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Running of spectral index for a hybrid inflationary model

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Abstract

A hybrid inflationary model with cubic potential where inflation ends in a different way, due to very rapid rolling of an auxiliary scalar field ψ is discussed. The slowly rolling inflaton field ϕ does not account for the majority of the energy density in hybrid inflation. Another field ψ takes this role, which is maintained in position by its interaction with ϕ until ϕ falls below a critical value ϕ_c. When this occurs, ψ has been destabilized and inflation comes to an end by rolling toward its true vacuum. In this model, the second derivative of the inflaton potential, which represents its effective mass, undergoes a sudden small change. The spectral indices related to density perturbations n_1 and n_2 just before and soon after the phase transition respectively are determined. It is found that the ensuing density perturbation has a power spectrum that is nearly flat with a step in n_s.

Mathematics Subject Classification (2010): 05C10

1. Introduction

Inflation, the early phase of rapid expansion, is the most desirable theory for the very early cosmos. Inflationary cosmology is not a substitute for the hot big bang concept, but rather an extension that happens at extremely early moments without interfering any of its previous findings. The inflationary theory has the potential to solve

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the problems with hot big bang cosmology (such as the flatness, horizon, and monopole concerns) [1]. According to Einstein's equation, negative pressure is necessary to generate such rapid expansion and in a scalar field theory, a negative pressure is easily accommodated [2]. In Guths's original inflation model, the scalar field was caught in a false potential minimum [3]. However, it was rapidly realized that this was not a realistic possibility. In order to solve the issue of the universe that may never attain its true vacuum state, future inflationary models employ a scalar field theory that predicted the universe would gradually roll down toward its true vacuum state. The majority of inflationary models are of the slow-roll kind [2]. In the slow-roll inflationary model, kinetic energy of the inflaton should be substantially lower than its potential energy [4].

Inflation's fundamental merit lies in the fact that it offers a hypothesis of inhomogeneities in the cosmos, which may account for the observable structure. These inhomogeneities are caused by quantum fluctuations of the inflaton field with respect to its vacuum state or vacuum fluctuations. Primordial density perturbations of the type Gaussian and adiabatic are produced by vacuum fluctuations. An essential parameter for characterising the origin of density fluctuations is the scalar spectral index n_s[5]. The spectral index of scalar perturbations is found to be $n_s = 0.9649 \pm 0.0042$ at 68% CL using data from the Planck satellite's temperature, polarisation and lensing observations. Planck's 2018 data in combination with BK15 strongly disagree with the monomial potential models $V(\phi) \propto \phi^p$ and p¿1[6], natural inflation[7], and low-scale SUSY models[8].

In some models, such as hybrid inflation, the termination of inflation occurs suddenly[9]. A general technique of introducing localized deviations in the essentially flat spectrum during the inflationary scenario is to add additional scalar fields to the inflaton, either directly (resulting in multiple inflation[10, 11]) or indirectly (by inducing a rapid change in the inflaton's effective potential[12]).

In the present study, we focus on a hybrid inflationary model in which the second derivative of the potential has a step-like discontinuity. In this model, there is an additional scalar field ψ and this field should not be in the slow-rolling regime. This scalar field ψ goes through a rapid phase change during the inflationary period due it is weak coupling to the inflaton field ϕ and it leaves an imprint in the spectrum of primordial density fluctuations. In Section 2 we discuss the details of the hybrid inflationary model considered. The scalar power spectrum with a local feature generated due to this model is presented in Section 3. In Section 4 we analyzed the running of the spectral index due to the phase transition and in Section 5 we conclude and discuss the results.

2. Hybrid Inflationary Model

The inflationary phase that occurred in the early universe can be explained by the homogeneous distribution of potential energy from a scalar field called the inflaton, denoted as ϕ, throughout the universe. In this study, we consider a hybrid inflationary model with a cubic potential,

$$V(\psi,\phi) = \frac{M^4\lambda}{4}(1+c\phi^p) - \frac{1}{2}\lambda\psi^2 M^2 + \frac{1}{4}\lambda\psi^4 + \frac{1}{2}\lambda'\psi^2\phi^2 \qquad (1)$$

with $p = 3$ and $c > 0$.

In Hybrid Inflation, the slowly rolling field ϕ is not the dominant contributor to the energy density. This function is played by another field ψ, which sustains its position through its interaction with ϕ until the value of ϕ falls below the critical value ϕ_c. At this point, ψ becomes destabilized and inflation terminates as it rolls towards its true vacuum state[13, 14]. Figure 1 depicts this potential.

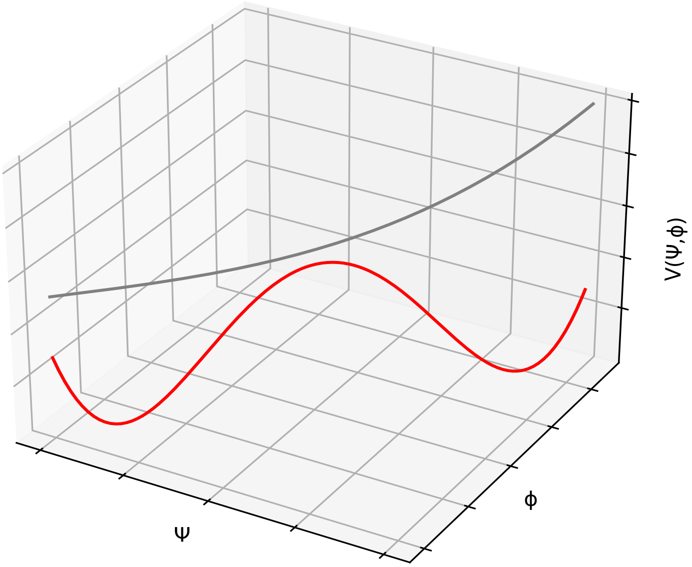

Figure 1: The hybrid inflationary potential

Our analysis focuses on the evolution of the effective mass and shows a distinctive step-like discontinuity. The application of the slow roll condition before and after the transition in our model reveals a simple observation of $\lambda' \ll \lambda$, which indicates that the inflaton field's self-coupling should be substantially greater than its coupling to the secondary field ψ [15, 16]. At the point of phase transition where $\phi = \phi_c$, the potential is,

$$V(\phi_c) \quad = \quad \frac{M^4 \lambda}{4}(1+\kappa) \tag{2}$$

with $\kappa = c\, M^3 \left(\frac{\lambda}{\lambda'}\right)^{3/2}$.

The viability of inflation as a hypothesis for the early universe is contingent upon the circumstance that the comoving Hubble radius at the onset of inflation was larger than the largest scales that can be observed presently. The universe grows by an exponential factor of almost 60 e-folds throughout the period of inflation. This condition is crucial because inflation causes the universe to expand rapidly, stretching out fluctuations on scales smaller than the Hubble radius at the beginning of inflation to scales larger than the current comoving Hubble radius, resulting in the large-scale structure we observe today. The amount of inflation produced by a scalar field can be calculated immediately via the slow-roll approximation, according to which the number of e-folds[17],

$$N \lesssim \frac{1}{M_{pl}^2} \int_{\phi_{end}}^{\phi_c} \frac{V}{V'}\, d\phi.$$

For the present hybrid model,

$$N \;=\; \frac{1}{128 M_{pl}^2} \Bigg[16\,\phi^2 \;-\; 8\kappa\phi_c\phi \;-\; \phi_c^2(3\kappa^2+16)\, ln \left|4\phi^2 - 3\kappa\phi_c\phi - 4\phi_c^2\right| \\ - \kappa\phi_c^2 \frac{(9\kappa^2+80)}{\sqrt{9\kappa^2+64}} ln \left| \frac{8\phi - 3\kappa\phi_c - \phi_c\sqrt{9\kappa^2+64}}{8\phi - 3\kappa\phi_c + \phi_c\sqrt{9\kappa^2+64}} \right| \Bigg]_{\phi_e}^{\phi_c} \tag{3}$$

where, ϕ_e is the value of inflaton field where the inflation ends.

One of the parameters that can be selected in the model is the exact value of the number of e-folds N in inflationary cosmology. We take $N = 60$, a value determined solely by the values of κ, ϕ_c, and ϕ_e. We can easily estimate the values of the parameters (λ, λ', M) in our model by comparing the observed CMB fluctuation measured by COBE or Planck with the inflationary curvature fluctuation on a larger scale. The values for the parameters κ, λ', c and M in a spatially flat LCDM universe with, $\Omega_m = 0.3153$ and $\Omega_\lambda = 0.6847$, after setting $N = 60$ and choosing $\lambda = 0.1$ are listed in Table 1. It is clear from the table that the value of M for this model is in the GUT range.

κ	$c \times M_{pl}^3$	λ	λ'	M/M_{pl}
0.18	0.0000307779	0.1	2.23523×10^{-8}	0.008518

Table 1. Typical parameter values for the hybrid inflationary potential given by (1).

3. Scalar Power spectrum

The spectrum of metric perturbations is produced in a de-sitter phase of accelerated expansion from quantum fluctuations and it is obtained that the spectrum is logarithmically dependent on its scale. These results were obtained for the initial working inflationary model that relies on R^2-*gravity*. This model is essentially equivalent to a model that involves a scalar field [18]. Another specific class of inflation known as new inflation also studied the quantum fluctuations [19, 20] and the results were in agreement with that of the R^2 – *gravity* model. The overall idea of the inflationary perturbations theory suggests that even though there are many different scenarios for inflation, the perturbation patterns in basic inflation models with limited free parameters are similar, showing a logarithmic dependence on the scale. These results are consistent with previous calculations [18]. As per the inflation theory, it is suggested that the spectrum of density perturbations remains nearly scale invariant [21, 5].

The scalar power spectrum is calculated for this hybrid inflationary model in the long wavelength limit around CMB observations. It is found to be nearly scale-invariant as seen in Figure 2. Observations at large scales have imposed a strong restriction on the amplitude of the power spectrum of scalar perturbations, which is approximately 2×10^{-9}. The small bump-like structure in the spectrum of curvature perturbation indicates a running of the spectral index at the transition point [5], however it diminishes from that point. The scale dependence of this hybrid model is characterized by a spectral index $n_s \approx 0.962$, which corresponds to a red tilt. The spectral index obtained for this model is consistent with the index of scalar perturbations determined by Planck temperature, polarization, and lensing data, $n_s = 0.9649 \pm 0.0042$ at 68% confidence level [22].

4. Running of Spectral index

The value of the scalar spectral index n_s is an important factor that characterizes the properties of initial density fluctuations in the universe. The spectral index value before (n_1) and after (n_2) the phase

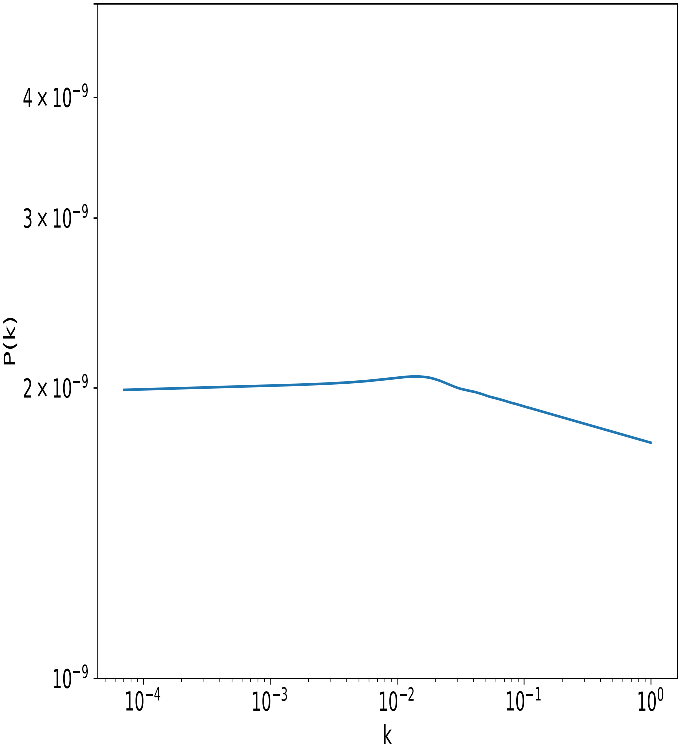

Figure 2: Scalar power spectrum for the hybrid inflationary model.

transition for the present model can be easily determined by the familiar formula,

$$n-1 = M_{pl}^2\left(-3\left(\frac{V'}{V}\right)^2+2\frac{V''}{V}\right) \tag{4}$$

Thus, we get

$$n_1-1 = 3\left(\frac{M_{pl}}{\phi_c}\right)^2\frac{\kappa(4-5\kappa)}{(1+\kappa)^2} \tag{5}$$

$$n_2-1 = -\left(\frac{M_{pl}}{\phi_c}\right)^2\frac{15\kappa^2+4\kappa+16}{(1+\kappa)^2} \tag{6}$$

We can deduce from (4.2) that the spectrum of inflationary perturbations has a red (blue) tilt on large scales if $\kappa > \frac{4}{5}(\kappa < \frac{4}{5})$ and $\kappa = \frac{4}{5}$ yielding an initial spectrum that is accurately scale invariant; n_1 equals 1.

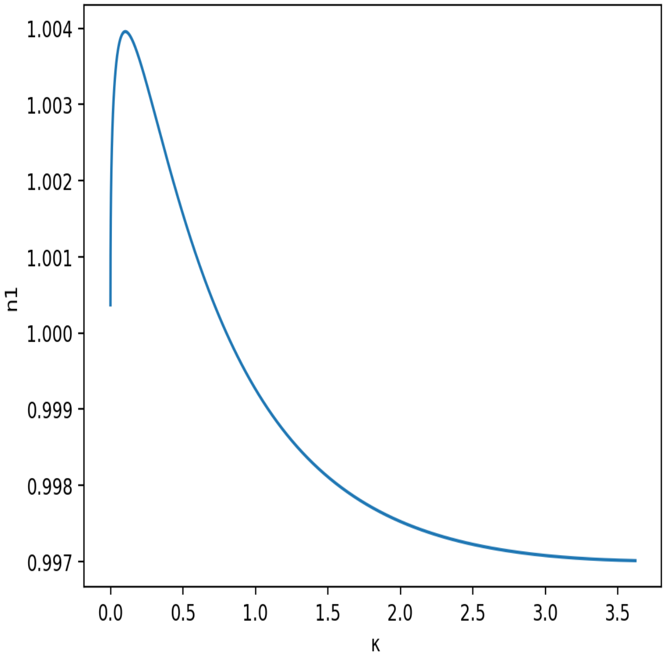

Figure 3. Spectral index just before (n_1) the phase transition in hybrid inflation as a function of the parameter κ.

During the transition, the total change in the spectral index is equal to

$$\Delta n \quad \equiv \quad n_1 - n_2 \quad = \quad 16\left(\frac{M_{pl}}{\phi_c}\right)^2 \frac{1}{1+\kappa} \tag{7}$$

Clearly, the value of κ must not be too large in order to be consistent with observations, because otherwise, $n_1 \simeq n_2$ and a thorough analysis of this model's predictions will be difficult. If we know the value of $\kappa\left(\equiv cM^3(\frac{\lambda}{\lambda'})^{\frac{3}{2}}\right)$ as well as ϕ_c we may find that n_1 and n_2 are totally determined using (4.2) and (4.3).

Figure 6 shows the relationship of how a step in the effective mass of the inflation m^2_{eff} results in a step in the scalar spectral index n_s. At the k value corresponding to the transition point ϕ_c, we can see rapid oscillations in $n_s(k)$ but these oscillations in n_s decrease slowly around an average value of $n_s = 0.962$.

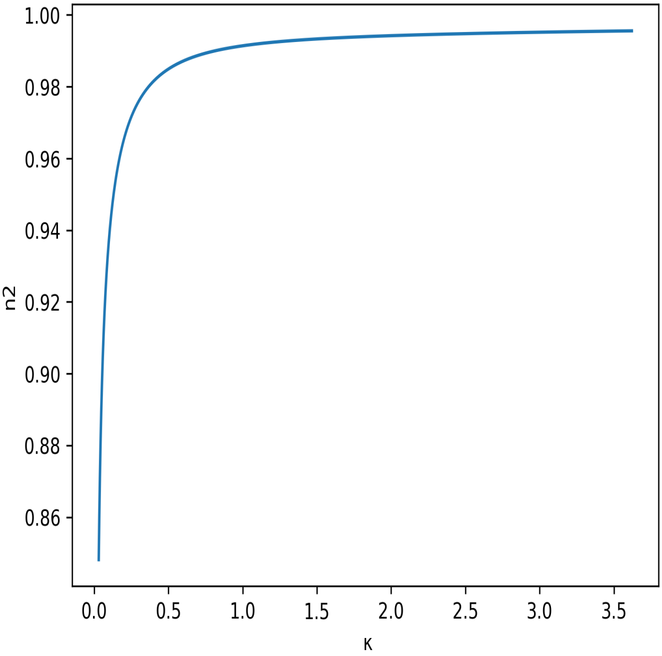

Figure 4. Spectral index just after (n_2) the phase transition in hybrid inflation as a function of the parameter κ.

5. Conclusion

Many parameters related to primordial inflation in the early Universe should have more precise measurements as a result of the improved cosmic observations. One such quantity is the spectral index of the scalar curvature of the primordial power spectrum, which has recently undergone enhanced measurement. In this work, we demonstrate that the hybrid inflationary potential generates a novel type of perturbation spectrum during the inflationary period. We determined the spectral feature of density fluctuations produced during inflation. The effective mass of the inflaton rapidly changes at the transition point ϕ_c during the inflationary epoch with this model. This modification imprints a local characteristic onto the spectrum of density perturbations in the early universe. This is accompanied by a significant running in the amplitude of the spectral index at the transition point. However, the running of the spectral index diminishes significantly when moving away from that point. According to the model described in this study, inflation may have been accompanied by a rapid second-order phase transition, which might account for the observed running of the spectral index in the Plancks 2018 data. This second-order phase

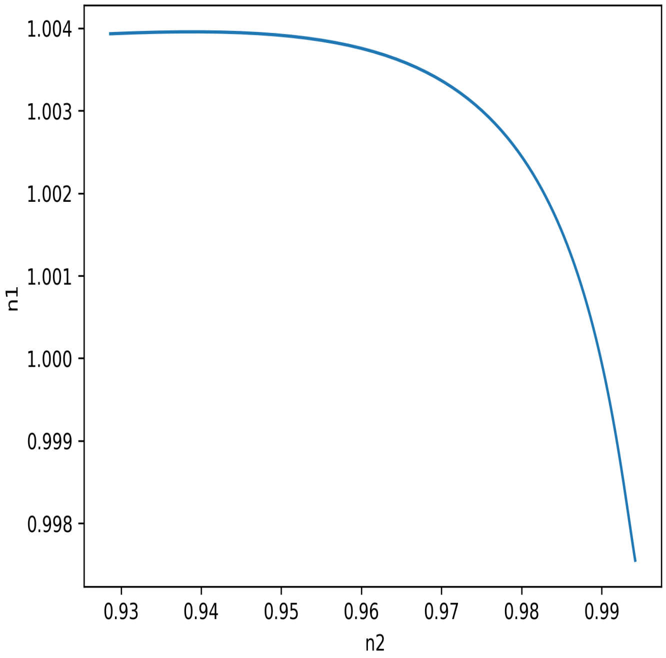

Figure 5. Spectral indices of the perturbations produced before (n_1) and after (n_2) the phase transition in hybrid inflation.

transition often results in the formation of topological defects. It has been shown, however, that topological defects may be inflated away by an e-folding of 60 during the phase transition[23].

Acknowledgments

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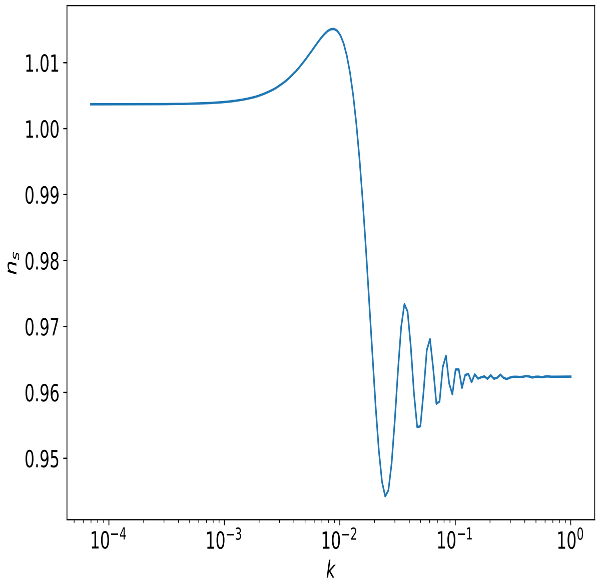

Figure 6. The primordial spectral index as a function of wavenumber *k* for this hybrid inflationary model.

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