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Classical Limit in the Double-Slit Experiment for Quantum Particles

B. A. Kagali* & K. S. Mallesh†

Abstract:

It is well known that the double-slit experiment, under proper conditions, reveals the wave nature of both light and quantum particles like electrons. The wave nature of quantum particles is demonstrated by assuming plane or spherical de Broglie waves to be associated with quantum particles. In this article, we deduce the wave nature of material particles starting with proper quantum mechanical amplitudes for propagation as arrived at in the path integral formulation. The classical particle limit of the interference pattern follows right away along the same lines as the corpuscular limit from wave optics.

Keywords: Duality, Path-integral Formalism, Classical limit

1. Introduction

The double slit experiment is the most widely studied [1, 2, 3, 4] experiment in the domain of quantum physics to illustrate the dual nature of microscopic (quantum) particles. The setup as shown in Figure (1), consists of a pair of apertures, each of width b, separated by a distance d. A monoenergetic beam of electrons from a source is directed towards these slits, and the electrons coming out of these holes are detected by a series of detectors aligned parallel to the line joining the apertures at a distance L. The distribution of electrons at various points along a line parallel to the slits is then studied and interpreted.

* Department of Physics, Bengaluru Central University, Bengaluru 560 001, Karnataka, India, bakagali@gmail.com

† Regional Institute of Education, Mysuru 570 006, Karnataka, India; ksmallesh@gmail.com

1.1. Classical particles incident on a double slit

Let us consider the effect of a double slit on a beam of monoenergetic particles incident on a double slit, as in Figure (1). By classical particle we mean objects with mass and whose position and momentum can be simultaneously specified accurately. A stream of bullets can be taken as an example of classical particles.

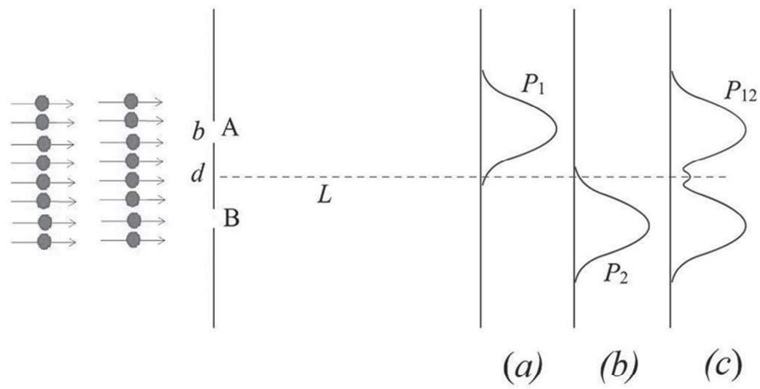

Figure 1. Classical particles through double slit: (*a*) when slit A is open and slit B is closed, (*b*) when slit A is closed and slit B is open, and (*c*) when both slits are open.

From Newtonian mechanics, we expect a distribution like P_1 when only slit *A* is open and a distribution like P_2 when slit *B* only is open on a screen kept far away from the slit. When both *A* and *B* are open, since the particles can travel through only one slit at a time, we expect a distribution like P_{12} on the screen such that,

$$P_{12} = P_1 + P_2. \tag{1.1}$$

1.2. Quantum particle incident on a double slit

When a beam of quantum particles such as electrons is incident on a double slit, the nature of their distribution on a faraway screen is illustrated in several textbooks [1, 2, 3, 4] as shown in Figure (2).

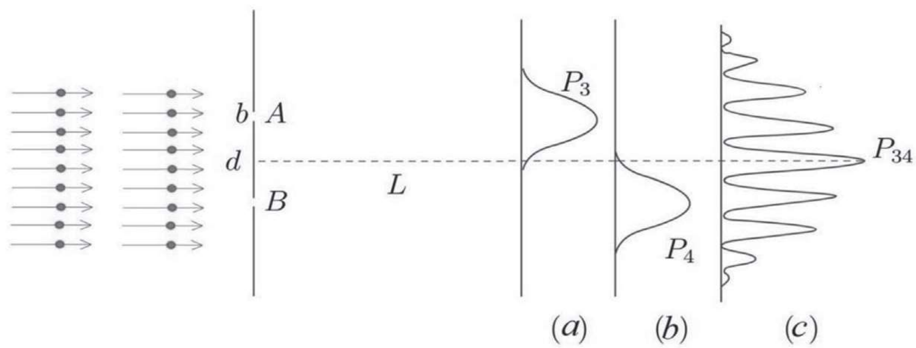

Figure 2: Quantum particles through double slit: (*a*) when slit A is open and slit B is closed, (*b*) when slit A is closed and slit B is open, and (*c*) when both slits are open.

It is generally stated that the quantum particle such as electron form a distribution like P_3, P_4 when only the slits *A* or *B* are kept open, respectively. No fringe-like structure is shown for P_3 and P_4. When both slits are open, the distribution on the screen is shown to have fringe-like structure as illustrated by P_{34}, indicating interference between the waves associated with the quantum particles. Clearly,

$$P_{34} \neq P_3 + P_4.$$

While deducing P_{34}, it is tacitly assumed that the quantum particles with momentum p are associated with plane waves or spherical waves having a wavelength given by the de Broglie formula: $\lambda = h/p$, and the distribution on the screen is proportional to $|\psi_A + \psi_B|^2$, where ψ_A and ψ_B are the waves arriving from the slits *A* and *B* respectively. This is done without proof in analogy with optical waves emerging from slits [5, 6, 7]. However, we can make use of the path integral formalism [8, 9, 10], to write down the amplitudes for the propagation. This formalism suggests that the amplitude of evolution of a system from one configuration to another in time is determined by the sum of all possible paths connecting the two. Compared to the other formalisms, this is known to be more elegant and universal. A detailed review of this formalism and its application to a good number of topics in basic quantum mechanics and in quantum field theory has been given by Robson et. al. [11]. On applying this formalism to the present problem, the amplitude for a free particle propagation from (x_0, t_0) to (x, t) is,

$$K\ (x,t;x_0,t_0)=\sqrt{\frac{m}{2\pi i\hbar\Delta t}}exp\left(\frac{i}{\hbar}\frac{m(x-x_0)^2}{2\Delta t}\right), \tag{1.2}$$

where m is the mass of the quantum particles and $\Delta t = t - t_0$. Using this amplitude for quantum particles, we evaluate the distribution of quantum particles on the screen kept away from a pair of slits of width *b* that are separated by a distance *d* by first evaluating the amplitudes ψ_A and ψ_B for the electrons arising from the slits *A* and *B* (See Figure (3)). The distribution on the screen will be proportional to $|\psi_A + \psi_B|^2$, according to the Born rule in quantum mechanics. The amplitude for the electrons to be present at an arbitrary point *P(L, y)* on the screen due to the slit *A* can be written by generalizing the formula (equation 1.2) for two-dimensional motion as,

$$\psi_A=\frac{m}{2\pi iht}\int_{\frac{(d-b)}{2}}^{\frac{(d+b)}{2}}\exp\left(\frac{i}{h}\frac{mL^2}{2t}\right)\exp\left(\frac{i}{h}\frac{m(y-\xi)^2}{2t}\right)d\xi \tag{1.3}$$

Here, we have assumed $t_0 = 0$ and that the incident electrons are uniformly distributed over the width of slit *A* etc. Assuming the separation *L* between the slits and

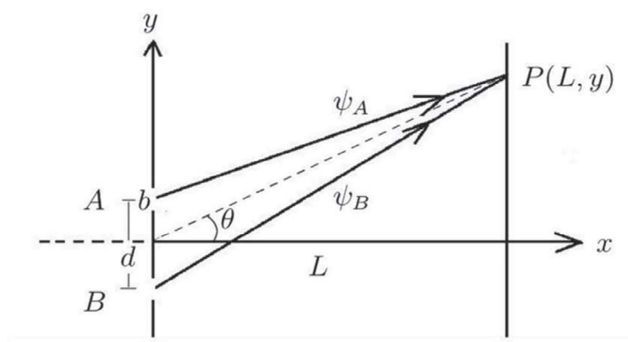

Figure 3: Quantum amplitudes ψ_A and ψ_B

the screen to be very much larger than *b* and *d* we get,

$$\psi_A \simeq \frac{m}{2\pi i\hbar t}exp\left(\frac{i}{\hbar}\frac{mL^2}{2t}\right)\int_{(d-b)/2}^{(d+b)/2} exp\left(-\frac{im}{\hbar t}y\xi\right)d\xi$$

$$\simeq\ C_1 exp\left(-\frac{i}{\hbar}\frac{myd}{2t}\right)\left(\frac{sin\frac{myb}{2\hbar t}}{\frac{my}{2\hbar t}}\right) \tag{1.4}$$

In the same approximation, the amplitude for electrons arriving at point P from the slit *B* can be written as

$$\psi_B \simeq\ C_1 exp\left(\frac{i}{\hbar}\frac{myd}{2t}\right)\left(\frac{sin\frac{myb}{2\hbar t}}{\frac{my}{2\hbar t}}\right) \tag{1.5}$$

Hence the amplitude for electrons arriving at P from both the slits will be,

$$\psi = \psi_A +\ \psi_B \qquad =\ 2C_1\, cos\, cos\left(\frac{m}{\hbar t}\frac{yd}{2}\right)\left(\frac{sin\frac{myb}{2\hbar t}}{\frac{my}{2\hbar t}}\right) \tag{1.6}$$

where, $C_1 = \frac{m}{2\pi i\hbar t}\, exp\, exp\left(\frac{imL^2}{2t}\right)$. It can also be seen that,

$$\frac{m}{\hbar t}\frac{yd}{2} = \frac{m\underline{L}}{\hbar t}\frac{yd}{2\underline{L}} = \frac{\pi d \quad sin\,\theta}{\lambda_d},$$

where, $\lambda_d = \frac{\hbar t}{m\underline{L}}$ = de Broglie wavelength of the particle arriving at P, as $\underline{L}/t$ is simply the speed of the particles reaching *P* from the slits. We can also write for the angular deviation of the point *P* from the incident direction θ as: $sin\,\theta = \frac{y}{\underline{L}}$ and $\underline{L}$= $\sqrt{L^2 + y^2}$. Similarly,

$$\frac{m}{\hbar t}\frac{yb}{2} = \frac{m\underline{L}}{\hbar t}\frac{yb}{2\underline{L}} = \frac{\pi b \quad sin\,\theta}{\lambda_d}.$$

Therefore, the distribution on the screen due to the double slit will be proportional to

$$I(L,\theta) = 4|C_1|^2 b^2 cos^2\left(\frac{\pi d \quad sin\,\theta}{\lambda_d}\right)\frac{sin^2\left(\frac{\pi b \quad sin\,\theta}{\lambda_d}\right)}{\left(\frac{\pi b \quad sin\,\theta}{\lambda_d}\right)^2}. \tag{1.7}$$

Very interestingly, this is exactly equal to the result one gets in the double slit interference of light of wavelength λ_d passing through a pair of slits of width b separated by a distance d [5, 6, 7] when we identify $|C_1|^2$ with the strength of the amplitude coming from each point on the slit. The distribution P_{34} will be like $I(L,\theta)$. The above derivation, based on proper quantum mechanics of material particles leads to the wavelike pattern on the screen. There is no need for assuming plane waves or spherical waves right in the beginning, as done in the references [1, 2, 3, 4] and then demonstrating the wave nature of quantum particles!

2. The Classical Limit of $I(L,\theta)$

$I(L,\theta)$ derived from proper quantum mechanical amplitude for material particles leads to a result similar to that for optical waves thereby demonstrating the wave nature of quantum particles. It is also to be noted that the distributions P_3 and P_4 should also exhibit fringe-like variations since the passage of any wave through a slit leads to a diffraction pattern with bright and dark fringes - this is not stated in the books mentioned earlier. It can also be obtained by using the generalization of equation (1.2) for two dimensions for the case of a single wide slit. Since we expect the results for quantum particles to reduce to those for classical particles in the actual limit of large masses or large momenta, we might ask the question of whether P_{34} reduces to P_{12} in the 'usual' classical limit. Recalling the results in Optics [5, 6, 7], we expect to obtain P_{12} from P_{34} as the slits become wider and their separation increases to values much larger than the wavelength of quantum particles, i.e.,

$$\lambda_d \ll b \ll d. \tag{2.1}$$

Under these conditions, the wave optical results reduce to those for geometrical optics (which support the corpuscular nature of light). This aspect is also not clarified in the mentioned books to take the discussion on quantum particles to its logical end!

3. Discussion

We have shown how the proper quantum mechanical amplitude derived from the path integral formalism leads to the observed interference pattern of the double slit experiment - thus deriving the result from a firm basis. Using optical analogy, we expect to get the classical particle limit as the slit width and slit separations become much larger than the the de Broglie wavelengths of the quantum particles. On the observational side, interference has been observed with large molecules, assembly of atoms such as C_{60} (Fullerenes) or even with clusters of thousands of atoms [12, 13, 14, 15]. It may be possible to see if the classical patterns are obtained when the slit widths and separations are increased in proper order. In fact, it may be possible to define a classical particle for a given set of experimental parameters using a double slit. There are great many technical challenges to be overcome in observing interference of larger and larger particles and, hence in observing the passage to the classical limit. We wish to mention here that the path integral formalism has been applied to double-slit phenomena in the literature to address some other interesting features. Jones et al. [16] have used this formalism to study and contrast the optical wave and the matter wave behavior, and Sawant et al. [17] have applied this to quantify contributions from non-classical paths.

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