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A Study on Cosmological Model of Anisotropic Universe with Electromagnetic Field and Cloud Strings in the Frame Work of General Relativity

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Abstract

In this study, we have investigated the Bianchi type-III anisotropic cosmological model in the presence of cloud strings with electro-magnetic field in the general theory of relativity. Exact solutions of field equations are obtained using the fact that shear scalar is proportional to scalar expansion and constant decelerating parameter, which is derived from the variation law of Hubble parameter proposed by Berman(Nuovo Cimento B 74, 182 (1983)) and linearly varying decelerating parameter proposed by Akarsu and Dereli(International Journal of Theoretical Physics, 51,612(2012)). The dynamics and significance of the physical parameters of the model are discussed using a graphical representation of these parameters. The dynamics and significance of the physical parameters of the models are discussed.

Keywords: Bianchi type-III, electro-magnetic field, linearly varying decelerating parameter, constant decelerating parameter.

1. Introduction:

In 1916, Einstein [1] proposed the general theory of relativity. It was effective in geometrizing the attraction by distinguishing the metric tensor by gravitational possibilities. Around then Einstein felt that our universe was static and built static models to clarify the advancement of the universe. After Hubble's observations [2] it is concluded that

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our universe is non-static. Then, all static models were ruled out, and non-static models came into the picture. Friedmann [3] was the first one to investigate the most general homogeneous isotropic non-static model described by the Robertson-Walker metric.

Spatially homogeneous and isotropic FRW models are the best-fit models to speak about the huge scope structure of the current day universe. This fact is confirmed by the analysis of cosmic microwave background fluctuations. But FRW models fail to explain the correct matter distribution in the beginning phases of the development of the universe and furthermore, there is proof of anisotropy in the beginning phases of the universe. Henceforth the models with an anisotropic foundation are appropriate to clarify the beginning phases of the universe. This way, the examination of Bianchi's models assume a crucial job in understanding the beginning phases of the development of the universe. Several authors studied spatially homogeneous anisotropic Bianchi models to get a relativistic picture of the early universe. In particular, Singh and Chaubey [4], Saha and Yadav [5], Adhav et al. [6], Akarsu and Kilnic [7], Yadav et al. [8], Pradhan et al. [9], have examined diverse Bianchi models with normal impeccable liquids.

The past two decades saw a raising importance to string cosmological models which have gotten extensive consideration from researchers in view of their significance in structure arrangement during the early universe. During the state change of the early universe, unconstrained balance breaking offers ascend to an arbitrary system of stable line like topological deformities known as cosmic strings. It is notable that massive strings are the principal components for the development of huge structures like the galaxies and galaxy clusters in the universe. Letlier [10], Stachel [11] and Sahoo [12], Hegazy et al. [13] examined the different significant highlights of string cosmological models either in the casing work of general relativity or in modified theories of gravitation. Pavon et al. [14], Pradhan et al. [9] are some of the authors who have investigated bulk viscous cosmological models in general relativity.

The magnetized cosmological model plays a vital role in the evolution of the universe and in the formation of galaxies and clusters of galaxies and other stellar bodies. The electromagnetic field that was generated during inflation is also one cause for the present period of accelerated expansion of the universe. The statistical breakdown of isotropy is additionally because of the magnetic field. The magnetic fields are hosted by the galaxies and cluster of galaxies. Subramanian [15] on his paper indicated that magnetic fields have a significant on the arrangement of stellar structures. Also so many authors have studied magnetized cosmological models. Some prominent in this context are Jimenez and Marato [16], Tripathy et al. [17], Parikh [18], and Grasso [19]. So many authors have studied string models with electromagnetic fields to understand the evolution of the universe in early phases. Rahman and Hegazy have studied Bianchi-type Vl_0 cosmological model with electromagnetic variable decelerating parameters in general relativity, mainly Tripathi et al. [20] have investigated the magnetized string model in the Bianchi type –III universe. Recently Priyokumar Singh et al. [21] have obtained a cloud string cosmological model with an electromagnetic field in Bianchi type-I universe in general relativity. Till date no one has obtained a magnetized cloud string model with constant & straightly changing decelerating parameters in the framework Bianchi type –III anisotropic universe in general relativity.

Inspired by the above discussion and investigations, in this paper, we have considered the magnetized cloud string cosmological model with constant and straightly changing deceleration parameters in Bianchi type-III space-time in the General hypothesis of relativity proposed by Einstein. This paper is sorted out as follows: in sect.2 field equations are determined in the Bianchi type-III universe. In sect.3 the solutions of field equations in two cases and the physical conversation of the model with the help of graphs are presented and the last segment contains a few conclusions of the acquired model.

2. Metric and Field Equations:

The spatially homogeneous anisotropic Bianchi type-III metric is of the form

$$ds^2=-dt^2+X^2dx^2+Y^2e^{-2x}dy^2+Z^2dz^2 \qquad (1)$$

Where $X=X(t)$, $Y=Y(t)$, $Z=Z(t)$.

The mixed tensor form of energy-momentum tensor for strings with electromagnetic field is

$$T_j^i = \rho u^i u_j - \lambda x^i x_j + E_j^i \tag{2}$$

where ρ denotes the density of strings, which is equal to $\rho = \rho_p + \lambda$, here ρ_p, λ denote the particle density and tension density of the string, respectively.

Here x_i, u_i satisfies

$$u^i u_i = -x^i x_i = -1 \tag{3}$$

and

$$u^i x_i = 0 \tag{4}$$

Also u^i and x^i in the direction of parallel to x-axis are given by

$$u^i = (0,0,0,1) \tag{5}$$

and

$$x^i = \left(\frac{1}{L}, 0,0,0\right) \tag{6}$$

The electromagnetic field E_{ij} in mixed tensor form considered as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \tag{7}$$

where F_{ij} is the electromagnetic field tensor.

By quantizing the magnetic field along x-axis, we will get only one non- vanishing component F_{14} in F_{ij} i.e. $F_{ij}=0$ *for* i,j=1,2,3,4 except F_{14}.

The non-vanishing components of E_i^j derived from equation (7) are

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{1}{2A^2}(F_{14})^2 \tag{8}$$

The field equations in general theory of relativity (with $\frac{8\pi G}{c^4} = 1$) is given by

$$G_j^i = -T_j^i \tag{9}$$

Where G_j^i is Einstein tensor.

By using equations (9) and (2) for the metric (1), the non-vanishing field equation can be obtained as

$$\frac{\ddot{Y}}{Y}+\frac{\ddot{Z}}{Z}+\frac{\dot{Y}\dot{Z}}{YZ}=\lambda-\frac{1}{2X^2}(F_{14})^2 \tag{10}$$

$$\frac{\ddot{Y}}{Y}+\frac{\ddot{Z}}{Z}+\frac{\dot{Y}\dot{Z}}{YZ}=\frac{1}{2X^2}(F_{14})^2 \tag{11}$$

$$\frac{\ddot{X}}{X}+\frac{\dot{X}\dot{Y}}{XY}+\frac{\ddot{Y}}{Y}-\frac{1}{X^2}=\frac{1}{2X^2}(F_{14})^2 \tag{12}$$

$$\frac{\ddot{X}}{X}+\frac{\dot{X}\dot{Y}}{XY}+\frac{\ddot{Y}}{Y}-\frac{1}{X^2}=\rho-\frac{1}{2X^2}(F_{14})^2 \tag{13}$$

$$\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}=0 \tag{14}$$

where overhead single, and double dots denote first and second-order derivatives w.r.t t respectively.

For equation (1), the scale factor a(t), spatial volume V, Hubble parameter H, Scalar expansion θ shear scalar σ^2 and average anisotropy parameter $A_\square$ are the phýsical and kinematical parameters that can be used to solve the above field equations. They are defined as follows

$$a(t)=(XYZ)^{\frac{1}{3}} \tag{15}$$

$$V=\left(Z(t)\right)^3=XYZ \tag{16}$$

$$H=\frac{1}{3}\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) \tag{17}$$

$$\theta=u^i_{;i}=3H=\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) \tag{18}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{2}\left(\frac{\dot{X}^2}{X^2}+\frac{\dot{Y}^2}{Y^2}+\frac{\dot{Z}^2}{Z^2}\right)-\frac{1}{6}(\theta^2) \tag{19}$$

$$A_\square=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{20}$$

3. Solutions and Model:

From equation (14) it is obtained that

$$X=kY \tag{21}$$

where k is the constant of integration. Without loss if generality, we can choose k=1, so

$$X=Y \tag{22}$$

Using eq. (22) the field equations (10)-(13) reduces to

$$\frac{\ddot{Y}}{Y}+\frac{\ddot{Z}}{Z}+\frac{\dot{Y}\dot{Z}}{YZ}=\frac{\lambda}{2} \tag{23}$$

$$\left(\frac{\dot{Y}}{Y}\right)^2+2\frac{\ddot{Y}}{Y}-\frac{1}{Y^2}=\frac{1}{2Y^2}(F_{14})^2 \tag{24}$$

$$\left(\frac{\dot{Y}}{Y}\right)^2+2\frac{\dot{X}\dot{Y}}{XY}-\frac{1}{Y^2}=\rho-\frac{1}{2Y^2}(F_{14})^2 \tag{25}$$

Clearly, this is a system of three differential equations in five unknowns *B,C,λ,ρ and* $F_{14.}$ To get the solution of these highly non-linear differential equations we use the following conditions, which are physically important.

Case (1): Model with Varying Decelerating Parameter

The shear scalar σ^2 is proportional to scalar expansion θ so that we can take

(Collins et al. [22]) $Y=Z^n$ (26)

Where $n\neq 1$ is a constant and preserves the anisotropic nature of the model.

A generalized linearly varying deceleration parameter (Akarsu and Dereli [23])

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-kt+m-1 \tag{27}$$

Where $k > 0$ and $m > 0$.

from equation (27) we can obtain $a(t)$ as follows

$$a(t)=c_2e^{\frac{2}{m}\left(\frac{k}{m}t-1\right)}=c_2\left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t}\right)^{\frac{1}{m}} \tag{28}$$

where c_2 is integration constant which can be chosen as unity.

from equations (28),(26) and (22) we can obtain the metric potentials are obtained as follows

$$Z = \left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t} \right)^{\frac{3}{m(2n+1)}} \tag{29}$$

$$Y = \left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t} \right)^{\frac{3n}{m(2n+1)}} = X \tag{30}$$

by using equations (29), (30) the metric eq. (1) can be written as

$$ds^2 = -dt^2 + \left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t} \right)^{\frac{6n}{m(2n+1)}} dx^2 + \left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t} \right)^{\frac{6n}{m(2n+1)}} e^{-2x} dy^2 + \left(\frac{\frac{k}{m}t}{2-\frac{k}{m}t} \right)^{\frac{6}{m(2n+1)}} dz^2 \tag{31}$$

Eq. (31) represents the magnetized cloud string cosmological model with linearly changing deceleration parameter in Einstein's theory of general relativity.

Physical Discussion of the Model:

The physical and kinematical parameters V, H, θ, σ^2 and A_h which are very important in physical discussion of the model are as follows

$$V = \left(\frac{\frac{kt}{m}}{2-\frac{kt}{m}} \right)^{\frac{3}{m}} \tag{32}$$

$$H = \frac{2}{t(2m-kt)} \tag{33}$$

$$\theta = \frac{6}{t(2m-kt)} \tag{34}$$

$$\sigma^2 = \frac{12(n-1)^2}{t^2(2n+1)^2(2m-kt)^2} \tag{35}$$

$$A_h = \frac{2(n-1)^2}{(2n+1)^2} \tag{36}$$

From (34) and (35), we have

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(2n+1)^2} = \quad constant \quad (\neq 0 \; from \;\; n \neq 1) \tag{37}$$

The string density, energy density, are obtained as follows

$$\lambda = \frac{72(n^2+n+1)+12(n+1)(2m-2kt)(2n+1)}{t^2(2n+1)^2(2m-kt)^2} \tag{38}$$

$$\rho = \frac{72n+24mn-24nkt}{t^2(2n+1)(2m-kt)^2} - 2\left(\frac{2m-kt}{kt}\right)^{\frac{6n}{m(2n+1)}} \tag{39}$$

Also, we obtain ${F_{14}}^2$ is obtained as

$${F_{14}}^2 = 2\left(\frac{kt}{2m-kt}\right)^{\frac{6n}{m(2n+1)}}\left(\frac{108n^2+24n(2n+1)(m-kt)}{t^2(2n+1)^2(2m-kt)^2}\right) - 2 \tag{40}$$

The particle density is obtained from $\rho_p=\rho-\lambda$ as follows

$$\rho_p = \frac{72n^2-48mn-24m-72+24k(2n+1)t}{t^2(2n+1)^2(2m-kt)^2} - 2\left(\frac{2m-kt}{kt}\right)^{\frac{6n}{m(2n+1)}} \tag{41}$$

Case (2): Model with Constant Decelerating Parameter

Variation of the Hubble's parameter proposed by Berman [24] which yields constant deceleration parameter models defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = m - 1 \quad \text{(where } m \text{ is a constant)} \tag{42}$$

from equation (42) we can obtain *a(t)* as follows

$$a(t) = (c_1mt + c_2)^{\frac{1}{m}} \tag{43}$$

where c_1, c_2 are integration constants.

from equations (26), (43) and (28) we can obtain the metric potentials are obtained as follows

$$a(t) = (c_1mt + c_2)^{\frac{1}{m}} \tag{44}$$

$$Z = (c_1mt + c_2)^{\frac{3}{m(2n+1)}} \tag{45}$$

by using equations (44),(45) the metric eq.(1) can be written as

$$ds^2 = -dt^2 + (c_1mt + c_2)^{\frac{6n}{m(2n+1)}}dx^2 + (c_1mt + c_2)^{\frac{6n}{m(2n+1)}}e^{-2x}dy^2 + (c_1mt + c_2)^{\frac{6}{m(2n+1)}}dz^2 \tag{46}$$

Eq. (46) represents the magnetized cloud string cosmological model with constant deceleration parameter in Einstein's theory of general relativity.

Physical Discussion of the Model:

The physical and kinematical parameters V, H, θ, σ^2 and A_h which are very important in physical discussion of the model are as follows

$$V = (c_1 mt + c_2)^{\frac{3}{m}} \tag{47}$$

$$H = \frac{c_1}{c_1 mt + c_2} \tag{48}$$

$$\theta = \frac{3c_1}{c_1 mt + c_2} \tag{49}$$

$$\sigma^2 = \frac{3(n-1)^2 {c_1}^2}{(2n+1)^2 (c_1 mt + c_2)^2} \tag{50}$$

$$A_h = \frac{2(n-1)^2}{(2n+1)^2} \tag{51}$$

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(2n+1)^2} = \quad constant \quad (\neq 0\, from\ n \neq 1) \tag{52}$$

The string density, energy density are obtained as

$$\lambda = \frac{6{c_1}^2[3(n^2+n+1) - m(2n^2+3n+1)]}{(2n+1)^2 (c_1 mt + c_2)^2} \tag{53}$$

$$\rho = \frac{6n{c_1}^2 (3-m)}{(2n+1)(c_1 mt + c_2)^2} - 2(c_1 mt + c_2)^{\frac{-6n}{m(2n+1)}} \tag{54}$$

Also, we obtain ${F_{14}}^2$ as

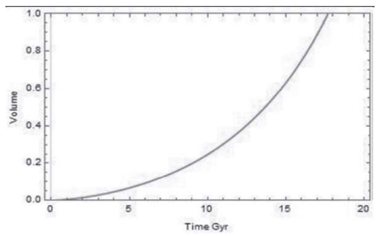
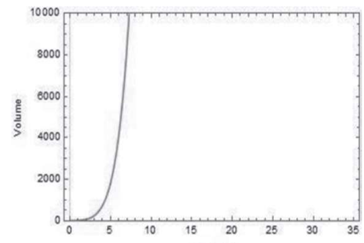
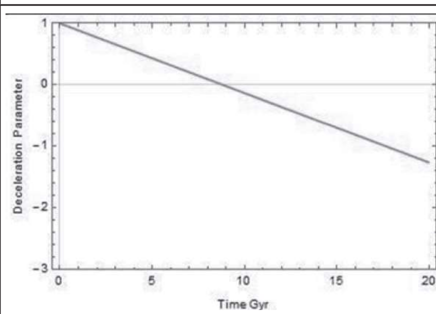
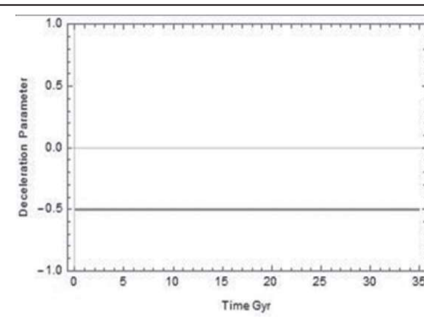
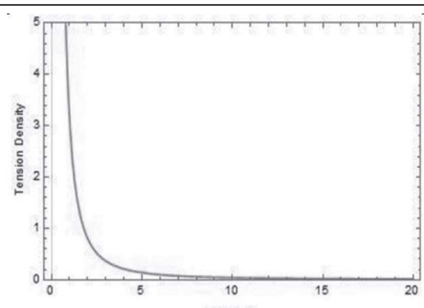
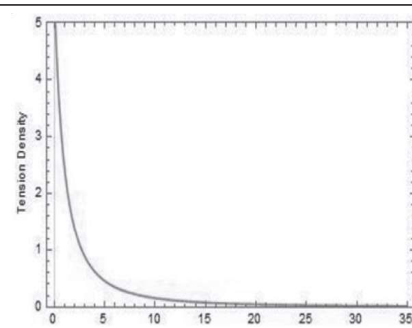
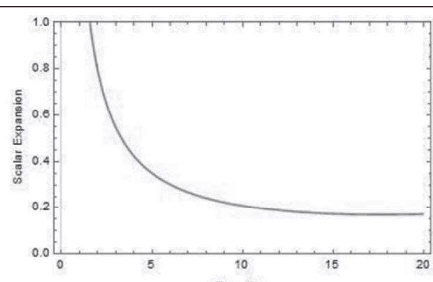
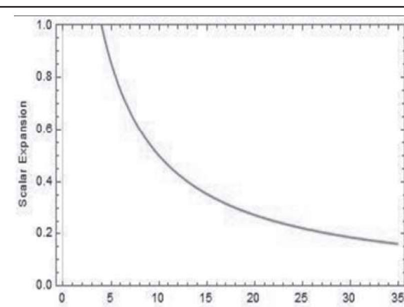
$${F_{14}}^2 = \frac{n{c_1}^2 (54n - 24mn - 12m)}{(2n+1)^2} (c_1 mt + c_2)^{\frac{6n-4mn-2m}{m(2n+1)}} - 2 \tag{55}$$

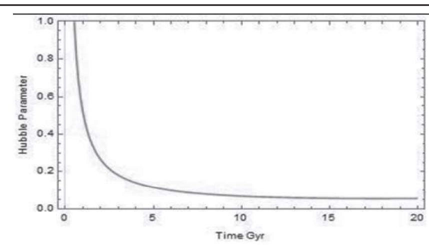
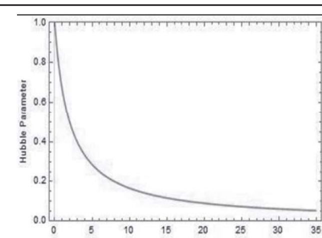
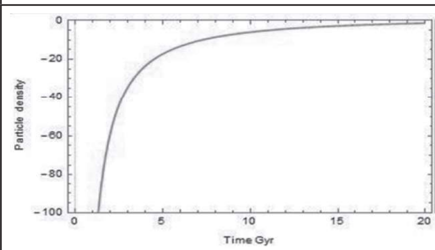
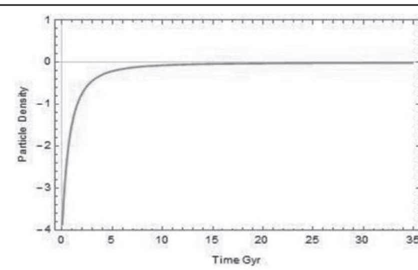
The particle density is obtained from $\rho_p = \rho - \lambda$ as follows

$$\rho_p = \frac{6{c_1}^2[m(2n+1) + 3(n^2-1)]}{(2n+1)^2 (c_1 mt + c_2)^2} - 2(c_1 mt + c_2)^{\frac{-6n}{m(2n+1)}} \tag{56}$$

For the physical discussion of the parameters the discussion about parameters and the graphs of parameters versus time t (Gyr) are as follows

The Graphical View of the parameters is as follows {Parameter v/s Time *t* (Gyr)}

Para-meter	Case (1) With Linearly Varying deceleration (*k*=0.113,*m*=2 and *n* = 2)	Case (2) With Constant deceleration (*m*=0.5, c_1=1, c_2=1 and *n*=0.5)
I Spatial Volume		
II Decele-ration Para-meter		
III Tension density		
IV Scalar Expan-sion		

Para-meter	Case (1) With Linearly Varying deceleration (k=0.113,m=2 and n = 2)	Case (2) With Constant deceleration (m=0.5, c_1=1, c_2=1 and n=0.5)
V Hubble Para-meter		
VI Particle density		

Conclusions

We have obtained the magnetized cloud string cosmological model in Bianchi type-III anisotropic universe in Einstein's theory of general relativity. To get the solutions of the field equations, we have taken the help of both linearly varying decelerating parameters proposed by Akarsu and Dereli [23] and constant decelerating parameters by Berman [24] in both cases. It is observed that the spatial volume increases w.r.t. time t and the parameters H,θ,σ^2 and ρ decreases as $t\to\infty$. In this model $H>0$ throughout the evolution of the universe and $q<0$ at present $t\simeq13.7168$. This shows that the present-day universe which is in accelerated expansion [25-27]. It is seen that $\frac{\sigma^2}{\theta^2}=\frac{(n-1)^2}{3(2n+1)^2}= constant\ (\neq 0\ from\ n\neq 1)$, so, both the obtained models in the two cases does not tend to isotropic nature for $t\to\infty$. It is seen that ρ_p has a large negative value at t = 0 and it approaches a constant positive finite value as $t\to\infty$. So, the universe will be dominated by particles for late time. We have observed the string tension density tends to zero as $t\to\infty$, so the obtained models represent the present-day matter dominated universe. So, we hope that these models have a good agreement with the present-day scenario of the universe with the recent observations [28-29].

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