

# Mathematical Modelling of Traffic Behaviour

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## Abstract

Mathematical modelling is a powerful tool that can be used to describe and understand complex real-world phenomena through the application of mathematical equations, algorithms, and simulations. Mathematical modelling of traffic flow plays a crucial role in understanding and predicting the dynamics of vehicular movement on roads. Usual modelling of traffic flow is restricted to treating traffic as an inorganic system. In this work, we bring in another layer of parameters that can affect traffic – human behaviour. Here we argue that a mathematical model with behavioural components could provide a more real-world understanding of traffic flow. The primary aim of the research is to create a mathematical model that incorporates human behaviour.

**Keywords:** Mathematical model; traffic congestion; psychological experiences; negative affect

## 1 Introduction

Travel adversities are the challenges that commuters face in traffic congestion and can cause negative affect [1], which can reduce the quality of life [2] and work productivity [3, 4]. Hormones such as adrenaline and cortisol are responsible for the negative affect [5, 6]. Constant exposure to travel adversities can induce driver stress, anger and aggression [7, 8, 9], tension, impatience, frustration and helplessness [10]. Driving in a congested environment can cause

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pollution, heart problems, anxiety, stress, and chronic stress, which can lead to physical health issues and psychological stress [11, 12].

When commuters act in some way to combat congestion, which can result in negative road occurrences such as physical damage to people and vehicles, traffic rule violations and road-traffic accidents cause travel adversities that lead to increased congestion. This demonstrates a cyclic relationship between travel adversities which causes negative affect, which leads to the narrowing of thought action repertoire, resulting in a fight or flight response [13], which causes negative road occurrences, in turn creating more congestion. A statistical model evaluating the probability of a vicious cycle between travel adversities and increased congestion could provide falsifiable insights [14].

## **2 Mathematical Model**

Mathematical modelling in traffic flow provides a quantitative framework to analyse and understand the complex dynamics of traffic systems. These traffic flow models have various applications, including forecasting congestions, evaluation of alternative transportation policies, and support for intelligent transportation systems. They can help in identifying bottlenecks, optimising signal timings, evaluating the impact of road expansions, and developing efficient traffic management strategies. Traffic models can be extended to represent complex road networks, by considering the interactions between different road segments, intersections, and traffic signals.

Existing mathematical models of real-world traffic flow rely heavily on theoretical foundations and do not take into account the behavioural components of traffic congestion and travel adversities [15, 16, 17]. A mathematical model with behavioural components could provide further insights into the existence of this correlation.

After establishing statistically that travel adversities cause negative affect, leading to fight and negative road occurrences, this can be modelled in the sequence as shown in figure 1.

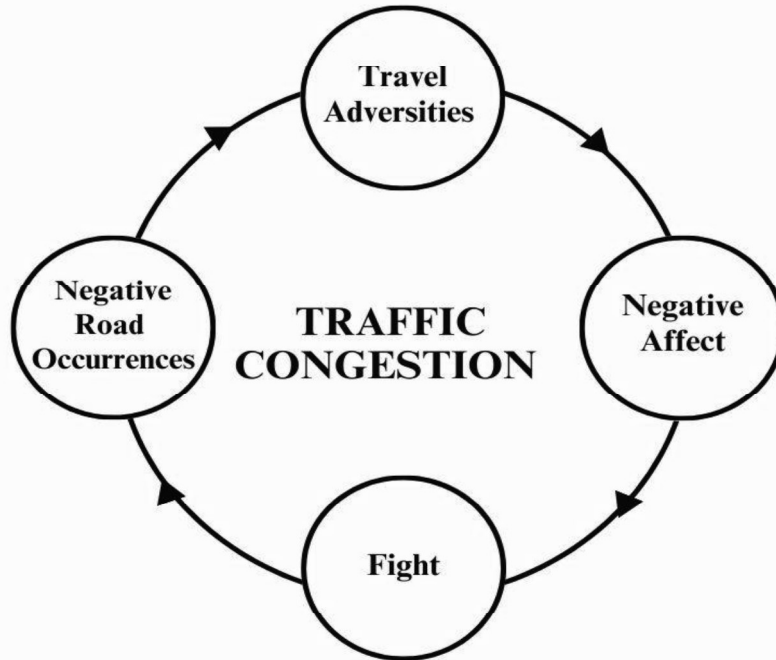


Figure 1: Nonlinear nature of traffic flow

The correlation between the four parameters, i.e. NA (Negative Affect), FT (Fight), NRO (Negative Road Occurrences), and TA (Travel Adversities), can be mathematically stated as:

$$NA=f_1(TA)+x \quad \dots (1)$$

$$FT=f_2(NA)+y \quad \dots (2)$$

$$NRO=f_3(FT)+z \quad \dots (3)$$

where the functions  $f_1, f_2, f_3$  incorporates the coefficients of TA, NA and FT respectively. These will be determined using study mentioned in section 3.  $x, y, z$  are the Gaussian errors in both the model and the data, assuming that the data is normally distributed leading us back to how NRO affects TA. This then completes the feedback loop:

$$TA=f_4(NRO)+w \quad \dots (4)$$

$f_4$  being the coefficient of NRO and  $w$ , the Gaussian error.

### 3 Method

The study will employ a sequential two-phase exploratory mixed methods design. The initial phase (qualitative approach) will investigate the psychological experiences of commuters as a

consequence of traffic congestion-related travel adversities. Using a validated semi-structured interview schedule, participants (men and women) between the age of 20 - 40 years will be interviewed in person until data saturation is reached. The interviews will be audio recorded, and transcripts will be checked and coded. Data will be analysed using thematic analysis. Phase 2 (quantitative approach) will statistically validate the themes that emerged in phase 1 by developing the themes related to psychological experiences and consequences into a checklist with Likert-type responses.

Data will be collected using online portals, and quantitative measures using descriptive statistics, frequency and percentage analysis, Mann Whitney U, Kruskal Wallis H and Dwass-Steel-Critchlow-Fligner pairwise comparisons will be used to explore data and SEM will be used to analyse and test models, leading to the creation of a mathematical model that could be used to evaluate the non-linear relationship between the various components.

#### **4. Discussion and Conclusion**

Travel adversities can cause negative affective states, leading to fight actions and negative road occurrences showing a non-linear relationship between negative road occurrences and travel adversities. These events can cause stress, aggression, and pressure, affecting professional and personal relationships. The fight response, including aggressive driving, altercations, and dangerous driving, can result in defiance, disobedience, and wilful driving violations. These negative road occurrences increase travel adversities and thereby increase congestion showing a non-linear cyclic pattern of occurrences.

The mathematical model offers a valuable tool for understanding traffic psychology and predicting real-world traffic behaviour. It reveals the importance of a behavioural component that is often overlooked in current models. The model can comprehend disparate and unrelated behavioural occurrences, allowing for the prediction of negative affect and road occurrences. Similar mathematical models can be set up and evaluated for other human behaviours (work in progress). Using this concept, we can establish an integrated, intelligent traffic management system that could alleviate congestion in densely populated cities.

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