit time
m	Expiration rate per unit
h	Holding cost per unit
d_c	Decaying cost per unit
t_c	Transportation cost per unit
e_c	Expiration cost per unit
T_1	The time at which lockdown starts, $T_1 < m$
T_2	The time at which lockdown terminates
T	The cycle time
$I_1(t)$	The level of inventory during the interval $[0, T_1]$
$I_2(t)$	The level of inventory during the interval $[T_1, T_2]$
$I_3(t)$	The level of inventory during the interval $[T_2, T]$

Assumptions

1. Both the demand and the rate of deterioration contribute to the volume of inventory being depleted.
2. Shortages are allowed in the scenario B, as the products move faster as per the need.
3. The selling price and expiration date influence the pace of demand.

4. There are expiration rates for all deteriorating products. The degradation rate turns 1 when time reaches the maximum life span m. The stocks of beverages will deteriorate if they are not kept at the right temperature. Likewise, there are many reasons for the deterioration of those beverages, such as microbial action, the effects of light, oxygen, etc., To enact the problem malleable, we seek the same assumptions as in Uthayakumar and Tharani [7] for the deterioration rate. (i.e)

$$\theta(t) = \frac{1}{1+m-t}, \qquad 0 \le t \le T \le m \tag{1}$$

5. Transportation cost is also considered to transfer the stock from the authorized dealer to an unauthorized dealer.
6. It is assumed that lockdown occurs at time T_1 and terminates at time T_2.
7. It is assumed that in the duration of lockdown, the authorized dealer is not allowed for sale while the unauthorized dealer is allowed.

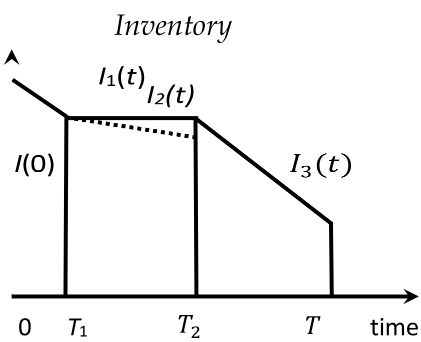

Figure 1: Inventory level during the lockdown situation with Scenario A

Model Development

Scenario A: When the authorized dealer holds the stock and waits for the unlock

At the very beginning, the authorized dealer buys the products for selling. He sells those inventories at p_1 cost to the customers during the interval $[0, T_1]$. Due to the pandemic situation, he has to store the unsold inventories himself and awaits for unlock in the scenario A. The inventory level during this situation is paraded in

Figure (1). The initial inventory level will remain the same when the lockdown occurs after the stock runs out because there was no estimate for this kind of scenario at the time of acquisition. i.e., $I_1(0) = I(0)$. Here are the differential equations for this system:

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -\frac{a}{p_1^b}\left(1 - \frac{t}{T}\right), \ 0 \le t \le T_1 \tag{2}$$

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = 0, \ T_1 \le t \le T_2 \tag{3}$$

$$\frac{dI_3(t)}{dt} + \theta(t)I_3(t) = -\frac{a}{p_1^b}\left(1 - \frac{t}{T}\right), \ T_2 \le t \le T \tag{4}$$

with the boundary conditions $I_1(0) = I(0)$, $I_1(T_1) = I_2(T_1)$, $I_2(T_2) = I_3(T_2)$. We have $I(0)$, by considering the normal condition case where there will be no shortage. Therefore, the initial inventory level which is nothing but ordered quantity of our system can be written as

$$Q = I_1(0) = \frac{a}{P_1^b}\left(1 - \frac{t}{T}\right)\left[\ln\left(\frac{1+m}{1+m-T_1}\right) - \frac{1+m}{T}\ln\left(\frac{1+m}{1+m-T_1}\right) + \frac{T_1}{T}\right] \tag{5}$$

On solving the eqns. (2), (3), (4), we get the following solutions.

$$I_1(t) = \frac{a(1+m-t)}{p_1^b}\left[\ln\left(\frac{1+m-t}{1+m}\right) - \frac{1+m}{T}\ln\left(\frac{1+m-t}{1+m}\right) - \frac{t}{T}\right] + c_1 \tag{6}$$

$$I_2(t) = c_2\frac{(1+m-t)}{(1+m)} \tag{7}$$

$$I_3(t) = \frac{a(1+m-t)}{p_1^b}\left[\ln\left(\frac{1+m-t}{1+m}\right) - \frac{1+m}{T}\ln\left(\frac{1+m-t}{1+m}\right) - \frac{t}{T}\right] + c_3 \tag{8}$$

where c_1, c_2, c_3 are constants. These constants can be found by using the boundary conditions of equations (2), (3), (4). The authorized dealer had some acquisition cost at the beginning of the cycle to buy the inventories. Thus, the acquisition cost can be given as

$$A_c = cQ \tag{9}$$

The holding cost in this case will be:

$$HC_1 = h\int_0^{T_1} I_1(t)dt + \int_{T_1}^{T_2} I_2(t)dt + \int_{T_2}^{T} I_3(t)dt \quad (10)$$

The authorized dealer keeps the unsold stock himself and awaits for unlock. Due to this he may get more amount of deterioration rate, as they are stored. In the course of time, he has to add up a cost called decaying cost. Therefore, the decaying cost can be written as

$$\mathrm{D_{cA}} = d_c\left\{\int_0^{T_1} I_1(t)dt + \int_{T_1}^{T_2} I_2(t)dt + \int_{T_2}^{T} I_3(t)dt\right\} \quad (11)$$

At the time T , there may be some products left due their expiration rate exceeds as the products are stored for a particular period of time. Thus, we have some expiration cost as follows

$$E_{cA} = e_c\{I_3(T)\} \quad (12)$$

The sales revenue of the authorized dealer is given by the following expression.

$$\mathrm{S_{pA}} = p_1\left(\frac{a(1+m-t)}{p_1^b}\left[\ln\left(\frac{1+m-t}{1+m}\right) - \frac{1+m}{T}\ln\left(\frac{1+m-t}{1+m}\right) - \frac{t}{T}\right]\right) \quad (13)$$

The total profit per unit time shall be indicated as follows.

$$max\ TP_1 = \frac{S_{pA}-(A_C+HC_1-D_{cA}+E_{cA})}{T} \quad (14)$$

subject to the constraints.

$$T_1 \geq 0$$
$$T_1 < T_2$$
$$T_2 < T \quad (15)$$

Scenario B: When the authorized dealer transfers the stock to an unauthorized dealer

The authorized dealer sells the unsold inventories to an unauthorized dealer at p_2 cost as the lockdown period starts. Also, he could get some shortages also as the lockdown continues and get less amount of deterioration cost rather the scenario A as the products moves faster. The scenario B is paraded in Figure(2). For this scenario, the inventory level in the interval $[0, T_1]$ is same as in the previous scenario. Then the remaining inventory level is decoded by the following differential equation:

$$\frac{dI_4(t)}{dt} = -\theta(t)I_4(t) - \frac{a}{p_2^b}\left(1 - \frac{t}{T_2}\right),\ T_1 \le t \le T_2 \qquad (16)$$

$$\frac{dI_5(t)}{dt} = -\frac{B}{1+\gamma(T_2-t)}\frac{a}{p_2^b}\left(1 - \frac{t}{T_2}\right),\ T_3 \le t \le T_2 \qquad (17)$$

$$\frac{dI_6(t)}{dt} = \left(1 - \frac{B}{1+\gamma(T_2-t)}\right)\frac{a}{p_1^b}\left(1 - \frac{t}{T}\right),\ T_3 \le t \le T_2 \qquad (18)$$

with the condition $I_1(T_1) = I_4(T_1)$. The inventory level during the interval $[T_1, T_3]$ is derived as

$$I_4(t) = \frac{a(1+m-t)}{p_2^b}\left[\ln\left(\frac{1+m-t}{1+m}\right) - \frac{1+m}{T_3}\ln\left(\frac{1+m-t}{1+m}\right) - \frac{t}{T_3}\right] + c_4 \qquad (19)$$

$$I_5(t) = \frac{aB}{T_2}[\log(1+\gamma(T_2-T_3)) - \gamma(T_2-T_3)] \qquad (20)$$

$$I_6(t) = (T_2 - T_3) - \frac{1}{2T_2}(T_2^2 - T_3^2) - \frac{aB}{T_2}\ [\log(1+\gamma(T_2-T_3)) - \gamma(T_2 - T_3)] \qquad (21)$$

where, c_4 is a constant from the condition of eqn (16).

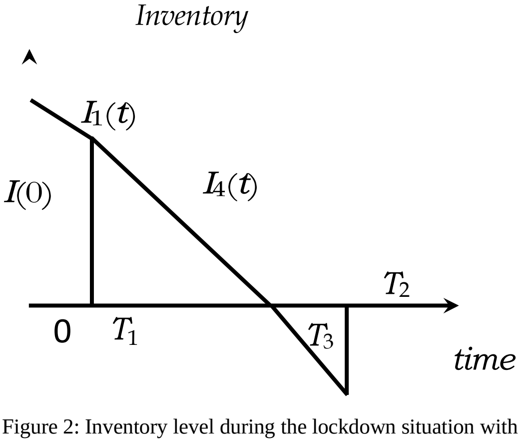

Figure 2: Inventory level during the lockdown situation with Scenario B

Methodology

Here, we have introduced the genetic algorithm approach to solve the gross profit. Genetic algorithm is the arbitrary search method that imitates the mechanism of natural selection (Goldberg [1]).

Solution Procedure for scenario A

Step 1: Begin with initial values of the variables in the determination of the fitness functions.

Step 2: Replace its values with the objective function for each variable. Now it is a function of T_1, T_2, T and p_1.

Step 3: In order to get the optimal values of T_1, T_2, T and p_1, solve the optimization problem for each variable subject to corresponding constraints for the objective function.

Step 4: Return the optimal value to the objective function from Step 3. From step 1 each variable conforms to a strategy with the cumulative cost of Step 3 equals the value of the fitness function.

Step 5: To add the mutation to them, pick variables from the previous value.

Step 6: Repeat Steps 2 to 5 until a stop criterion is achieved. There's a negligible optimum objective function value. Once this requirement is met, stop the process and get the optimal objective value and the optimal variable.

Solution procedure of scenario A is similar to scenario B.

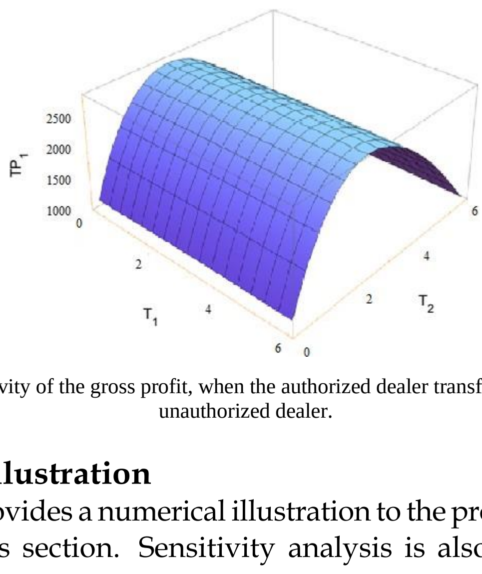

Figure 3: Concavity of the gross profit, when the authorized dealer transfers the stock to an unauthorized dealer.

Numerical Illustration

This section provides a numerical illustration to the proposed problem in the previous section. Sensitivity analysis is also carried out to illustrate the behavior of the gross profit due to the f l uctuations in each parameter. The parameters applied in the model are incurred as follows $a = 25000$ units, $b = 1.1, h = 10$ rupees/unit, $c = 15$ rupees/unit, $d_c = 3$ rupees/unit, $e_r = 0.03$ per unit, $t_c = 2.5$ rupees/unit, $s = 2$ rupees/unit, $c_l = 1.5$ rupees/unit, $\gamma = .5$ per unit, $m = 12$ months. By using the genetic algorithm, the optimal values are found as $T_1 = 1\, month, T_2 = 2\, months, T = 6\, months, p_1 = 72\, rupees, TP_1 = 2739.31$ rupees for the scenario A and $T_1 = 1.5\, months, T_2 = 5.2\, months,\ p_1 = 75\, rupees, p_2 = 62\, rupees, TP_2 = 3245.85$ rupees for the scenario B.

Table 1: Optimal values of the proposed model for both the cases

T_1	T_2	p_1	TP_1	T_1	p_1	p_1	TP_2	% difference
1	2	72	2739.31	1.5	75	62	3245.82	5.07

Sensitivity analysis

Here in this model, we addressed the various choices available for an authorized dealer to opt. We have found the optimum order quantity for the dealer with the aid of the normal case. Since nobody knows about these lockdown conditions at the time of ordering stock, the inventory will be ordered according to the usual conditions.

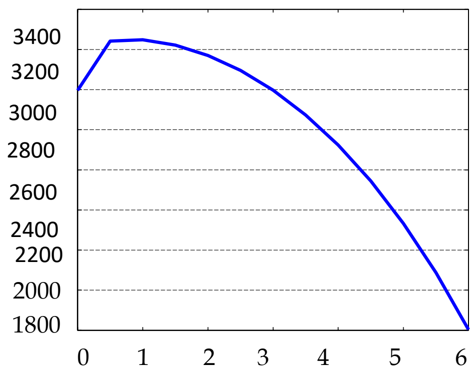

Figure 4: Concavity of the gross profit, when the authorised dealer holds the stock and waits for the unlock

The variation in different system parameters have also been presented to show the effect of variation on unit time both T_1 and T_2. From Figure1, we can observe that (i) the holding cost for the first case will be rapidly increasing due to the increase in T_1, (ii) The decaying cost also increases as the time T_1 increases, (iii) the acquisition cost will remain constant as the authorized dealer bought the products with normal condition, and (iv) the holding cost for the second case will increase as the time T_2 increases.

Conclusion

The various conditions for the authorized dealer with fast deteriorating inventories have been addressed ~~by~~ depending on the time of lockdown and the inventory available. Here, we have derived an inventory model with lockdown situation for deteriorating food products. Two different scenarios have been discussed and at last we have concluded that the scenario B will be more profitable than A from the numerical results. In addition, we shall expand the pattern of price and expiration demand to a function of time, advertisement, and commodity consistency of demand. The model is found quite suitable to meet the conditions which arise due to a sudden disruption.

Parameter	-40%	-20%	+20%	+40%
A	2075	2082	2096	2103
D	1267	1678	2499	2910
c_1	1419	1790	2387	2686
H	1965	2027	2150	2212
S	2092	2090	2087	2085
c_2	1985	2037	2140	2192
R	2086	2088	2090	2091
M	2227	2142	2052	2024
B	2051	2070	2108	2127
I_e	2139	2114	2063	2038
I_r	2089	2089	2089	2089
T	2086	2088	2090	2091
Λ	2086	2088	2090	2091
H	2089	2089	2089	2088
Γ	2103	2095	2083	2078

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