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Interference and Diffraction of Quantum Particles Revisited

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Abstract

Interference and diffraction phenomena are the basic properties shown by waves—de-Broglie concept of matter waves brought in the extended version of wave properties of material particles. Wave properties of quantum particles are important to understand various phenomena in physics. Matter waves are a core part of quantum theory. Now wave-particle duality is exploited for many applications, such as imaging microscopic particles using electron microscopes etc. Here we deduce the interference and diffraction of quantum particles on the basis of the propagation amplitude according to the path integral formulation of quantum mechanics. The intensity distribution due to interference and diffraction of quantum particles are compared with those obtained from the formal methods. The results agree with the traditional approach assuming a de Broglie wave associated with particles.

Keywords: Interference, Diffraction & Path Integral Formulation

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Introduction

Quantum particles such as electrons and neutrons are shown to exhibit wave-like properties such as interference and diffraction as they pass through various kinds of slits on the assumption of de Broglie waves (Mater waves) associated with material particles. Once deBroglie waves are assumed to emerge from each point of a slit in the form of spherical waves or plane waves, just like optical waves, interference and diffraction effects arise naturally on the observation screen kept at a distance from the slits [1- 4]. Hence such an approach is not satisfactory as providing a proof of the wave nature of quantum particles.

In this article, we deduce the emergence of interference on the proper basis of the quantum mechanical amplitude for the propagation of material particles as given by the path integral formulation of quantum mechanics. de Broglie wavelength for material particles emerges naturally in the analysis.

Interference due to two-point slits

Let us consider a beam of monoenergetic particles, such as electrons incident on a pair of point slits as shown in Figure 1.

The path integral formalism [5,6] gives the amplitude for a free particle of mass m to go from $(x_0, 0)$ to $(x,\ t)$ as

$$K(x,t;x_0,0)=\sqrt{\frac{m}{2\pi\hbar ti}}\exp\left[\frac{i}{\hbar}\frac{m(x-x_0)^2}{2t}\right] \quad (1)$$

For a two-dimensional motion, this generalises to

$$K(x,y,t;x_0,y_0,0) = \left(\frac{m}{2\pi\hbar ti}\right) exp\left[\frac{i}{\hbar}\frac{m\{(x-x_0)^2+(y-y_0)^2\}}{2t}\right] \quad (2)$$

For the geometry of the slits considered in Figure 1, the amplitude for the quantum particles to go from S_1 to the observation point $P(L,0)$ on the screen can be written as

$$\psi_1 = \left(\frac{m}{2\pi\hbar ti}\right) exp\left[\frac{im}{2\hbar t}\left\{L^2 + \left(y - {}^{b}/_{2}\right)^2\right\}\right] \quad (3)$$

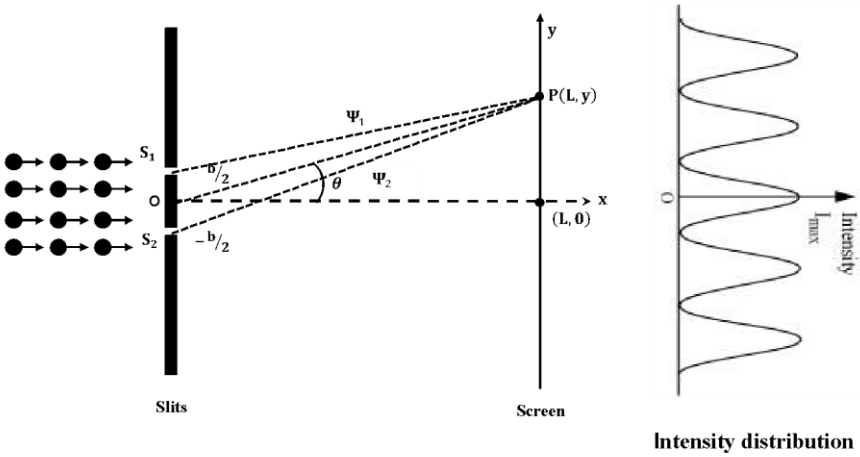

Figure 1: Interference due to two-point slits S_1 and S_2 that are separated by a distance b

Similarly, the quantum mechanical amplitude for particles to go from slit S_2 to point P on the screen can be written as

$$\psi_2 = \left(\frac{m}{2\pi\hbar t i}\right) exp\left[\frac{im}{2\hbar t}\left\{L^2 + \left(y + {}^{b}/_{2}\right)^2\right\}\right] \quad (4)$$

As per the laws of quantum mechanics the probability of observing a particle at P is given by

$$I(L, y) = |\psi_1 + \psi_2|^2 \quad (5)$$

Using Ψ_1 and Ψ_2 we get, after simplification

$$I(L, y) = \left(\frac{m}{2\pi\hbar t}\right)^2 \left[1 + 1 + 2\cos\left\{\frac{m}{2\hbar t}(2yb)\right\}\right] = \left(\frac{m}{2\pi\hbar t}\right)^2 \left[2\left(1 + \cos\left\{\frac{mby}{\hbar t}\right\}\right)\right] \quad (6)$$

But $\frac{mby}{\hbar t} = \frac{2\pi}{h}\left(\frac{m\bar{L}}{t}\right)\left(\frac{y}{\bar{L}}\right) b$, where $\bar{L} = \left(L^2 + y^2\right)^{\frac{1}{2}}$

We can note that $\frac{m\bar{L}}{t}$ is nothing but the momentum of the particle arriving at P and $\frac{y}{\bar{L}}$ = sin(θ), θ being the angle made by the point P at the center of the slits. We can note that $\frac{h}{\left(\frac{m\bar{L}}{t}\right)}$ is nothing but the usual de Broglie wavelength (λ) of the particles. Using all these, we can write

$$I(L,y) = \left(\frac{m}{2\pi\hbar t}\right)^2 \left[2\left(1 + cos\left(\frac{2\pi b \sin\theta}{\lambda}\right)\right)\right] = \left(\frac{m}{2\pi\hbar t}\right)^2 \left[4\, cos^2(\beta)\right] \quad (7)$$

Where $\beta = \frac{\pi b \sin\theta}{\lambda}$.

This is nothing but the well-known intensity distribution on a screen due to a pair of slits separated by a distance b when wave has wavelength λ incident on the slits.

Single Slit Diffraction

Let us consider the effect of a single slit of finite width on a beam of quantum particles, as shown in Figure 2.

The quantum mechanical amplitude at a point $P(L,y)$ on the screen is obtained by summing over all the amplitudes coming from each point of a finite slit. Hence, the amplitude at a general point P can be written as

$$\psi(L,y) = \left(\frac{m}{2\pi\hbar t i}\right) \int_{-\frac{b}{2}}^{\frac{b}{2}} exp\left[\frac{im}{2\hbar t}\{L^2 + (y-\xi)^2\}\right] d\xi \quad (8)$$

Where ξrefers to a point on the slit.

For large L in comparison to the slit width $L^2 + (y-\xi)^2 \cong L^2 - 2y\xi$. Hence,

$$\psi(L,y) \cong \left(\frac{m}{2\pi\hbar t i}\right) e^{\frac{imL^2}{2\hbar t}} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-\frac{imy\xi}{\hbar t}} d\xi \quad (9)$$

$$\psi(L,y) \cong \left(\frac{m}{2\pi\hbar t i}\right) \frac{sin\left(\frac{myb}{2\hbar t}\right)}{\left(\frac{myb}{2\hbar t}\right)} (b) \quad (10)$$

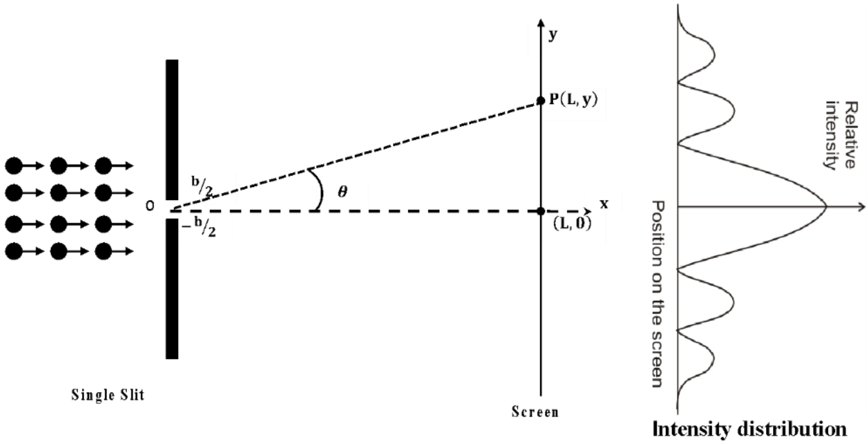

Figure 2: Single slit diffraction

We can write $\frac{myb}{2ht} = \pi \left[\frac{m\bar{L}}{ht}\right]\left[\frac{y}{L}\right] b$, where $\bar{L} = (L^2 + y^2)^{\frac{1}{2}}$. Noting that $\frac{ht}{m\bar{L}} = \lambda$, the deBroglie wavelength of the particles and $\frac{y}{L} = sin\theta$ where θ is the angle of the point on the screen with respect to the center of the slit. We get

$$\psi(L, y) = \left(\frac{m}{2\pi\hbar ti}\right)\frac{sin(\beta)}{(\beta)}(b) \tag{11}$$

Where $\beta = \frac{(\pi b \sin\theta)}{\lambda}$.

The intensity of the particles on the screen will be

$$I = |\psi(L, y)|^2 = \left(\frac{m}{2\pi\hbar t}\right)^2 b^2 \left[\frac{sin^2(\beta)}{\beta^2}\right] \tag{12}$$

This is exactly the kind of the result one gets in optics for single slit diffraction if one equates $\left(\frac{m}{2\pi\hbar t}\right)^2$ as the intensity due to a point source on the screen [7-10].

Diffraction due to a double slit

Let us consider the effect of a pair of slits of finite width on a beam of quantum particles incident normally on them, as shown in Figure 3. Following the arguments used previously the amplitude for particles coming from the slit of width b at a distance $\frac{d}{2}$ from the center as shown Figure 3 can be expressed as

$$\psi_1 = \left(\frac{m}{2\pi\hbar ti}\right) \int_{\frac{d}{2}-\frac{b}{2}}^{\frac{d}{2}+\frac{b}{2}} exp\left[\frac{im}{2\hbar t}\{L^2 + (y-\xi)^2\}\right] d\xi \tag{13}$$

In the approximation of a small slit width and large separation between the slits and the screen

$$\psi_1 \cong \left(\frac{m}{2\pi\hbar ti}\right) e^{\frac{imL^2}{2\hbar t}} \int_{\frac{d}{2}-\frac{b}{2}}^{\frac{d}{2}+\frac{b}{2}} e^{-\frac{imy\xi}{\hbar t}} d\xi \tag{14}$$

This can be simplified to

$$\psi_1 \cong b\left(\frac{m}{2\pi\hbar ti}\right) e^{\frac{imL^2}{2\hbar t}} e^{-\frac{iymd}{2\hbar t}} \left(\frac{\sin\beta}{\beta}\right) \tag{15}$$

Where $\beta = \frac{myb}{2\hbar t}$

But, once again, $\beta = \pi\left[\frac{m\frac{L}{t}}{h}\right]\left(\frac{y}{L}\right) b$, where $\bar{L} = \sqrt{(L^2+y^2)}$. Again, $\frac{ht}{m\bar{L}} = \lambda$, the deBroglie wavelength of the particles and $\frac{y}{L} = sin\theta$, θ being the angle of deviation of the observation point P from the centre of the slits (see figure 3).

In a similar manner the amplitude of the quantum particles arriving from the slit S_2 can be written as

$$\psi_2 = \left(\frac{m}{2\pi\hbar ti}\right) \int_{-\frac{d}{2}-\frac{b}{2}}^{-\frac{d}{2}+\frac{b}{2}} exp\left[\frac{im}{2\hbar t}\{L^2 + (y-\xi)^2\}\right] d\xi \tag{16}$$

This can be simplified to

$$\psi_2 \cong b\left(\frac{m}{2\pi\hbar ti}\right) e^{\frac{imL^2}{2\hbar t}} e^{+\frac{iymd}{2\hbar t}} \left(\frac{\sin\beta}{\beta}\right) \tag{17}$$

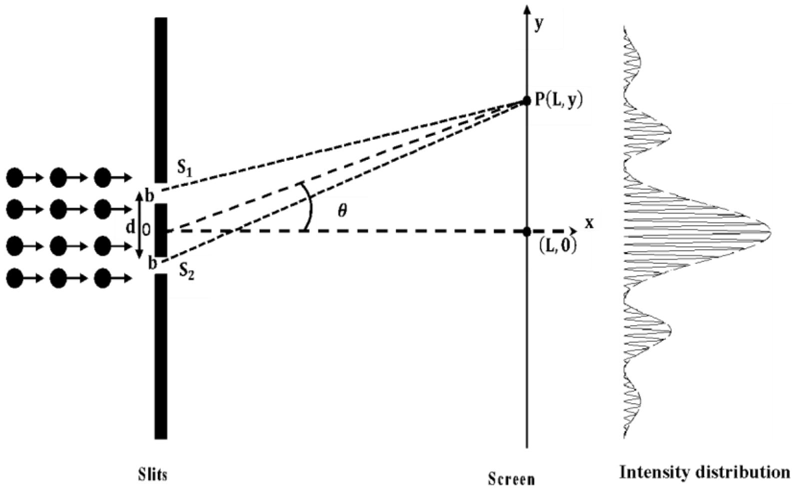

Figure 3: Double slit diffraction

Hence, the net amplitude at $P(L,y)$ will be $\Psi = \Psi_1 + \Psi_2$ and the intensity of the particles will be

$$I = |\psi(L,y)|^2 = 4b^2\left(\frac{m}{2\pi\hbar t}\right)^2\left[cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)^2\frac{sin^2(\beta)}{\beta^2}\right] \qquad (18)$$

This agrees with the corresponding optical result for a wavelength λ when $\left(\frac{m}{2\pi\hbar t}\right)^2$ is taken as the intensity due to a point source. This factor arises in the propagation amplitude for going from x_0 to x and ensures that the sum of such amplitudes over all possible x values is equal to unity for any time t.

Conclusions

We have shown here the emergence of wavelike properties of quantum particles emerging from the proper quantum mechanical amplitude for particles within the framework of the path integral formulation. Similar calculations can be done for more complicated geometries and compared with similar results in optics. The intensity distributions reduce to those for classical particles in the limit of $\lambda \ll$ slit width << slit separations.

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