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Neutrosophic Nano Extremal Disconnectedness

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Abstract

This paper delves into the exploration of extremally disconnected spaces within the context of neutrosophic nano topological space. It introduces a novel space termed neutrosophic nano mixed space, created through the fusion of neutrosophic nano minimal structure and neutrosophic nano topology. The primary objective is to investigate extremal disconnectedness within this new space, shedding light on its properties and implications. Furthermore, the research examines the characterization of various types of open sets within the neutrosophic nano mixed space. As an extension, we gave a bio-mathematical application of neutrosophic nano extremal disconnectedness.

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Keywords: $\mathcal{NN}$ -extremally disconnectedness, $\mathcal{NN}$ -minimal structure, $\mathcal{NN}$ -mixed space.

1. Introduction

In 1965, the concept of fuzzy sets was introduced by L.A. Zadeh [1], in which each element has an associated value from 0 to 1. In 1986, Atanassov [2] introduced intuitionistic fuzzy set, each element of this set contains components with both associated and non- associated values, it was the next development in fuzzy sets. The neutrosophic set was introduced by F. Smarandache [3]. He introduced the degree of indeterminacy as independent component in his 1995 manuscript, that was published in 1998. Chang [4] invented fuzzy topological space and established basic concepts including open and closed sets and continuity in 1968. Further, an intuitionistic fuzzy topological space was first proposed by Coker [5], in 1997. In 2016, Serkan Karatas [6] introduced neutrosophic topology and investigate some related properties such as neutrosophic closure, neutrosophic interior, neutrosophic exterior, neutrosophic boundary and neutrosophic subspace.

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Nano topological space was first conceptualized by L. Thivagar et. al. In 2018[7], he connectively studied the concept of nano topology with neutrosophic sets as well as fuzzy and intuitionistic sets. Extremally disconnectedness of a topological space was introduced by Gillman and Jerison [7]. In 2018, L. Thivagar [8] gave some characterizations of nano extremally disconnectedness in terms of nano semi-open and nano regular open sets. In 2022, Gour Pal [9], introduced the notion of minimal structure in neutrosophic topological space. A. Pandi [10], introduced the concept of Nano Minimal Structure Spaces. Ahmad Al-Omari [11], revealed that a mixed space is a combination of a topological space τ on X with minimal structure mX and it is said to be m-extremally disconnected if the m-closure of every τ – open set is τ – open.

In classical set theory, any two sets are either comparable or disjoint, also a set and its complement are always disjoint. But in neutrosophic set logic, there exist sets which are not comparable, also a set and its complement may be comparable. Because of the peculiar behaviour of neutrosophic sets, types of neutrosophic nano topological space slightly differ from types of nano topological space on classical set. The extremal disconnectedness was studied on each type of neutrosophic nano topological space. In the next section, we introduced the neutrosophic nano minimal structure and combined it with neutrosophic nano topology to generate neutrosophic nano mixed pace. Some weaker forms of open sets were defined in the newly defined space and analyzed the extremal disconnectedness. In the last section, a bio-mathematical application of neutrosophic nano extremal disconnectedness was given.

2. Preliminaries

Some vital requisites for our work were given below

Definition 2.1 [12] A neutrosophic set A on a non-empty set X is of the form $A = \{< x, [C_{Ax}, I_{Ax}, W_{Ax}] >: x \in X\}$, where $C_{Ax}, I_{Ax}, W_{Ax} \in [0, 1]$ and $0 \leq C_{Ax} + I_{Ax} + W_{Ax} \leq 3$, $\forall x \in X$. Here C_{Ax} - the correctness value, I_{Ax} - the indeterminacy value and W_{Ax} - the wrongness value, $\forall x \in X$.

Definition 2.2 [12] Let $X/_E$ be an equivalence relation on X and Y be a neutrosophic subset of X. The neutrosophic nano minor, neutrosophic nano major approximation and neutrosophic nano boundary of Y are denoted by $\mathcal{NN}_{min}(Y)$, $\mathcal{NN}_{maj}(Y)$ and $\mathcal{NN}_B(Y)$ respectively, defined as follows:

(i) $\mathcal{NN}_{min}(Y) = \{< x, C_{min(Y)x}, I_{min(Y)x}, W_{min(Y)x} > / y \in [x]_E, x \in X\}$

(ii) $\mathcal{NN}_{maj}(Y) = \{< x, C_{maj(Y)x}, I_{maj(Y)x}, W_{maj(Y)x} > / y \in [x]_E, x \in X\}$

(iii) $\mathcal{NN}_B(Y) = \mathcal{NN}_{maj}(Y) - \mathcal{NN}_{min}(Y)$, where $C_{min(Y)x} = \wedge_{y \in [x]_E} C_Y(y)$, $I_{min(Y)x} = \vee_{y \in [x]_E} I_Y(y)$, $W_{min(Y)x} = \vee_{y \in [x]_E} W_Y(y)$ and

$C_{maj(Y)x} = \vee_{y \in [x]_E} C_Y(y)$, $I_{maj(Y)x} = \wedge_{y \in [x]_E} I_Y(y)$, $W_{maj(Y)x} = \wedge_{y \in [x]_E} W_Y(y)$.

Definition 2.3 [12] Let Y be a neutrosophic subset of a non-empty set X and if the collection $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y), \mathcal{NN}_B(Y)\}$, where $0_{\mathcal{N}} = \{< x, 0, 1, 1 >: x \in X\}$ and $1_{\mathcal{N}} = \{< x, 1, 0, 0 >: x \in X\}$ forms a topology then it is said to be a neutrosophic nano topology. Neutrosophic nano open sets ($\mathcal{NNO}$) are the members of $\mathcal{NN}\tau_E(Y)$ and its complements are called neutrosophic nano closed sets ($\mathcal{NNC}$ sets in short).

Definition 2.4 [3] For neutrosophic sets U and V union, intersection, subset and complement are defined as,

(i) $U \cup V = \{< x, C_{Ux} \vee C_{Vx}, I_{Ux} \wedge I_{Vx}, W_{Ux} \wedge W_{Vx} >: \forall x \in X\}$

(ii) $U \cap V = \{< x, C_{Ux} \wedge C_{Vx}, I_{Ux} \vee I_{Vx}, W_{Ux} \vee W_{Vx} >: \forall x \in X\}$

(iii) $U \subseteq V$ iff $C_{Ux} \leq C_{Vx}, I_{Ux} \geq I_{Vx}, W_{Ux} \geq W_{Vx}; \forall x \in X$

(iv) $U' = \{< x, W_{Ux}, 1 - I_{Ux}, C_{Ux} >: \forall x \in X\}$

Definition 2.5 [12] Let $(X, \mathcal{NN}\tau_E(Y))$ be a $\mathcal{NN}$ space and let $A \subseteq X$, then the neutrosophic nano interior of A is defined as the union of all neutrosophic nano open subsets of A and it is denoted by $\mathcal{NN}_{int}A$. The neutrosophic nano closure of A is defined as the intersection of all neutrosophic nano closed sets containing A and it is denoted by $\mathcal{NN}_{cl}(A)$

3. Different forms of $\mathcal{NN}$ space

In this section, some types of neutrosophic nano topological space($\mathcal{NN}$ space) were derived, with respect to the novel behavior of neutrosophic sets.

Proposition 3.1 Let X be a non-void universal set and let E be an equivalence relation on X and Y be a neutrosophic subset of X.

i. If $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}\}$ is the indiscrete $\mathcal{NN}$ space.

ii. If $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and

  a) either $Y \subseteq Y'$ or $Y \cap Y' = 0_{\mathcal{N}}$, then $NN\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y)\}$

  b) either $Y' \subset Y$ or Y is not comparable with Y', then

  $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_B(Y)\}$

iii. If $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{maj}(Y)\}$

iv. If $\mathcal{NN}_{min}(Y) \neq 0_{\mathcal{N}}$; $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$ and

a) $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]'$ (or) $[\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{min}(Y)$ (or) $\mathcal{NN}_{min}(Y) \cap [\mathcal{NN}_{min}(Y)]' = 0_{\mathcal{N}}$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_B(Y)\}$

b) $\mathcal{NN}_{min}(Y) = [\mathcal{NN}_{min}(Y)]'$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y)\}$

v. If $\mathcal{NN}_{min}(Y) \neq \mathcal{NN}_{maj}(Y)$ where $\mathcal{NN}_{min}(Y) \neq 0_{\mathcal{N}}$; $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$ and

a) $[NN_{min}(Y)]' = \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$ (or) $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{maj}(Y)$ (or) $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{min}(Y)]'$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y)\}$

b) $[\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$ (or) $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{maj}(Y)$, then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y), \mathcal{NN}_B(Y)\}$

Proof. (i) Let $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$, then $\mathcal{NN}_B(Y) = 1_{\mathcal{N}}$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}\}$.

(ii) Let $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$. (a) Suppose $Y \subseteq Y'$ then $\mathcal{NN}_B(Y) = Y$ and suppose $Y \cap Y' = 0_{\mathcal{N}}$ then $\mathcal{NN}_B(Y) = 0_{\mathcal{N}}$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y)\}$. (b) Suppose $Y' \subset Y$ or Y is not comparable with Y′ then $\mathcal{NN}_B(Y) = Y'$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_B(Y)\}$

(iii) Let $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$. Take $\mathcal{NN}_{maj}(Y) = A$, then $\mathcal{NN}_B(Y) = A$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{maj}(Y)\}$.

(iv) Let $\mathcal{NN}_{min}(Y) \neq 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$. (a) Suppose $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]'$ (or) $[\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{min}(Y)$ (or) $\mathcal{NN}_{min}(Y) \cap [\mathcal{NN}_{min}(Y)]' = 0_{\mathcal{N}}$, then $\mathcal{NN}_B(Y) = [\mathcal{NN}_{min}(Y)]'$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_B(Y)\}$. (b) Suppose $\mathcal{NN}_{min}(Y) = [\mathcal{NN}_{min}(Y)]'$, then $\mathcal{NN}_B(Y) = \mathcal{NN}_{min}(Y)$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y)\}$.

(v) Let $\mathcal{NN}_{min}(Y) \neq \mathcal{NN}_{maj}(Y)$ where $\mathcal{NN}_{min}(Y) \neq 0_{\mathcal{N}}$; $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$. (a) Suppose $[NN_{min}(Y)]' = \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$ then $\mathcal{NN}_B(Y) = \mathcal{NN}_{min}(Y)$. Suppose $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{maj}(Y)$ (or) $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{min}(Y)]'$ then $\mathcal{NN}_B(Y) = \mathcal{NN}_{maj}(Y)$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y)\}$. (b) Suppose $[\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$ (or) $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' \subset \mathcal{NN}_{maj}(Y)$, then $\mathcal{NN}_B(Y) = [\mathcal{NN}_{min}(Y)]'$. Hence $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y), \mathcal{NN}_B(Y)\}$

Remark 3.2 If there is a singleton set in the equivalence class then for any neutrosophic subset of X, there does not exist indiscrete $\mathcal{NN}$ space

4. Extremally disconnected $\mathcal{NN}$ space

In this section, we have analyzed the extremal disconnectedness in each type of neutrosophic nano topological spaces.

Definition 4.1 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) is said to be neutrosophic nano extremally disconnected ($\mathcal{NN}$ -extremally disconnected) if the neutrosophic nano closure of each neutrosophic nano open set is neutrosophic nano open in X.

Example 4.2 Let X = { ς, ζ, ξ } and $X/_E$ = {{ ς, ζ }, { ξ } be the equivalence relation on X. Let Y = {< ς, $\frac{7}{10},\frac{1}{2},\frac{3}{5}$>, < ζ, $\frac{2}{5},\frac{1}{2},\frac{3}{10}$ >, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >} be a neutrosophic subset of X. Then $\mathcal{NN}_{min}(Y)$ = {< ς, $\frac{2}{5},\frac{1}{2},\frac{3}{5}$ >, < ζ, $\frac{2}{5},\frac{1}{2},\frac{3}{5}$ >, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >}; $\mathcal{NN}_{maj}(Y)$ = {< ς, $\frac{7}{10},\frac{1}{2},\frac{3}{10}$ >, < ζ, $\frac{7}{10},\frac{1}{2},\frac{3}{10}$ >, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >}; $\mathcal{NN}_B(Y)$ = {< ς, $\frac{3}{5},\frac{1}{2},\frac{2}{5}$ >, < ζ, $\frac{3}{5},\frac{1}{2},\frac{2}{5}$ >, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >}. Now $\mathcal{NN}\tau_E(Y)$ = {$0_{\mathcal{N}}$, $1_{\mathcal{N}}$, {< ς, $\frac{2}{5},\frac{1}{2},\frac{3}{5}$ >, < ζ, $\frac{2}{5},\frac{1}{2},\frac{3}{5}$>, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$> }, {< ς, $\frac{7}{10},\frac{1}{2},\frac{3}{10}$ >, < ζ, $\frac{7}{10},\frac{1}{2},\frac{3}{10}$ >, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >}, {< ς, $\frac{3}{5},\frac{1}{2},\frac{2}{5}$ >, < ζ, $\frac{3}{5},\frac{1}{2},\frac{2}{5}$>, < ξ, $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ >}} is a $\mathcal{NN}$ topology on X, which is $\mathcal{NN}$ - extremally disconnected.

Theorem 4.3 A $\mathcal{NN}$ space (X, $NN\tau_E(Y)$) is $\mathcal{NN}$ - extremally disconnected if $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$.

Proof. Assume $\mathcal{NN}_{maj}(Y) = 1_{\mathcal{N}}$.
Case (i): If $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$, then $\mathcal{NN}_B(Y) = 1_{\mathcal{N}}$. Now, $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}\} = [\mathcal{NN}\tau_E(Y)]^c$. So, the $\mathcal{NN}$ closure of each $\mathcal{NNO}$ set is $\mathcal{NNO}$ in X. Hence (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$ -extremally disconnected.

Case(ii): If $\mathcal{NN}_{min}(Y) \neq 0_{\mathcal{N}}$ (say A), then $\mathcal{NN}_B(Y) = A'$. Now $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, A, A'\} = [\mathcal{NN}\tau_E(Y)]^c$. Since, $\mathcal{NN}_{cl}(0_{\mathcal{N}}) = 0_{\mathcal{N}}$, $\mathcal{NN}_{cl}(1_{\mathcal{N}}) = 1_{\mathcal{N}}$, $\mathcal{NN}_{cl}(A) = A$, $\mathcal{NN}_{cl}(A') = A'$, so the $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$ - extremally disconnected.

Remark 4.4 A nano topological space on a non-void universal set U is extremally disconnected if and only if $U_R(X) = U$ [13]. But in $\mathcal{NN}$ space if (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$-extremally disconnected, then $\mathcal{NN}_{maj}(Y)$ need not be equal to $1_{\mathcal{N}}$, which can be revealed by the example 4.2.

Theorem 4.5 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$-extremally disconnected if $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and if either $Y' \subseteq Y$ or $Y \cap Y' = 0_{\mathcal{N}}$.

Proof. Let $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and $Y' \subseteq Y$, then by proposition 3.1 (ii) (b), $\mathcal{NN}\tau_E(Y) = \{0_N, 1_N, Y, Y'\}$. So, $\mathcal{NN}\tau_E(Y) = [\mathcal{NN}\tau_E(Y)]'$. Hence, $\mathcal{NN}$-closure of every $\mathcal{NNO}$ set is $\mathcal{NNC}$. Now let, $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and $Y \cap Y' = 0_{\mathcal{N}}$, then by proposition 3.1 (ii) (a), $\mathcal{NN}\tau_E(Y) = \{0_N, 1_N, Y\}$. So, $[\mathcal{NN}\tau_E(Y)]' = \{0_N, 1_N, Y'\}$. Now, $\mathcal{NN}_{cl}(Y) = 1_N$.

Theorem 4.6 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) is not $\mathcal{NN}$-extremally disconnected if $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and $Y \subset Y'$.

Proof. Let $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and $Y \subset Y'$, then by proposition 3.1 (ii) (a), $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, Y\}$ and $[\mathcal{NN}\tau_E(Y)]' = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, Y'\}$. Here $\mathcal{NN}_{cl}(Y) = Y'$. Hence (X, $\mathcal{NN}\tau_E(Y)$) is not $\mathcal{NN}$-extremally disconnected.

Theorem 4.7 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) is not $\mathcal{NN}$ -extremally disconnected if $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and Y is not comparable with Y′.

Proof. Let $\mathcal{NN}_{min}(Y) = \mathcal{NN}_{maj}(Y) = Y$ and Y is not comparable with Y′, then $\mathcal{NN}_B(Y) = Y \cap Y'$ (say A). Now $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, Y, A\}$ and $[\mathcal{NN}\tau_E(Y)]^C = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, Y', A'\}$. Here $\mathcal{NN}_{cl}(Y) = A'$, which is not $\mathcal{NNO}$ and $\mathcal{NN}_{cl}(A) = Y'$, which is not $\mathcal{NNO}$. Hence (X, $\mathcal{NN}\tau_E(Y)$) is not $\mathcal{NN}$ -extremally disconnected.

Theorem 4.8 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) with $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$ is $\mathcal{NN}$-extremally disconnected if either of the cases below is true:

i) $\mathcal{NN}_{maj}(Y) = [\mathcal{NN}_{maj}(Y)]'$.

ii) $[\mathcal{NN}_{maj}(Y)]' \subset \mathcal{NN}_{maj}(Y)$.

iii) $\mathcal{NN}_{maj}(Y)$ is not comparable with $[\mathcal{NN}_{maj}(Y)]'$.

iv) $\mathcal{NN}_{maj}(Y) \cap [\mathcal{NN}_{maj}(Y)]' = 0_{\mathcal{N}}$.

Proof. Take $\mathcal{NN}_{maj}(Y) = A$. (i) Let $\mathcal{NN}_{maj}(Y) = [\mathcal{NN}_{maj}(Y)]'$, then by proposition 3.1 (iii), $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, A\} = [\mathcal{NN}\tau_E(Y)]'$. Clearly, (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$-extremally disconnected. For (ii), (iii) and (iv) $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, A\}$ and $[\mathcal{NN}\tau_E(Y)]' = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, A'\}$. Now $\mathcal{NN}_{cl}(A) = 1_{\mathcal{N}}$, which is $\mathcal{NNO}$. Hence (X, $\mathcal{NN}\tau_E(Y)$) is $\mathcal{NN}$-extremally disconnected

Theorem 4.9 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) with $\mathcal{NN}_{min}(Y) = 0_{\mathcal{N}}$ and $\mathcal{NN}_{maj}(Y) \neq 1_{\mathcal{N}}$ is not $\mathcal{NN}$-extremally disconnected if $\mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{maj}(Y)]'$.

Proof. Let $\mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{maj}(Y)]'$ then by proposition 3.1 (iii), $\mathcal{NN}\tau_E(Y) = \{0_N, 1_N, \mathcal{NN}_{maj}(Y)\}$ and $[\mathcal{NN}\tau_E(Y)]' = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, [\mathcal{NN}_{maj}(Y)]'\}$. Now $\mathcal{NN}_{cl}(\mathcal{NN}_{maj}(Y)) = [\mathcal{NN}_{maj}(Y)]'$, which is not $\mathcal{NNO}$. Hence (X, $\mathcal{NN}\tau_E(Y)$) is not $\mathcal{NN}$-extremally disconnected.

Theorem 4.10 A $\mathcal{NN}$ space (X, $\mathcal{NN}\tau_E(Y)$) = $\{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y)\}$ is $\mathcal{NN}$-extremally disconnected in either of the following cases.

i) $[\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$.

ii) $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{maj}(Y)$.

iii) $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) = [\mathcal{NN}_{maj}(Y)]' \subset [\mathcal{NN}_{min}(Y)]'$

Proof. (i) Let $[\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$, then $[\mathcal{NN}_{maj}(Y)]' \subset [\mathcal{NN}_{min}(Y)]'$. Hence $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = \mathcal{NN}_{min}(Y)$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = 1_{\mathcal{N}}$.

(ii) Let $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]' = \mathcal{NN}_{maj}(Y)$, then $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = \mathcal{NN}_{min}(Y)$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = \mathcal{NN}_{maj}(Y)$.

(iii) Let $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) = [\mathcal{NN}_{maj}(Y)]' \subset [\mathcal{NN}_{min}(Y)]'$ then $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = \mathcal{NN}_{maj}(Y)$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = \mathcal{NN}_{maj}(Y)$. Hence $\mathcal{NN}$ closure of every $\mathcal{NNO}$ set is $\mathcal{NNO}$ in X.

Theorem 4.11 A $\mathcal{NN}$ space $(X, \mathcal{NN}\tau_E(Y)) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y)\}$ is not $\mathcal{NN}$-extremally disconnected if either $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{maj}(Y)]' \subset [\mathcal{NN}_{min}(Y)]'$ or $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{maj}(Y)]' \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{min}(Y)]'$.

Proof. If $\mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{maj}(Y)]' \subset [\mathcal{NN}_{min}(Y)]'$, then $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = [\mathcal{NN}_{maj}(Y)]'$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = [\mathcal{NN}_{maj}(Y)]'$. If $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{maj}(Y)]' \subset \mathcal{NN}_{maj}(Y) \subset [\mathcal{NN}_{min}(Y)]'$, then $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = [\mathcal{NN}_{maj}(Y)]'$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = [\mathcal{NN}_{min}(Y)]'$. This follows the proof.

Theorem 4.12 A $\mathcal{NN}$ space $(X, \mathcal{NN}\tau_E(Y)) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y), \mathcal{NN}_B(Y)\}$ is $\mathcal{NN}$-extremally disconnected.

Proof. $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \mathcal{NN}_{min}(Y), \mathcal{NN}_{maj}(Y), \mathcal{NN}_B(Y)\}$ is only if either $[\mathcal{NN}_{min}(Y)]^C \subset \mathcal{NN}_{min}(Y) \subset \mathcal{NN}_{maj}(Y)$ or $\mathcal{NN}_{min}(Y) \subset [\mathcal{NN}_{min}(Y)]^C \subset \mathcal{NN}_{maj}(Y)$. In both cases $\mathcal{NN}_{cl}[\mathcal{NN}_{min}(Y)] = \mathcal{NN}_{min}(Y)$, $\mathcal{NN}_{cl}[\mathcal{NN}_B(Y)] = \mathcal{NN}_B(Y)$ and $\mathcal{NN}_{cl}[\mathcal{NN}_{maj}(Y)] = 1_{\mathcal{N}}$. This completes the proof.

5. Neutrosophic nano mixed space

Definition 5.1 A collection $\mathcal{NNM}_X$ of subsets of X is called a neutrosophic nano minimal structure ($\mathcal{NNM}_X$-structure in short) on X if $0_{\mathcal{N}} \in \mathcal{NNM}_X$ and $1_{\mathcal{N}} \in \mathcal{NNM}_X$. If $A \in \mathcal{NNM}_X$, then it is called $\mathcal{NNM}_X$ -open set (denoted by $\mathcal{NNM}_X\mathcal{O}$) and A′ is $\mathcal{NNM}_X$-closed set (denoted by $\mathcal{NNM}_X\mathcal{C}$).

Definition 5.2 Let $\mathcal{NNM}_X$ be a neutrosophic nano minimal structure on X. For $A \subseteq X$, the $\mathcal{NNM}_X$ -closure of A (denoted by $\mathcal{NNM}_{Xcl}(A)$) and $\mathcal{NNM}_X$ interior of A (denoted by $\mathcal{NNM}_{Xint}(A)$) are defined as follows.

i. $\mathcal{NNM}_{Xcl}(A) = \cap \{F : A \subset F, F' \in \mathcal{NNM}_X\}$.

ii. $\mathcal{NNM}_{Xint}(A) = \cup \{U : U \subset A, U \in \mathcal{NNM}_X\}$.

Lemma 5.3 Let $\mathcal{NNM}_X$ be a neutrosophic nano minimal structure on X. For A, $B \subseteq X$, the following holds.

i. $\mathcal{NNM}_{Xcl}(A') = [\mathcal{NNM}_{Xint}(A)]'$ and $\mathcal{NNM}_{Xint}(A') = [\mathcal{NNM}_{Xcl}(A)]'$

ii. If $A' \in \mathcal{NNM}_X$, then $\mathcal{NNM}_{Xcl}(A) = A$ and if $A \in \mathcal{NNM}_X$, then $\mathcal{NNM}_{Xint}(A) = A$

iii. $\mathcal{NNM}_{Xcl}(0_{\mathcal{N}}) = 0_{\mathcal{N}}$; $\mathcal{NNM}_{Xcl}(1_{\mathcal{N}}) = 1_{\mathcal{N}}$
$\mathcal{NNM}_{Xint}(0_{\mathcal{N}}) = 0_{\mathcal{N}}$; $\mathcal{NNM}_{Xint}(1_{\mathcal{N}}) = 1_{\mathcal{N}}$

iv. If $A \subseteq B$, then $\mathcal{NNM}_{Xcl}(A) \subseteq \mathcal{NNM}_{Xcl}(B)$ and $\mathcal{NNM}_{Xint}(A) \subseteq \mathcal{NNM}_{Xint}(B)$

v. $\mathcal{NNM}_{Xint}(A) \subseteq A \subseteq \mathcal{NNM}_{Xcl}(A)$

vi. $\mathcal{NNM}_{Xcl}[\mathcal{NNM}_{Xcl}(A)] = \mathcal{NNM}_{Xcl}(A)$ and $\mathcal{NNM}_{Xint}(\mathcal{NNM}_{Xint}(A)) = \mathcal{NNM}_{Xint}(A)$

Definition 5.4 A neutrosophic nano minimal structure on X is said to have property $\mathcal{B}$ if the union of any family of subsets in $\mathcal{NNM}_X$ is also in $\mathcal{NNM}_X$.

Lemma 5.5 Let X be a non-void universal set and $\mathcal{NNM}_X$, a neutrosophic nano minimal structure on X having property $\mathcal{B}$. For a neutrosophic subset A of X, the following properties hold.

i. $A \in \mathcal{NNM}_X$ iff $\mathcal{NNM}_{Xint}(A) = A$

ii. A is $\mathcal{NNM}_X$-closed iff $\mathcal{NNM}_{Xcl}(A) = A$

iii. $\mathcal{NNM}_{Xint}(A) \in \mathcal{NNM}_X$ and $\mathcal{NNM}_{Xcl}(A)$ is $\mathcal{NNM}_X$- closed

Definition 5.6 A $\mathcal{NN}$ space $(X, \mathcal{NN}\tau_E(Y))$ with neutrosophic nano minimal structure $\mathcal{NNM}_X$ on X is called $\mathcal{NN}$ mixed space and it is denoted by $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$.

Definition 5.7 A $\mathcal{NN}$ mixed space $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ is said to be $\mathcal{NNM}_X$ - extremally disconnected if $\mathcal{NNM}_{Xcl}(A) \in \mathcal{NN}\tau_E(Y)$ for each $\mathcal{NNO}$-set A.

Definition 5.8 A $\mathcal{NN}$ mixed space $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ is said to be $\mathcal{NNM}_X$- hyper connected if $\mathcal{NNM}_{Xcl}(A) = 1_{\mathcal{N}}$ for each $\mathcal{NNO}$-set A.

Proposition 5.9 In a neutrosophic nano mixed space $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$, if X is $\mathcal{NNM}_X$- hyper connected, then X is $\mathcal{NNM}_X$- extremally disconnected.

Proof. Let $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ be $\mathcal{NNM}_X$- hyper connected. Then $\mathcal{NNM}_{Xcl}(A) = 1_{\mathcal{N}}$ for each $\mathcal{NNO}$-set A. Since $1_{\mathcal{N}} \in \mathcal{NN}\tau_E(Y)$, X is $\mathcal{NNM}_X$-extremally disconnected.

Remark 5.10 The concepts $\mathcal{NN}$- extremally disconnected and $\mathcal{NNM}_X$ - extremally disconnected are independent.

Example 5.11 Let X = {ς, ζ, ξ} be the universal set and $X/_E$ = {{ς, ζ}, {ξ}} be the equivalence relation on X and let Y = {< ς, $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ >, < ζ, $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ >, < ξ, $\frac{3}{10}, \frac{1}{2}, \frac{3}{10}$>}. Then $\mathcal{NN}\tau_E(Y) = \{0_N, 1_N,$ {< ς, $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$>, < ζ, $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$>, < ξ, $\frac{3}{10}, \frac{1}{2}, \frac{3}{10}$ >}}. Let $\mathcal{NNM}_X = \{0_{\mathcal{N}}, 1_{\mathcal{N}},$ {< ς, $\frac{3}{10}, \frac{1}{2}, \frac{1}{5}$>, < ζ, $\frac{3}{10}, \frac{1}{2}, \frac{1}{5}$ >, < ξ, $\frac{3}{10}, \frac{1}{2}, \frac{3}{10}$ >}}. Then $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ is $\mathcal{NNM}_X$ - extremally disconnected but $(X, \mathcal{NN}\tau_E(Y))$ is not $\mathcal{NN}$-extremally disconnected.

Example 5.12 Let X = {ς, ζ, ξ, υ} and $X/_E$ = {{ς, ζ}, {ξ, υ}} be the equivalence relation on X. Take $Y = \{< \varsigma, 0, \frac{1}{2}, 1 >, < \zeta, \frac{3}{10}, 1, \frac{1}{5} >, < \xi, 0, \frac{1}{2}, 1 >, < \upsilon, \frac{2}{5}, 1, \frac{3}{5} >\}$. Then $\mathcal{NN}\tau_E(Y) = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \{< \varsigma, 0, \frac{1}{2}, 1 >, < \zeta, \frac{3}{10}, 1, \frac{1}{5} >, < \xi, 0, \frac{1}{2}, 1 >, < \upsilon, \frac{2}{5}, 1, \frac{3}{5} >\}\}$. Let $\mathcal{NNM}_X = \{0_{\mathcal{N}}, 1_{\mathcal{N}}, \{< \varsigma, \frac{1}{5}, \frac{1}{2}, \frac{3}{5} >, < \zeta, \frac{1}{10}, \frac{1}{2}, \frac{1}{5} >, < \xi, \frac{1}{5}, \frac{1}{2}, \frac{4}{5} >, < \upsilon, \frac{3}{10}, \frac{1}{2}, \frac{7}{10} >\}\}$. Then $(X, \mathcal{NN}\tau_E(Y))$ is $\mathcal{NN}$-extremally disconnected but $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ is not $\mathcal{NNM}_X$ -extremally disconnected.

Definition 5.13 A neutrosophic subset A of a neutrosophic nano mixed space $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ is said to be:

i. $\alpha - \mathcal{NNM}_X$ -open if $A \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A])$
ii. semi- $\mathcal{NNM}_X$ -open if $A \subseteq \mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A]$
iii. pre- $\mathcal{NNM}_X$ -open if $A \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,A)$
iv. $\beta - \mathcal{NNM}_X$ -open if $A \subseteq \mathcal{NN}_{cl}\,[\mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,A)]$
v. strongly-$\beta - \mathcal{NNM}_X$ -open if $A \subseteq \mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,A)]$

Proposition 5.14 If A is $\alpha - \mathcal{NNM}_X$ -open, then it is semi- $\mathcal{NNM}_X$ -open.

Proof. Let A be $\alpha - \mathcal{NNM}_X$ -open, then $A \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A]) \subseteq \mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A]$. Hence A is semi- $\mathcal{NNM}_X$ -open.

Proposition 5.15 If A is $\alpha - \mathcal{NNM}_X$ -open, then it is pre- $\mathcal{NNM}_X$ -open.

Proof. Let A be $\alpha - \mathcal{NNM}_X$ -open, then $A \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A])$. Also, $\mathcal{NN}_{int}A \subseteq A \Rightarrow \mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A] \subseteq \mathcal{NNM}_{Xcl}A \Rightarrow \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A]) \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Therefore, $A \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}A)$. Hence A is pre- $\mathcal{NNM}_X$ -open.

Proposition 5.16 If A is semi- $\mathcal{NNM}_X$ -open and pre- $\mathcal{NNM}_X$ -open, then A is $\alpha - \mathcal{NNM}_X$ -open.

Proof. Suppose A is both semi- $\mathcal{NNM}_X$ -open and pre- $\mathcal{NNM}_X$ -open. Then $A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A] \Rightarrow \mathcal{NNM}_{Xcl}A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A] \Rightarrow \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A) \subseteq \mathcal{NN}_{int}\,(\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A]) \Rightarrow A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}\,[\mathcal{NN}_{int}A])$. Hence A is $\alpha - \mathcal{NNM}_X$ -open.

Theorem 5.17 Let $(X, \mathcal{NN}\tau_E(Y), \mathcal{NNM}_X)$ be a neutrosophic nano mixed space, the following properties are equivalent.

i. X is $\mathcal{NNM}_X$ -extremally disconnected
ii. $\mathcal{NNM}_{Xint}(A)$ is $\mathcal{NNC}$ for every $\mathcal{NN}$-closed subset A of X
iii. $\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A] \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A), \forall A \subseteq X$
iv. Each semi- $\mathcal{NNM}_X$ -open set is pre- $\mathcal{NNM}_X$ -open

v. $\mathcal{NNM}_{Xcl}A \in \mathcal{NN}\tau_E(Y)$ for every strongly-$\beta - \mathcal{NNM}_X$ -open set A

vi. Each strongly-$\beta - \mathcal{NNM}_X$ -open set is pre- $\mathcal{NNM}_X$ -open

vii. A is $\alpha - \mathcal{NNM}_X$-open iff it is semi- $\mathcal{NNM}_X$ -open $\forall A \subseteq X$

Proof. (i) ⇒ (ii). Let $A \subseteq X$, such that A is $\mathcal{NNC}$ set. Then A' is $\mathcal{NNO}$ set in X. Since X is $\mathcal{NNM}_X$ -extremally disconnected, $\mathcal{NNM}_{Xcl}(A') = [\mathcal{NNM}_{Xint}(A)]'$, which is $\mathcal{NNO}$. Thus, $\mathcal{NNM}_{Xint}(A)$ is $\mathcal{NNC}$.

(ii) ⇒ (iii). Let A be any neutrosophic subset of X. Then $[\mathcal{NN}_{int}(A)]'$ is $\mathcal{NNC}$ in X. By (ii) $\mathcal{NNM}_{Xint}([\mathcal{NN}_{int}(A)]')$ is $\mathcal{NNC}$ in X ⇒ $[\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)]]'$ is $\mathcal{NNC}$ in X ⇒ $\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)]$ is $\mathcal{NNO}$ in X. Clearly, $\mathcal{NN}_{int}(A) \subseteq A \Rightarrow \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)] \subseteq \mathcal{NNM}_{Xcl}A \Rightarrow \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)]) \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A) \Rightarrow \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)] \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$.

(iii) ⇒ (iv). Let A be semi-$\mathcal{NNM}_X$-open then $A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)]$. By (iii), $\mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(A)] \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Therefore, $A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Hence A is pre-$\mathcal{NNM}_X$-open.

(iv) ⇒ (v). Let A be strongly-$\beta - \mathcal{NNM}_X$ -open set. Then $A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)] \Rightarrow \mathcal{NNM}_{Xcl}A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)]$. Hence $\mathcal{NNM}_{Xcl}A$ is semi- $\mathcal{NNM}_X$-open. By (iv), $\mathcal{NNM}_{Xcl}A$ is pre- $\mathcal{NNM}_X$-open. Therefore, $\mathcal{NNM}_{Xcl}A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}[\mathcal{NNM}_{Xcl}A]) \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Hence, $\mathcal{NNM}_{Xcl}A$ is $\mathcal{NNO}$.

(v) ⇒ (vi). Let A be strongly-$\beta - \mathcal{NNM}_X$ -open set. By (v), $\mathcal{NNM}_{Xcl}A = \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Thus $A \subseteq \mathcal{NNM}_{Xcl}A = \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. Therefore, A is pre-$\mathcal{NNM}_X$-open.

(vi) ⇒ (vii). Let A be semi-$\mathcal{NNM}_X$-open set. Then $A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A]$. Since, $A \subseteq \mathcal{NNM}_{Xcl}A \Rightarrow \mathcal{NN}_{int}A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A) \Rightarrow \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}A] \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)]$. Therefore $A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)]$. Therefore A is strongly-$\beta - \mathcal{NNM}_X$-open. Now by (vi), A is pre-$\mathcal{NNM}_X$-open. Since A is semi-$\mathcal{NNM}_X$-open and pre-$\mathcal{NNM}_X$-open, it is $\alpha - \mathcal{NNM}_X$-open.

(vii) ⇒ (i). Let A be $\mathcal{NNO}$ on X. Then $\mathcal{NN}_{int}A = A$. Now, $A \subseteq \mathcal{NNM}_{Xcl}A \Rightarrow A = \mathcal{NN}_{int}A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A) \Rightarrow \mathcal{NNM}_{Xcl}A \subseteq \mathcal{NNM}_{Xcl}[\mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)]$. Hence $\mathcal{NNM}_{Xcl}A$ is semi-$\mathcal{NNM}_X$-open. By (vii), $\mathcal{NNM}_{Xcl}A$ is $\alpha-\mathcal{NNM}_X$-open. By proposition 5.15, $\mathcal{NNM}_{Xcl}A$ is pre-$\mathcal{NNM}_X$-open. Therefore, $\mathcal{NNM}_{Xcl}A \subseteq \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}[\mathcal{NNM}_{Xcl}A]) = \mathcal{NN}_{int}(\mathcal{NNM}_{Xcl}A)$. So, $\mathcal{NNM}_{Xcl}A$ is $\mathcal{NNO}$. Hence X is $\mathcal{NNM}_X$-extremally disconnected.

6. Application

In real life situation, the extremal disconnectedness describes that, if there is an inverse change in the present state, that affects nothing of its basic properties. For example, if we collect some crops grew in our district, as a universal set. According to their biological types, split them into a partition, which gives us an equivalence class. Based on farmer's speculation (if all the growth factors are good), the yield from the crop are given as a neutrosophic values. We consider it as a neutrosophic subset of the universal set. For this, we generate the neutrosophic nano topology. The neutrosophic nano open sets, represent the lower, upper and boundary approximation of the given neutrosophic set. The neutrosophic nano closed sets give the approximation values of the growth of crops, in the reverse situation (if the growth factors are in bad condition). Now, if the $\mathcal{NN}$- closure of $\mathcal{NNO}$ -set is $\mathcal{NNO}$, then the farmer's expectation is fulfilled. Otherwise, if there is a $\mathcal{NNO}$ -set, whose closure is not $\mathcal{NNO}$, then there may be a loss in yield.

Example 6.1 Let X = {ς, ζ, ξ} be an universal set, where ς - tomato, ζ - brinjal and ξ - drum stick. X/E = {{ς, ζ}, {ξ}}. Let Y = {< ς, $\frac{7}{10}, \frac{1}{2}, \frac{3}{10}$ >, < ζ, $\frac{3}{5}, \frac{1}{2}, \frac{2}{5}$ >, < ξ, $\frac{3}{5}, \frac{1}{2}, \frac{3}{10}$ >} be a neutrosophic subset of X, represents the expected yield. Now, $NN\tau_E(Y)$ = {0_N, 1_N, {< ς, $\frac{3}{5}, \frac{1}{2}, \frac{2}{5}$ >, < ζ, $\frac{3}{5}, \frac{1}{2}, \frac{2}{5}$ >, < ξ, $\frac{3}{5}, \frac{1}{2}, \frac{3}{10}$ >}, {< ς, $\frac{7}{10}, \frac{1}{2}, \frac{3}{10}$ >, < ζ, $\frac{7}{10}, \frac{1}{2}, \frac{3}{10}$ >, < ξ, $\frac{3}{5}, \frac{1}{2}, \frac{3}{10}$ >}, {< ς, $\frac{2}{5}, \frac{1}{2}, \frac{3}{5}$ >, < ζ, $\frac{2}{5}, \frac{1}{2}, \frac{3}{5}$ >, < ξ, $\frac{3}{10}, \frac{1}{2}, \frac{3}{5}$ >}}. (X, $NN\tau_E(Y)$) is NN-extremally disconnected. Hence, we conclude that, these crops will satisfy the farmer's expectation, even though if there exists an unexpected climatic change.

Conclusion

This research has unveiled the intriguing properties of extremal disconnectedness within the newly introduced Neutrosophic Nano Mixed Space. Through the investigation of various types of open sets within this context, we have gained valuable insights into the structure and behavior of the Neutrosophic Nano Mixed Space. Moreover, the application of neutrosophic nano extremal disconnectedness in a bio-mathematical context highlights the potential practical significance of our findings. This study opens avenues for further research into the rich landscape of neutrosophic nano topological spaces and their applications across diverse fields.

References

[1] L. A. Zadeh, "Fuzzy Sets," *Information and Control,* vol. 8, pp. 338-353, 1965.

[2] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems,* vol. 20, no. 1, pp. 87-96, 1986.

[3] Smarandache.F, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic, Rehoboth : American Research Press, 1998.

[4] Chang.C.L, "Fuzzy Topological Spaces," *Journal of Mathematical Analysis and*

Applications, vol. 24, pp. 182-190.

[5] Coker and Dogan, "An Introduction to intuitionistic fuzzy topological spaces," *Fuzzy Sets and Systems,* vol. 88, no. 1, pp. 81-89, 1997.

[6] K. Serkan and K. Cemil, "Neutrosophic Topology," *Neutrosophic sets and Systems,* vol. 13, pp. 90-95, 2016.

[7] L. Gillman and M. Jerison, Rings of continuous functions, New York: Univ. Series in Higher Math., Van Nostrand, Princeton, 1960.

[8] T. Lellis and R. Carmel, "On Nano Forms of Weakly Open sets," *Internat.j.Math.and stat.Inv.,,* vol. 1, no. 1, pp. 31-37, 2013.

[9] P. Gour, T. Binod Chandra and D. Runu, "On Continuity in Minimal Structure Neutrosophic Topological Space," *Neutrosophic Sets and Systems,* vol. 51, pp. 360-370, 2022.

[10] A. Pandi, K. Banumathi and V. Vijeyarani, "Lies Between Nano Closed Sets And Nano g*-Closed Sets in Nano Minimal Structure Spaces," *International Journal of Research in Advent Technology,* vol. 6, no. 10, pp. 2568-2572, 2018.

[11] A.-. O. Ahmad, A. S. Hanan and N. Takashi, "On extremally disconnected spaces via m-structure," *Commun.Korean Math.Soc.,* vol. 34, no. 1, pp. 351-359, 2019.

[12] M. Lellis Thivagar, J. Saeid, V. Sutha Devi and V. Antony Samy, "A novel approach to nano topology via neutrosophic sets," *Neutrosophic Sets and Systems,* vol. 20, pp. 86-94, 2018.

[13] M. Lellis Thivagar, V. Ramesh and V. Antonysamy, "Extremally Disconnectedness in Nano Topology," *International Journal of Pure and Applied Mathematics,* vol. 120, no. 8, pp. 197-205, 2018.

[14] T. Witczak, "Interior and closure in anti-minimal and anti-biminimal spaces in the frame of anti-topology," *Neutrosophic Sets and Systems,* vol. 56, pp. 429-440, 2023.

[15] P. Gour, D. Runu and T. Binod Chandra, "Minimal Structures and Grill in Neutrosophic Topological Spaces," *Neutrosophic Sets and Systems,* vol. 51, pp. 134-145, 2022.

[16] P. Kalaivani and R. Nithyakala, "An Application of Nano Topological Spaces in Agriculture," *International Journal of Scientific Research in Science and Technology,* vol. 11, no. 5, pp. 292-300, 2024.

[17] A. Salama and S. Alblowi, "Generalized neutrosophic set and generalized neutrosophic topological spaces," *Journal of Computer Sci.Engineering,* vol. 2, no. 7, pp. 129-132, 2012.