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Topological Cordial Labelling of Some Graphs

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Abstract

Topological cordial labelling of a graph $G = (V(G), E(G))$ with $|V(G)| = n$ is an injective function $f: V(G) \to 2^X$, where X is any non – empty set such that $|X| < n$ and $\{f(V(G))\}$ forms a topology on X, that induces a function $f^*: E(G) \to \{0,1\}$ defined by $f^*(uv) = 1 \; if \; f(u) \cap f(v)$ is not an empty set and not a singleton set and 0 otherwise for all $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges labelled with 0 and $e_f(1)$ is the number of edges labelled with 1. A graph that admits topological cordial labelling is called a topological cordial graph. In this paper, topological cordial labelling of some special graphs are discussed.

Keywords: Topological cordial graph, coconut tree, cycle, semi - udukkai graph, graph operations.

INTRODUCTION

In this paper we consider only simple and undirected graphs. The graph G has a vertex set $V = V(G)$ and the edge set $E = E(G)$. For notations and terminology, we refer to Bondy and Murthy[1].

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Acharya[2] established another link between graph theory and point set topology. The topological cordial labelling was introduced by S. Selestin Lina S, Asha S and they proved vertex switching of cycle C_n [3] and some special graphs are topological cordial graph[4]. In this paper, we discuss topological cordial labelling for some special graphs and graph operations.

BASIC DEFINITIONS

Definition 2.1

A **topological cordial labelling** of a graph $G = (V(G), E(G))$ with $|V(G)| = n$ is an injective function $f: V(G) \rightarrow 2^X$ where X is a non-empty set such that $|X| < n$ and $\{f(V(G))\}$ forms a topology on X that induces a function $f^*: E(G) \rightarrow \{0,1\}$ defined by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges labelled with 0 and $e_f(1)$ is the number of edges labelled with 1. Any graph G that admits topological cordial labelling is called a topological cordial graph.

Definition 2.2

A graph in which any two distinct vertices are adjacent is called a complete graph. Complete graph with n vertices is denoted by K_n.

Definition 2.3

The corona of two graphs $G_1 and\ G_2$ is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Definition 2.4

A Coconut tree $CT(m, n)$, for all positive integer n and m ≥ 2 is obtained from the path P_m by appending 'n' new pendant edges at an end vertex of P_m.

Definition 2.5

An udukkai graph $U_n, n \geq 2$ is a graph constructed by joining two fan graphs $F_n, n \geq 2$ with two paths $P_n, n \geq 2$ that share a common vertex at the centre.

Definition 2.6

A semi – udukkai graph $U_{m,n}, m, n \geq 2$ is a graph constructed by joining two fan graphs $F_m, m \geq 2$ with two paths $P_n, n \geq 2$ by sharing a common vertex at the centre.

MAIN RESULTS

Theorem 3.1

The coconut tree $CT(m, n)$ is a topological cordial graph for $m = n + 2$ and for any $n \in \mathbb{N}$.

Proof:

Let $G = CT(n + 2, n)$ be a coconut tree.

Let $V(G) = \{u_1, u_2, \ldots, u_{n+2}, v_1, \ldots, v_n\}$be the vertex set of G, where $u_1, u_2, \ldots, u_{n+2}$ are the vertices of the path and $v_1, \ldots, v_n$ be the pendant vertices attached with the end vertex of the Path P_{n+2}.

Then $|V(G)| = 2n + 2$

Let $E(G) = \{u_i u_{i+1} / 1 \leq i \leq n + 1\} \cup \{u_1 v_i / 1 \leq i \leq n\}$ be the edge set of G.

Then $|E(G)| = 2n + 1$

Let $X = \{1,2, \ldots, |V(G)| - 1\}$

Define the function $f: V(G) \to 2^X$ as follows:

$f(u_1) = \phi, \qquad f(u_i) = \{1,2, \ldots, n + i - 1\}$ for $2 \leq \mathrm{i} \leq \mathrm{n} + 2,$

$$f(v_i) = \{1,2, \ldots, i\} \ for \ 1 \leq \mathrm{i} \leq \mathrm{n}$$

Then the induced function is given as follows.

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(u_1u_2) = 0$, $f^*(u_1v_i) = 0$ for $1 \leq i \leq n$ and $f^*(u_iu_{i+1}) = 1$ for $2 \leq i \leq n+2$.

Then $e_f(0) = n+1$ and $e_f(1) = n$

Therefore $\left|e_f(0) - e_f(1)\right| \leq 1$.

Hence f is a topological cordial labelling and G is a topological cordial graph.

Example 3.2

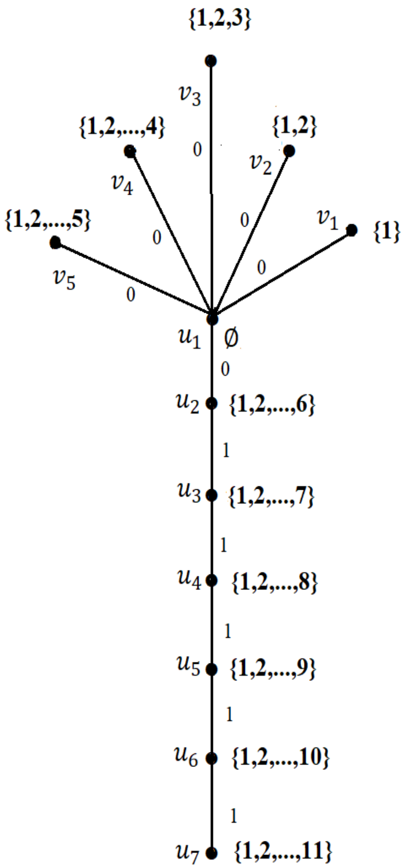

Fig 3.2 Topological cordial labeling of CT(7,5)

Theorem 3.3

For any cycle $C_n, the\ graph\ (n-3)K_1, n \geq 4$ attached to any vertex of C_n is a topological cordial graph.

Proof:

Let $u_1, u_2, \dots, u_n$ be the vertices of C_n and let $v_1, \dots, v_{n-3}$ be the new vertices.

Let u_1 be the vertex to which $(n-3)K_1$ will be attached and let the resulting graph be G.

Then $V(G) = \{u_1, u_2, \dots, u_n, v_1, \dots, v_{n-3}\}$ and $|V(G)| = 2n-3$

Also, $E(G) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_1 v_i / 1 \leq i \leq n-3\}$ and

$|E(G)| = 2n-3$.

Let $X = \{1,2, \dots, |V(G)|-1\}$

Define the function $f: V(G) \to 2^X$ as follows:

$f(u_1) = \phi$, $f(u_i) = \{1,2, \dots, n+i-4\}$ for $2 \leq i \leq n$,

$f(v_i) = \{1,2, \dots, i\}$ for $1 \leq i \leq n-3$

Then the induced function is given as follows.

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(u_i u_{i+1}) = 1$ for $2 \leq i \leq n-1$, $f^*(u_1 v_i) = 0$ for $1 \leq i \leq n-3$, $f^*(u_n u_1) = 0$ and $f^*(u_1 u_2) = 0$

Then $e_f(0) = n-1$ and $e_f(1) = n-2$

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence f is a topological cordial labelling and G is a topological cordial graph.

Example 3.4

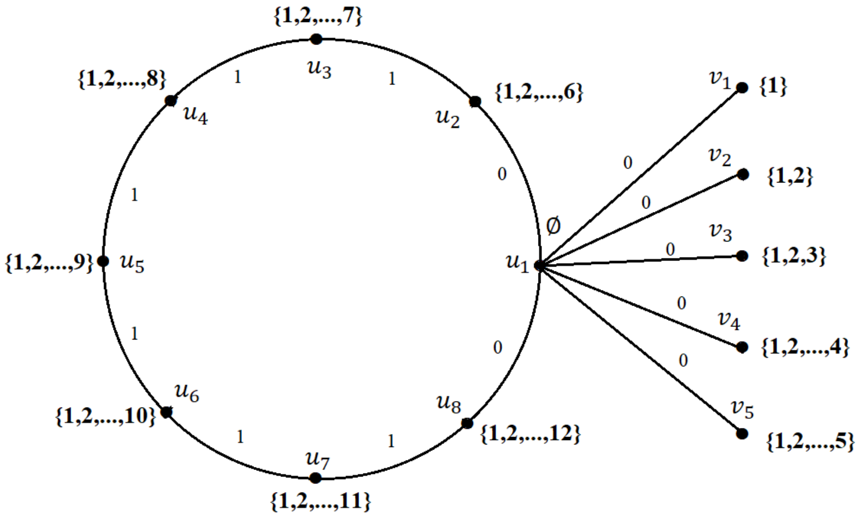

Fig 3.3 Topological cordial labeling of $5K_1$ attached to C_8

Theorem 3.5

The corona graph $P_2 \odot nK_1$, for any $n \in \mathbb{N}$ is a topological cordial graph.

Proof:

Let $G = P_2 \odot nK_1$ and let $V(G) = \{u_1, u_2, v_1, \dots, v_{2n}\}$ be the vertex set of G

Then $|V(G)| = 2n + 2$

Let $E(G) = \{u_1u_2\} \cup \{u_1v_i / 1 \leq i \leq n\} \cup \{u_2v_{i+n} / 1 \leq i \leq n\}$ be the edge set of G.

Then $|E(G)| = 2n + 1$

Let $X = \{1,2, \dots, |V(G)| - 1\}$

Define the function $f: V(G) \to 2^X$ as follows:

$f(u_1) = \phi$, $f(u_2) = \{1,2,\dots,2n+1\}$, $f(v_i) = \{1,2,\dots,i\}$ $for\ 1 \leq i \leq 2n$

Then the induced function is given as follows.

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \epsilon E(G)$.

Here $f^*(u_1u_2) = 0$, $f^*(u_1v_i) = 0\ for\ 1 \leq i \leq n$, $f^*(u_2v_{i+n}) = 1\ for\ 1 \leq i \leq n$

Then $e_f(0) = n+1$ and $e_f(1) = n$

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence f is a topological cordial labelling and G is a topological cordial graph.

Example 3.6

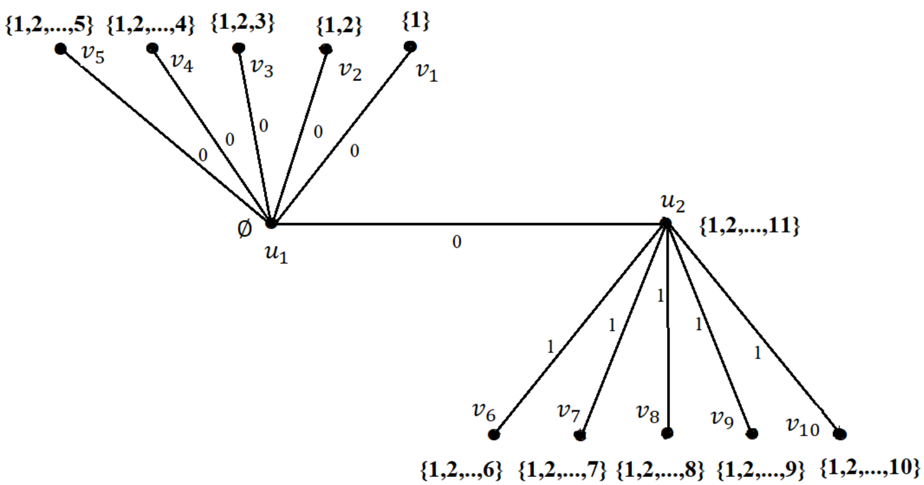

Fig 3.4 Topological cordial labeling of $P_2 \odot 5K_1$

Theorem 3.7

A semi – udukkai graph $U_{m,n}, m, n \geq 2$ is a topological cordial graph for $n = 6$ and for any $n \in \mathbb{N}$.

Proof:

Let $G = U_{m,6}$, $m, \geq 2$

Let u_1 be the common centre vertex of G

Let $V(G) = \{u_1, u_2, \dots, u_{2m+1}, v_1, \dots, v_{10}\}$ be the vertices of G, then $|V(G)| = 2m + 11$

And $E(G) = \{u_1 u_i / 2 \leq i \leq 2m + 1\} \cup \{u_i u_{i+1}/2 \leq i \leq m, m + 2 \leq i \leq 2m\} \cup \{v_i v_{i+1}/1 \leq i \leq 4, 6 \leq i \leq 9\} \cup \{u_1 v_1\} \cup \{u_1 v_6\}$

Then $|E(G)| = 4m + 8$

Let $X = \{1,2, \dots, |V(G)| - 2\}$

Define the function $f: V(G) \rightarrow 2^X$ as follows:

`$f(u_1) = \phi$, $f(v_1) = \{1\}$, $f(v_2) = \{2\}$,

$f(u_i) = \{1,2, \dots, 8 + i\}$ $for\ 2 \leq \mathrm{i} \leq 2\mathrm{m} + 1$, $f(v_i) = \{1,2, \dots, i - 1\}$ for $3 \leq \mathrm{i} \leq 10$

Then the induced function is given as follows.

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(u_1 u_i) = 0\ for\ 2 \leq i \leq 2m + 1, f^*(u_1 v_1) = 0, f^*(v_1 v_2) = 0,$

$f^*(v_2 v_3) = 0,\ f^*(u_1 v_6) = 0, f^*(u_i u_{i+1}) = 1\ for\ 2 \leq i \leq m, f^*(u_i u_{i+1}) = 1$

$for\ m + 2 \leq i \leq 2m, f^*(v_i v_{i+1}) = 1\ for\ 3 \leq i \leq 4\ and\ 6 \leq i \leq 9.$

Then $e_f(0) = 2m + 4$ and $e_f(1) = 2m + 4$

Therefore $|e_f(0) - e_f(1)| \leq 1.$

Hence f is a topological cordial labelling and G is a topological cordial graph.

Example 3.8

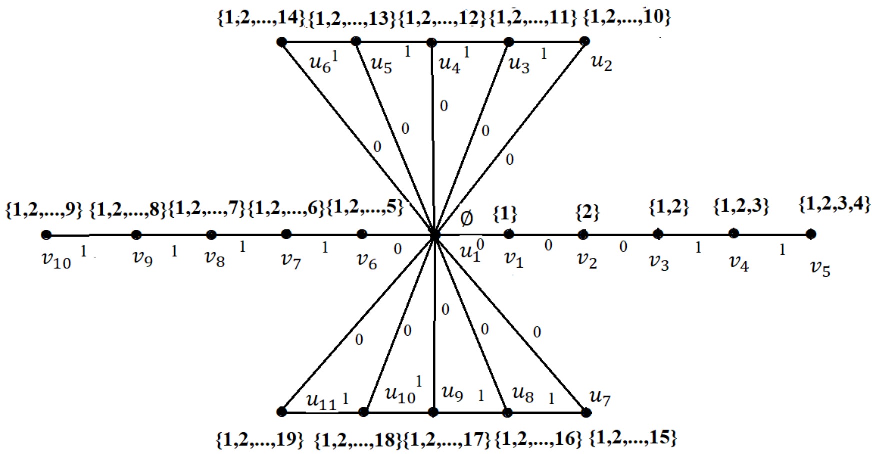

Fig 3.5 Topological cordial labeling of $U_{5,6}$

Theorem 3.9

Let G be a topological cordial graph with atleast three vertices. Then the graph obtained by attaching $mK_1$for any $m \in \mathbb{N}$ to two vertices of G in which one of the vertex label is an empty set or a singleton set and the other vertex label is a non-empty set and a singleton set is a topological cordial graph.

Proof:

Let G be a topological cordial graph

Then there exists a non-empty set $X_1 = \{1,2,\ldots,n_1\}$ *such that* $n_1 < |V(G)|$

Also there exists the function $f_1: V(G) \to 2^{X_1}$ and from the induced function

$f_1^*: E(G) \to \{0,1\}$, we have $\left|e_{f_1}(0) - e_{f_1}(1)\right| \leq 1$.

Let G^* be the graph obtained by attaching $mK_1$to two vertex of G namely, the vertex whose label is ϕ or a singleton set, say u and the with the vertex whose label is other than ϕ and a singleton set say w.

Let $V(G^*) = V(G) \cup \{v_1, \dots, v_{2m}\}$ be the vertex set of G^*,then $|V(G^*)| = |V(G)| + 2m$

Also $E(G^*) = E(G) \cup \{uv_i / 1 \le i \le m\} \cup \{wv_i / \ m+1 \le i \le 2m\}$and

$|E(G^*)| = |E(G)| + 2m$

Here, take $X = X_1 \cup \{n_1+1, n_1+2, \dots, n_1+2m\}$ then $|X| = n_1 + 2m < |V(G^*)|$

Define $f: V(G^*) \to 2^X$ as $f\big(V(G)\big) = f_1(V(G))$ and

$f(v_i) = \{1,2, \dots, n_1+i\} \ for\ 1 \le i \le 2m$

Then the induced function is given as follows.

$$f^*(uv) = \begin{cases} 1 \ \ if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 \qquad\qquad otherwise \end{cases}$$

for all $uv \in E(G^*)$.

Here $f^*(uv) = f_1^*(uv)$ for all $uv \in E(G)$.

$f^*(uv_i) = 0$ for $1 \le i \le m$ and $f^*(wv_i) = 1$ for $m+1 \le i \le 2m$.

Now $\left|e_f(0) - e_f(1)\right| = |e_{f_1}(0) + m - (e_{f_1}(1) + m)|$

$$= \left|e_{f_1}(0) - e_{f_1}(1)\right|$$

≤ 1

Hence f is a topological cordial labelling and $G^*$is a topological cordial graph.

Example 3.10

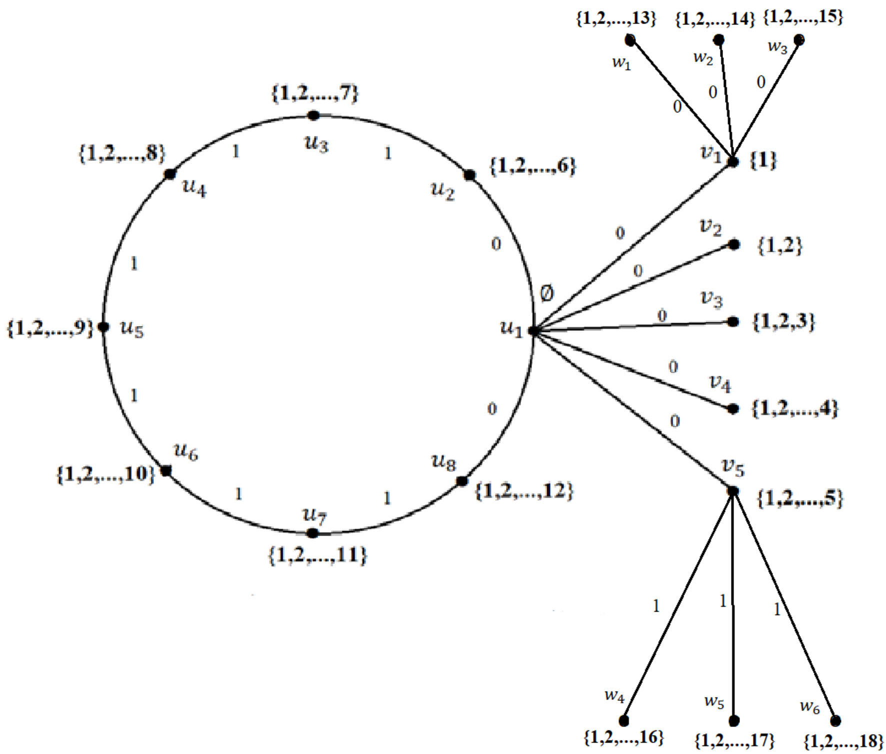

Fig 3.5 Topological cordial labelling of the graph obtained by attaching $3K_1$ to the vertex $v_1$and u_3 of example 3.4

CONCLUSION

Topological cordial labelling for some special graphs and graph operations are discussed. Since all the graphs do not admit topological cordial labelling, it is very interesting to find the graphs that admit topological cordial labelling.

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