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Bounds on Distance Difference Dominating Energy of a Graph

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Abstract

This work defines distance difference dominating matrix of graph and estimates characteristic values of distance difference dominating matrix. The summation of absolute eigen/characteristic values of graph's distance difference dominating matrix yields distance difference dominating energy of simple connected graph. The properties of distance difference dominating eigen values are analysed. The boundaries of distance difference dominating energy are established.

Keywords: Eigen values, distance difference dominating matrix, distance difference dominating energy.

AMS Subject Classification: 05C50, 05C69

1. Introduction

The concept of graph energy, which was first put forth by Gutman[6] in the framework of molecular orbital theory, has developed into a substantial area of research within spectral graph theory. Let $\mathcal{G}$ be simple un-directed graph with vertex set $\mathbb{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \cdots, \mathfrak{v}_\ell\}$ and edge set $\mathbb{E} = \{\mathfrak{e}_1, \mathfrak{e}_2, \cdots, \mathfrak{e}_t\}$. The adjacency matrix of $\mathcal{G}$ is given by $A = (a_{x\wp})$. Assuming in decreasing sequence, the characteristic values of $\mathcal{G}$ are $\lambda_1, \lambda_2, \ldots, \lambda_\ell$ of A. Given that A is

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symmetric and real matrix, eigen/characteristic values of matrix are real and have a total of 0. The summation of the absolute eigen/characteristic values of graph defines the energy $E(\mathcal{G})$ which is given by

$$E(\mathcal{G}) = \sum_{x=1}^{\ell} |\lambda_x|.$$

The fundamentals of domination in graphs are discussed by Teresa W. Haynes, Stephen Hedetniemi, and Peter Slater[7]. A set $M \subseteq \mathbb{V}$ is a dominating set if each vertex in the set $\mathbb{V} \backslash M$ is connected to a vertex in M. Dominating set with least number of vertices is known as minimum dominating (min-dominating) set. Domination number, denoted by $\gamma_{D^2}(\mathcal{G})$, is least number of vertices within every dominating set of a graph. The $\gamma_{D^2}(\mathcal{G})$-set of $\mathcal{G}$ is the minimal dominating set with $\gamma_{D^2}(\mathcal{G})$ vertices.

In recent decades, energies based on domination have attracted notable attention due to their capacity to represent both localized control and broader influence within networks. This research builds upon earlier foundational studies in distance energy [9], dominating energy[4,8] as well as it bounds[10], and recent developments concerning distance difference parameters [2,13].

The aim of this article is to investigate the theoretical computation of characteristic polynomial and energy of distance difference dominating matrix $A_{D^2}(\mathcal{G})$, analyse its characteristics across different types of graphs, and present significant findings related to bounds on $E_{D^2}(\mathcal{G})$ along with properties of eigen values. $E_{D^2}(\mathcal{G})$ not only extends classical energy concepts from adjacency and Laplacian matrices to dominating matrices but also introduces a distance-aware fairness perspective into domination theory, making it a powerful tool for both theoretical exploration and practical optimization in spatially-sensitive network design.

2. Distance Difference (D^2)-Dominating Energy

Let $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ denotes a simple graph with ℓ vertices and t edges. Let M be min-dominating set of a graph. A path of $\mathcal{G}$ is walk with distinct terminal vertices in which every vertex appears only once. The distance between $\mathfrak{v}_x$ and $\mathfrak{v}_\wp$ is the least number of edges that connect them. Let $D(\mathfrak{v}_x, \mathfrak{v}_\wp)$ be detour distance between $\mathfrak{v}_x$ and $\mathfrak{v}_\wp$ which is distance of longest $\mathfrak{v}_x\mathfrak{v}_\wp$-path connecting two vertices $\mathfrak{v}_x$ and $\mathfrak{v}_\wp$. We denote $d(\mathfrak{v}_x, \mathfrak{v}_\wp)$ as the geodesic distance between $\mathfrak{v}_x$ and $\mathfrak{v}_\wp$ which is distance of shortest $\mathfrak{v}_x\mathfrak{v}_\wp$-path in terms of edges connecting $\mathfrak{v}_x$ and $\mathfrak{v}_\wp$ vertices.

Distance difference dominating matrix $A_{D^2}(\mathcal{G})$ of graph $\mathcal{G}$ is $\ell \times \ell$ square matrix, denoted by $(p_{x\wp})$ whose $(x, \wp)^{\text{th}}$ entry is defined by

$$(p_{x\wp}) = \begin{cases} D(\mathfrak{v}_x, \mathfrak{v}_\wp) - d(\mathfrak{v}_x, \mathfrak{v}_\wp) & if\ \mathfrak{v}_x \neq \mathfrak{v}_\wp \\ 1 & if\ x = \wp, \mathfrak{v}_x \in M \\ 0 & otherwise \end{cases}$$

The characteristic polynomial of distance difference dominating matrix $A_{D^2}(\mathcal{G})$ is found by $\psi(\mathcal{G},\vartheta) := det(\vartheta I - A_{D^2}(\mathcal{G}))$ where I is ℓ order unit matrix and ϑ is any scalar. Characteristic equation of $A_{D^2}(\mathcal{G})$ is $\psi(\mathcal{G},\vartheta) = 0$. The roots of characteristic equation of $A_{D^2}(\mathcal{G})$ are the distance difference dominating eigen values or characteristic values of $\mathcal{G}$.

The distance difference dominating energy $E_{D^2}(\mathcal{G})$ of an undirected simple finite graph $\mathcal{G}$ is determined by adding absolute eigen/characteristic values of its distance difference dominating matrix $A_{D^2}(\mathcal{G})$.i.e. if $\vartheta_1, \vartheta_2, \dots, \vartheta_\ell$ are eigen/characteristic values of $A_{D^2}(\mathcal{G})$, distance difference dominating energy of $\mathcal{G}$ is

$$E_{D^2}(\mathcal{G}) = \sum_{x=1}^{\ell} |\vartheta_x|.$$

Since the distance difference dominating matrix $A_{D^2}(\mathcal{G})$ is symmetric and real matrix, its eigen/characteristic values are real values and are in decreasing sequence with labels $\vartheta_1 \geq \vartheta_2 \geq \dots \geq \vartheta_\ell$.

3. Distance Difference (D^2)-Dominating Energy of Some Graphs

Theorem 3.1 The distance difference dominating energy of complete graph $K_\ell(\ell \geq 3)$ is $E_{D^2}(K_\ell) = (\ell\text{-}2)^2 + \sqrt{(\ell-2)^2(\ell^2-2)+1}$.

Proof. Let $\mathbb{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \cdots, \mathfrak{v}_\ell\}$ be vertex set of a complete graph $K_\ell(\ell \geq 3)$. The min- dominating set of K_ℓ is $M = \{\mathfrak{v}_1\}$. Then D^2-dominating matrix of K_ℓ is given by

$$A_{D^2}(K_\ell) = \begin{bmatrix} 1 & \ell-2 & \ell-2 & \cdots & \ell-2 \\ \ell-2 & 0 & \ell-2 & \cdots & \ell-2 \\ \ell-2 & \ell-2 & 0 & \cdots & \ell-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \ell-2 & \ell-2 & \ell-2 & \cdots & 0 \end{bmatrix}_{\ell\times\ell}$$

The complete graph has characteristic polynomial $(\vartheta + (\ell-2))^{\ell-2}[\vartheta^2 - [(\ell-2)^2+1]\vartheta - (\ell-2)^3]$. The characteristic equation of K_ℓ is $(\vartheta + (\ell-2))^{\ell-2}[\vartheta^2 - [(\ell-2)^2+1]\vartheta - (\ell-2)^3] = 0$. The D^2 -dominating eigen values are $\vartheta = [-(\ell-2)][(\ell-2)$ times], $\vartheta = \frac{(\ell-2)^2 \pm \sqrt{(\ell\text{-}2)^2(\ell^2-2)+1}}{2}$. The distance difference dominating energy of

$K_\ell(\ell \geq 3)$ is $E_{D^2}(K_\ell) = |-(\ell-2)|(\ell\text{-}2) + \left|\frac{(\ell\text{-}2)^2+\sqrt{(\ell-2)^2(\ell^2-2)+1}}{2}\right| + \left|\frac{(\ell\text{-}2)^2-\sqrt{(\ell-2)^2(\ell^2-2)+1}}{2}\right|$

$E_{D^2}(K_\ell) = (\ell\text{-}2)^2 + \sqrt{(\ell-2)^2(\ell^2-2)+1}$

Theorem 3.2 The distance difference dominating energy of a star graph $S_{1,\ell\text{-}1}(\ell \geq 2)$ is $E_{D^2}(S_{1,\ell\text{-}1}) = 1$.

Proof. Consider a star graph $S_{1,\ell\text{-}1}$ of order ℓ with vertex set $\mathbb{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \cdots, \mathfrak{v}_\ell\}$. The min-dominating set is $M = \{\mathfrak{v}_1\}$. Then D^2-dominating matrix of a star graph $S_{1,\ell\text{-}1}$ is given by

$$A_{D^2}(S_{1,\ell-1}) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{\ell\times\ell}$$

The star graph has characteristic polynomial $\vartheta^{\ell-1}(\vartheta-1)$. The characteristic equation of $S_{1,\ell\text{-}1}$ is $\vartheta^{\ell-1}(\vartheta-1) = 0$. The D^2-dominating eigen values are $\vartheta = 0$ [$(\ell-1)$ times], $\vartheta = 1$. The distance difference dominating energy of a star graph $S_{1,\ell\text{-}1}(\ell \geq 2)$ is $E_{D^2}(S_{1,\ell\text{-}1}) = |0|(\ell-1) + |1| = 1$.

Theorem 3.3 The distance difference dominating energy of cocktail party graph $CP_{\ell\times2}(\ell \geq 3)$ is $E_{D^2}(\mathrm{CP}_{\ell\times2}) = 4\ell^2 - 8\ell + 1 + \sqrt{\kappa^4 + 4\kappa^3 + 2\kappa^2 + 4\kappa + 1}$ where $\kappa = 2\ell - 2$.

Proof. Let $CP_{\ell\times2}(\ell \geq 3)$ be cocktail party graph with vertex set $\mathbb{V} = \bigcup_{x=1}^{\ell}\{f_x, g_x\}$. The min-dominating set of $CP_{\ell\times2}$ is $M = \{f_1, g_1\}$. Then D^2-dominating matrix of $CP_{\ell\times2}$ is given by

$$A_{D^2}(CP_{\ell\times2}) = \begin{bmatrix} 1 & 2(\ell-1) & \cdots & 2(\ell-1) & 2\ell-3 & 2(\ell-1) & \cdots & 2(\ell-1) \\ 2(\ell-1) & 0 & \cdots & 2(\ell-1) & 2(\ell-1) & 2\ell-3 & \cdots & 2(\ell-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2(\ell-1) & 2(\ell-1) & \cdots & 0 & 2(\ell-1) & 2(\ell-1) & \cdots & 2\ell-3 \\ 2\ell-3 & 2(\ell-1) & \cdots & 2(\ell-1) & 1 & 2(\ell-1) & \cdots & 2(\ell-1) \\ 2(\ell-1) & 2\ell-3 & \cdots & 2(\ell-1) & 2(\ell-1) & 0 & \cdots & 2(\ell-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2(\ell-1) & 2(\ell-1) & \cdots & 2\ell-3 & 2(\ell-1) & 2(\ell-1) & \cdots & 0 \end{bmatrix}_{2\ell\times2\ell}$$

The cocktail party graph has characteristic polynomial $(\vartheta+(2\ell-4))(\vartheta+(2\ell-3))^{\ell-1}(\vartheta+(2\ell-1))^{\ell-2}[\vartheta^2-(4\ell^2-8\ell+3)\vartheta-(8\ell^3-20\ell^2+18\ell-6)]$. The characteristic equation of $CP_{\ell\times 2}$ is $(\vartheta+(2\ell-4))(\vartheta+(2\ell-3))^{\ell-1}(\vartheta+(2\ell-1))^{\ell-2}[\vartheta^2-(4\ell^2-8\ell+3)\vartheta-(8\ell^3-20\ell^2+18\ell-6)]=0$. The D^2-dominating eigen values are $\vartheta=-(2\ell-4)$, $\vartheta=-(2\ell-3)[(\ell-1)$ times], $\vartheta=-(2\ell-1)[(\ell-2)$ times], $\vartheta=\frac{(4\ell^2-8\ell+3)\pm\sqrt{\kappa^4+4\kappa^3+2\kappa^2+4\kappa+1}}{2}$ where $\kappa=2\ell-2$. The distance difference dominating energy of $CP_{\ell\times 2}(\ell\geq 3)$ is

$$E_{D^2}(\mathrm{CP}_{\ell\times 2}) = |-(2\ell-4)| + |-(2\ell-3)|(\ell\text{-}1) + |-(2\ell-1)|(\ell\text{-}2) + \left|\frac{(4\ell^2-8\ell+3)+\sqrt{\kappa^4+4\kappa^3+2\kappa^2+4\kappa+1}}{2}\right| + \left|\frac{(4\ell^2-8\ell+3)-\sqrt{\kappa^4+4\kappa^3+2\kappa^2+4\kappa+1}}{2}\right|$$

$E_{D^2}(\mathrm{CP}_{\ell\times 2})=4\ell^2-8\ell+1+\sqrt{\kappa^4+4\kappa^3+2\kappa^2+4\kappa+1}$ where $\kappa=2\ell-2$.

Theorem 3.4 The distance difference dominating energy of a $(2,\ell)$-Barbell graph $\mathcal{G}(K_\ell,K_\ell)(\ell\geq 3)$ is $E_{D^2}(\mathcal{G}(K_\ell,\mathrm{K}_\ell))=3\ell^2-10\ell+9+\sqrt{(\ell-1)(9\ell^3-35\ell^2+27\ell+15)}$

Proof. Let $\mathcal{G}(K_\ell,K_\ell)(\ell\geq 3)$ be a $(2,\ell)$-Barbell graph with vertex set $\mathbb{V}=\cup_{x=1}^{\ell}\{f_x,g_x\}$. The min-dominating set of $\mathcal{G}(K_\ell,K_\ell)$ is $M=\{f_1,g_1\}$. Then D^2-dominating matrix of $(2,\ell)$ -Barbell graph is given by

$$A_{D^2}(\mathcal{G}(K_\ell,K_\ell)) = \begin{bmatrix} 1 & \ell-2 & \ell-2 & \cdots & \ell-2 & 0 & 2\ell-4 & 2\ell-4 & \cdots & 2\ell-4 \\ \ell-2 & 0 & \ell-2 & \cdots & \ell-2 & \ell-2 & 2\ell-4 & 2\ell-4 & \cdots & 2\ell-4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \ell-2 & \ell-2 & \ell-2 & \cdots & 0 & \ell-2 & 2\ell-4 & 2\ell-4 & \cdots & 2\ell-4 \\ 0 & \ell-2 & \ell-2 & \cdots & \ell-2 & 1 & \ell-2 & \ell-2 & \cdots & \ell-2 \\ \ell-2 & 2\ell-4 & 2\ell-4 & \cdots & 2\ell-4 & \ell-2 & 0 & \ell-2 & \cdots & \ell-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \ell-2 & 2\ell-4 & 2\ell-4 & \cdots & 2\ell-4 & \ell-2 & \ell-2 & \ell-2 & \cdots & \ell-2 \end{bmatrix}_{2\ell\times 2\ell}$$

The $(2,\ell)$ -Barbell graph has characteristic polynomial $(\vartheta-1)(\vartheta+(\ell^2-2\ell))(\vartheta+(\ell-2))^{2\ell-4}[\vartheta^2-(3\ell^2-10\ell+9)\vartheta-(\ell-2)(4\ell^2-15\ell+12]$. The characteristic equation of $\mathcal{G}(K_\ell,K_\ell)$ is $(\vartheta-1)(\vartheta+(\ell^2-2\ell))(\vartheta+(\ell-2))^{2\ell-4}[\vartheta^2-(3\ell^2-10\ell+9)\vartheta-(\ell-2)(4\ell^2-15\ell+12]=0$. The D^2 -dominating eigen values are $\vartheta=1$, $\vartheta=-(\ell^2-2\ell)$, $\vartheta=-(\ell-2)[(2\ell-4)$

times], $\vartheta = \frac{(3\ell^2-10\ell+9)\pm\sqrt{9\ell^4-44\ell^3+62\ell^2-12\ell-15}}{2}$. The distance difference dominating energy of a $(2,\ell)$-Barbell graph $\mathcal{G}(K_\ell,K_\ell)(\ell \geq 3)$ is

$$E_{D^2}(\mathcal{G}(K_\ell,\mathrm{K}_\ell)) = |1| + |-(\ell^2-2\ell)| + |-(\ell-2)|(2\ell\text{-}4) + \left|\frac{(3\ell^2-10\ell+9)+\sqrt{9\ell^4-44\ell^3+62\ell^2-12\ell-15}}{2}\right| + \left|\frac{(3\ell^2-10\ell+9)-\sqrt{9\ell^4-44\ell^3+62\ell^2-12\ell-15}}{2}\right|$$

$$E_{D^2}(\mathcal{G}(K_\ell,\mathrm{K}_\ell)) = 3\ell^2-10\ell+9+\sqrt{(\ell-1)(9\ell^3-35\ell^2+27\ell+15)}$$

Theorem 3.5 The distance difference dominating energy of a globe graph $Gl(\ell)(\ell \geq 2)$ is $E_{D^2}(Gl(\ell)) = \begin{cases} \sqrt{2^{(\ell-2)}}+\sqrt{1+4\ell^4}, & \ell = 2 \\ 1+2\sqrt{2^{(\ell-2)}}+\sqrt{(2\ell+1)(2\ell+9)}, & \ell = 4,6 \\ (2\ell-1)+\sqrt{(2\ell+1)(2\ell+9)}, & \ell \neq 2,4,6 \end{cases}$

Proof. Consider a globe graph $Gl(\ell)$ of order $\ell \geq 2$ with vertex set $\mathbb{V} = \{f_1, f_2, g_1, g_2, \cdots, g_\ell\}$ where $deg(f_1) = deg(f_2) = \ell$. The minimum dominating set is $M = \{f_1, f_2\}$. Then D^2-dominating matrix of a globe graph $Gl(\ell)$ is given by

$$A_{D^2}(Gl(\ell)) = \begin{bmatrix} 1 & 0 & 2 & 2 & \cdots & 2 \\ 0 & 1 & 2 & 2 & \cdots & 2 \\ 2 & 2 & 0 & 2 & \cdots & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & 2 & \cdots & 0 \end{bmatrix}_{(\ell+2)\times(\ell+2)}$$

Case(i) $\boldsymbol{\ell = 2}$

The globe graph of order 2 has characteristic polynomial $\vartheta(\vartheta-1)^{\sqrt{2^{(\ell-2)}}}(\vartheta^2-\vartheta-s^4)$. The characteristic equation of $Gl(2)$ is $\vartheta(\vartheta-1)^{\sqrt{2^{(\ell-2)}}}(\vartheta^2-\vartheta-s^4) = 0$. The D^2-dominating eigen values are $\vartheta = 0$, $\vartheta = 1$ [$(\sqrt{2^{\ell-2}})$ times], $\vartheta = \frac{1\pm\sqrt{1+4\ell^4}}{2}$. The distance difference dominating energy of a globe graph $Gl(2)$ is

$$E_{D^2}(Gl(2)) = |0| + |1|\left(\sqrt{2^{(\ell-2)}}\right) + \left|\frac{1+\sqrt{1+4\ell^4}}{2}\right| + \left|\frac{1-\sqrt{1+4\ell^4}}{2}\right| = \sqrt{2^{(\ell-2)}}+\sqrt{1+4\ell^4}$$

Case (ii) $\boldsymbol{\ell = 4, 6}$

The globe graph has characteristic polynomial $\vartheta(\vartheta-1)(\vartheta+2)^{\sqrt{2^{(\ell-2)}}}(\vartheta^2-(2\ell-1)\vartheta-(6\ell+2))$. The characteristic equation of $Gl(\ell)$ is $\vartheta(\vartheta-1)(\vartheta+2)^{\sqrt{2^{(\ell-2)}}}(\vartheta^2-(2\ell-1)\vartheta-(6\ell+2))=0$. The D^2-dominating eigen values are $\vartheta=0$, $\vartheta=1$, $\vartheta=-2[(\sqrt{2^{(\ell-2)}})$ times], $\vartheta=\frac{(2\ell-1)\pm\sqrt{(2\ell+1)(2\ell+9)}}{2}$. The distance difference dominating energy of a globe graph $Gl(\ell)$ is

$$E_{D^2}(Gl(\ell)) = |0|+|1|+|-2|(\sqrt{2^{(\ell-2)}})+\left|\frac{(2\ell-1)+\sqrt{(2\ell+1)(2\ell+9)}}{2}\right| + \left|\frac{(2\ell-1)-\sqrt{(2\ell+1)(2\ell+9)}}{2}\right| = 1+2\sqrt{2^{\ell-2}}+\sqrt{(2\ell+1)(2\ell+9)}$$

Case(iii) $\boldsymbol{\ell \neq 2, 4, 6}$

The globe graph has characteristic polynomial $(\vartheta-1)(\vartheta+2)^{\ell-1}(\vartheta^2-(2\ell-1)\vartheta-(6\ell+2))$. The characteristic equation of $Gl(\ell)$ is $(\vartheta-1)(\vartheta+2)^{\ell-1}(\vartheta^2-(2\ell-1)\vartheta-(6\ell+2))=0$. The D^2-dominating eigen values are $\vartheta=1$, $\vartheta=-2[(\ell-1)$ times], $\vartheta=\frac{(2\ell-1)\pm\sqrt{(2\ell+1)(2\ell+9)}}{2}$. The distance difference dominating energy of globe graph $Gl(\ell)$ is $E_{D^2}(Gl(\ell))=|1|+|-2|(\ell-1)+\left|\frac{(2\ell-1)+\sqrt{(2\ell+1)(2\ell+9)}}{2}\right|+\left|\frac{(2\ell-1)-\sqrt{(2\ell+1)(2\ell+9)}}{2}\right|$ $E_{D^2}(Gl(\ell))=(2\ell-1)+\sqrt{(2\ell+1)(2\ell+9)}$

4. Properties of Distance Difference (D^2)-Dominating Eigen Values

Proposition 4.1 Let $\mathcal{G}=(\mathbb{V},\mathbb{E})$ be graph. Let M be a min-dominating set. Let $\psi(\mathcal{G},\vartheta)=c_0\vartheta^{\ell}+c_1\vartheta^{\ell-1}+\ldots+c_{\ell}$ be the characteristic polynomial of graph $\mathcal{G}$. Then

i. $c_0=1$

ii. $c_1=-|M|=-\gamma_{D^2}(\mathcal{G})$

Proof.

i. Proof of (i) follows immediately after definition of $\psi(\mathcal{G},\varepsilon)$.

ii. Since total of diagonal elements of $A_{D^2}(\mathcal{G})$ equals $\gamma_{D^2}(\mathcal{G})$, the total of determinants of all 1×1 leading principal submatrices of $A_{D^2}(\mathcal{G})$ is trace of $A_{D^2}(\mathcal{G})$ and this is efficiently the equal of $\gamma_{D^2}(\mathcal{G})$. Thus $(-1)^1c_1=\gamma_{D^2}(\mathcal{G})$.

Theorem 4.2 Let $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ be graph. If eigen/characteristic values of $A_{D^2}(\mathcal{G})$ are $\vartheta_1, \vartheta_2, \ldots, \vartheta_\ell$ then

a. $\sum_{x=1}^{\ell} \vartheta_x = \gamma_{D^2}(\mathcal{G})$

b. $\sum_{x=1}^{\ell} {\vartheta_x}^2 = 2Q + \gamma_{D^2}(\mathcal{G})$

where $Q = \sum_{x<\wp} \left[D(\mathfrak{v}_x, \mathfrak{v}_\wp) - d(\mathfrak{v}_x, \mathfrak{v}_\wp)\right]^2$ and $\gamma_{D^2}(\mathcal{G}) = |M|$.

Proof:

a. It is commonly known that the total of characteristic values of $A_{D^2}(\mathcal{G})$ equals trace of $A_{D^2}(\mathcal{G})$. As a result,

$$\sum_{x=1}^{\ell} \vartheta_x = \sum_{x=1}^{\ell} p_{xx} = \gamma_{D^2}(\mathcal{G})$$

b. Analogously, total of square of characteristic values of $A_{D^2}(\mathcal{G})$ equals trace of $[A_{D^2}(\mathcal{G})]^2$. Hence

$$\sum_{x=1}^{\ell} {\vartheta_x}^2 = \sum_{x=1}^{\ell} \sum_{\wp=1}^{\ell} (p_{x\wp} p_{\wp x})$$

$$= \sum_{x \neq \wp} (p_{x\wp} p_{\wp x}) + \sum_{x=1}^{\ell} (p_{xx})^2$$

$$= 2 \sum_{x<\wp} \left[D(\mathfrak{v}_x, \mathfrak{v}_\wp) - d(\mathfrak{v}_x, \mathfrak{v}_\wp)\right]^2 + \sum_{x=1}^{\ell} (p_{xx})^2$$

$$\sum_{x=1}^{\ell} {\vartheta_x}^2 = 2Q + \gamma_{D^2}(\mathcal{G})$$

5. Bounds on Distance Difference (D^2)-Dominating Energy

Here the lower and upper boundaries of distance difference (D^2) dominating energy are computed. The next few lemmas are applied to determine the bounds on distance difference (D^2) dominating energy.

Lemma 5.1[1] If $\alpha_1, \ldots, \alpha_\ell$ and $\beta_1, \ldots, \beta_\ell$ are positive real values, then real constants $\alpha, \beta, U,$ and V exists such that, for each $x = 1, \ldots, \ell$, $\alpha \leq \alpha_x \leq U$ and $\beta \leq \beta_x \leq V$. Then inequality that follows is true

$$\left| \ell \sum_{x=1}^{\ell} \alpha_x \beta_x - \sum_{x=1}^{\ell} \alpha_x \sum_{x=1}^{\ell} \beta_x \right| \leq \sigma(\ell)(U - \alpha)(V - \beta)$$

where $\sigma(\ell) = \ell \left\lfloor \frac{\ell}{2} \right\rfloor \left(1 - \frac{\ell}{2} \left\lfloor \frac{\ell}{2} \right\rfloor\right)$. This equality is true provided that $\alpha_1 = \alpha_2 = \ldots = \alpha_\ell$ and $\beta_1 = \beta_2 = \ldots = \beta_\ell$.

Lemma 5.2[11] Let α_1,..., α_ℓ and β_1,…, β_ℓ be non-negative real values, then

$$\sum_{x=1}^{\ell} {\alpha_x}^2 \sum_{x=1}^{\ell} {\beta_x}^2 - \left(\sum_{x=1}^{\ell} \alpha_x \beta_x\right)^2 \leq \frac{\ell^2}{4}(Z_1 Z_2 - z_1 z_2)^2$$

where $Z_1 = \max_{1\leq x\leq \ell}(\alpha_x)$; $Z_2 = \max_{1\leq x\leq \ell}(\beta_x)$; $z_1 = \min_{1\leq x\leq \ell}(\alpha_x)$ and $z_2 = \min_{1\leq x\leq \ell}(\beta_x)$.

Lemma 5.3[3] *If α_1,..., α_ℓ and β_1,..., β_ℓ are non-negative real values, then real constants r and R exists, such that, for all $x = 1,2,\ldots,\ell$, $r\alpha_x \leq \beta_x \leq R\alpha_x$. Then inequality that follows is true*

$$\sum_{x=1}^{\ell} \beta_x^2 + rR\sum_{x=1}^{\ell} \alpha_x^2 \leq (r+R)\sum_{x=1}^{\ell} \alpha_x \beta_x$$

This equality is true provided that $r\alpha_x = \beta_x = R\alpha_x$ for at least one x, $x = 1,2,\ldots,\ell$.

Lemma 5.4[12] Let α_1,…, α_ℓ and β_1,…, β_ℓ be positive real numbers, then

$$\sum_{x=1}^{\ell} {\alpha_x}^2 \sum_{x=1}^{\ell} {\beta_x}^2 \leq \frac{1}{4}\left(\sqrt{\frac{Z_1 Z_2}{z_1 z_2}} - \sqrt{\frac{z_1 z_2}{Z_1 Z_2}}\right)^2 \left(\sum_{x=1}^{\ell} \alpha_x \beta_x\right)^2$$

where $Z_1 = \max_{1\leq x\leq \ell}(\alpha_x)$; $Z_2 = \max_{1\leq x\leq \ell}(\beta_x)$; $z_1 = \min_{1\leq x\leq \ell}(\alpha_x)$ and $z_2 = \min_{1\leq x\leq \ell}(\beta_x)$.

Lemma 5.5[5] Let α_1,…, α_ℓ, β_1,…, β_ℓ, ς_1,…, ς_ℓ and θ_1,..., θ_ℓ be sequences of real values and p_1,…, p_ℓ, q_1,…, q_ℓ are nonnegative. Then inequality that follows is true

$$\sum_{x=1}^{\ell} p_x \alpha_x^2 \sum_{x=1}^{\ell} q_x \beta_x^2 + \sum_{x=1}^{\ell} p_x \varsigma_x^2 \sum_{x=1}^{\ell} q_x \theta_x^2 \geq 2\sum_{x=1}^{\ell} p_x \alpha_x \varsigma_x \sum_{x=1}^{\ell} q_x \beta_x \theta_x$$

Theorem 5.6 Suppose that $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(G) = \eta$ and $Q = \sum_{x<\wp}\left[D(\mathfrak{v}_x, \mathfrak{v}_\wp) - d(\mathfrak{v}_x, \mathfrak{v}_\wp)\right]^2$. Let $|\vartheta_1| \geq |\vartheta_2| \geq \ldots \geq |\vartheta_\ell|$ be decreasing sequence of characteristic values of $A_{D^2}(\mathcal{G})$. Then inequality that follows is true

$$E_{D^2}(\mathcal{G}) \geq \sqrt{2Q\ell + \eta\ell - \sigma(\ell)(|\vartheta_1| - |\vartheta_\ell|)^2}$$

where $\sigma(\ell) = \ell\left\lfloor\frac{\ell}{2}\right\rfloor\left(1 - \frac{\ell}{2}\left\lfloor\frac{\ell}{2}\right\rfloor\right)$, while $\lfloor\omega\rfloor$ indicates the integer portion of real value ω. Equality is preserved provided that $\mathcal{G} \cong \overline{K}_\ell$.

Proof. Let $\vartheta_1, \vartheta_2, \ldots, \vartheta_\ell$ be the eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\alpha_x = \beta_x := |\vartheta_x|$, $\alpha = \beta := |\vartheta_\ell|$ and $U = V := |\vartheta_1|$, $x = 1,2,\ldots,\ell$, in lemma 5.1, then the inequality becomes

$$\left|\ell\sum_{x=1}^{\ell}|\vartheta_x|^2-\left(\sum_{x=1}^{\ell}|\vartheta_x|\right)^2\right|\leq\sigma(\ell)(|\vartheta_1|-|\vartheta_\ell|)^2$$

Since $E_{D^2}(\mathcal{G})=\sum_{x=1}^{\ell}|\vartheta_x|$ and $\sum_{x=1}^{\ell}|\vartheta_x|^2=\sum_{x=1}^{\ell}{\vartheta_x}^2=2Q+\gamma_{D^2}(\mathcal{G})=2Q+\eta$,

$$\ell(2Q+\eta)-(E_{D^2}(\mathcal{G}))^2\leq\sigma(\ell)(|\vartheta_1|-|\vartheta_\ell|)^2$$

$$E_{D^2}(\mathcal{G})\geq\sqrt{2Q\ell+\eta\ell-\sigma(\ell)(|\vartheta_1|-|\vartheta_\ell|)^2}$$

Therefore, equality is preserved provided that $|\vartheta_1|=|\vartheta_2|=\ldots=|\vartheta_\ell|$.

Theorem 5.7 Suppose that $\mathcal{G}=(\mathbb{V},\mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(\mathcal{G})=\eta$ and $Q=\sum_{x<\wp}\left[D(\mathfrak{v}_x,\mathfrak{v}_\wp)-d(\mathfrak{v}_x,\mathfrak{v}_\wp)\right]^2$. Let $|\vartheta_1|\geq|\vartheta_2|\geq\ldots\geq|\vartheta_\ell|$ be decreasing sequence of characteristic values of $A_{D^2}(\mathcal{G})$. Then $E_{D^2}(\mathcal{G})\geq\sqrt{2Q\ell+\eta\ell-\frac{\ell^2}{4}(\vartheta_\ell-\vartheta_1)^2}$.

Proof. Let $\vartheta_1,\vartheta_2,\ldots,\vartheta_\ell$ be the eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\alpha_x=1$ and $\beta_x:=|\vartheta_x|$, $x=1,2,\ldots,\ell$, in lemma 5.2, then the inequality becomes

$$\sum_{x=1}^{\ell}1^2\sum_{x=1}^{\ell}|\vartheta_x|^2-\left(\sum_{x=1}^{\ell}|\vartheta_x|\right)^2\leq\frac{\ell^2}{4}(\vartheta_\ell-\vartheta_1)^2$$

Since $E_{D^2}(\mathcal{G})=\sum_{x=1}^{\ell}|\vartheta_x|$, $\sum_{x=1}^{\ell}1=\ell$ and $\sum_{x=1}^{\ell}|\vartheta_x|^2=\sum_{x=1}^{\ell}{\vartheta_x}^2=2Q+\gamma_{D^2}(\mathcal{G})=2Q+\eta$,

$$\ell(2Q+\eta)-(E_{D^2}(\mathcal{G}))^2\leq\frac{\ell^2}{4}(\vartheta_\ell-\vartheta_1)^2$$

$$E_{D^2}(\mathcal{G})\geq\sqrt{2Q\ell+\eta\ell-\frac{\ell^2}{4}(\vartheta_\ell-\vartheta_1)^2}$$

Theorem 5.8 Suppose that $\mathcal{G}=(\mathbb{V},\mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(\mathcal{G})=\eta$ and $Q=\sum_{x<\wp}\left[D(\mathfrak{v}_x,\mathfrak{v}_\wp)-d(\mathfrak{v}_x,\mathfrak{v}_\wp)\right]^2$. Let $|\vartheta_1|\geq|\vartheta_2|\geq\ldots\geq|\vartheta_\ell|>0$ be decreasing sequence of eigen values of $A_{D^2}(\mathcal{G})$. Then $E_{D^2}(\mathcal{G})\geq\frac{|\vartheta_1||\vartheta_\ell|\ell+2Q+\eta}{|\vartheta_1|+|\vartheta_\ell|}$. Equality is preserved provided that $\mathcal{G}\cong\overline{K}_\ell$.

Proof. Let $\vartheta_1,\vartheta_2,\ldots,\vartheta_\ell$ be the eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\beta_x:=|\vartheta_x|$, $\alpha_x=1$, $r:=|\vartheta_\ell|$ and $R:=|\vartheta_1|$, $x=1,2,\ldots,\ell$, in lemma 5.3, then the inequality becomes

$$\sum_{x=1}^{\ell}|\vartheta_x|^2+|\vartheta_1||\vartheta_\ell|\sum_{x=1}^{\ell}1^2\le(|\vartheta_1|+|\vartheta_\ell|)\sum_{x=1}^{\ell}|\vartheta_x|$$

Since $E_{D^2}(\mathcal{G})=\sum_{x=1}^{\ell}|\vartheta_x|$, $\sum_{x=1}^{\ell}1=\ell$ and $\sum_{x=1}^{\ell}|\vartheta_x|^2=\sum_{x=1}^{\ell}{\vartheta_x}^2=2Q+\gamma_{D^2}(\mathcal{G})=2Q+\eta$, we get

$$2Q+\eta+|\vartheta_1||\vartheta_\ell|\ell\le(|\vartheta_1|+|\vartheta_\ell|)E_{D^2}(\mathcal{G})$$

$$E_{D^2}(\mathcal{G})\ge\frac{|\vartheta_1||\vartheta_\ell|\ell+2Q+\eta}{|\vartheta_1|+|\vartheta_\ell|}$$

For some x, if it is true that $r\alpha_x=\beta_x=R\alpha_x$, then it follows that $\beta_x=r=R$ for that same x. This indicates that $|\vartheta_x|\le|\vartheta_\wp|\le|\vartheta_x|$ for every $\wp\neq x$. Hence equality in theorem is preserved provided that $|\vartheta_1|=|\vartheta_2|=\ldots=|\vartheta_\ell|$.

Theorem 5.9 Suppose that $\mathcal{G}=(\mathbb{V},\mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(\mathcal{G})=\eta$ and $Q=\sum_{x<\wp}\left[D(\mathfrak{v}_x,\mathfrak{v}_\wp)-d(\mathfrak{v}_x,\mathfrak{v}_\wp)\right]^2$. Let $|\vartheta_1|\ge|\vartheta_2|\ge\ldots\ge|\vartheta_\ell|>0$ be the eigenvalues of $A_{D^2}(\mathcal{G})$. Then $E_{D^2(\mathcal{G})}\ge\frac{2\sqrt{(2Q+\eta)\ell}\sqrt{\vartheta_1\vartheta_\ell}}{\vartheta_1+\vartheta_\ell}$ where ϑ_1 and ϑ_ℓ represent the lowest and highest of absolute values of ϑ_x's.

Proof. Let $\vartheta_1,\vartheta_2,\ldots,\vartheta_\ell$ be eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\alpha_x:=|\vartheta_x|$ and $\beta_x=1, x=1,2,\ldots,\ell$ in lemma 5.4, then the inequality becomes

$$\sum_{x=1}^{\ell}|\vartheta_x|^2\sum_{x=1}^{\ell}1^2\le\frac{1}{4}\left(\sqrt{\frac{\vartheta_\ell}{\vartheta_1}}+\sqrt{\frac{\vartheta_1}{\vartheta_\ell}}\right)\left(\sum_{x=1}^{\ell}|\vartheta_x|\right)^2$$

Since $E_{D^2}(\mathcal{G})=\sum_{x=1}^{\ell}|\vartheta_x|$, $\sum_{x=1}^{\ell}1=\ell$ and $\sum_{x=1}^{\ell}|\vartheta_x|^2=\sum_{x=1}^{\ell}{\vartheta_x}^2=2Q+\gamma_{D^2}(\mathcal{G})=2Q+\eta$, we get

$$\ell(2Q+\eta)\le\frac{1}{4}\left(\frac{(\vartheta_1+\vartheta_\ell)^2}{\vartheta_1\vartheta_\ell}\right)(E_{D^2}(\mathcal{G}))^2$$

$$E_{D^2(\mathcal{G})}\ge\frac{2\sqrt{(2Q+\eta)\ell}\sqrt{\vartheta_1\vartheta_\ell}}{\vartheta_1+\vartheta_\ell}$$

Theorem 5.10 Suppose that $\mathcal{G}=(\mathbb{V},\mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(\mathcal{G})=\eta$ and $Q=\sum_{x<\wp}\left[D(\mathfrak{v}_x,\mathfrak{v}_\wp)-d(\mathfrak{v}_x,\mathfrak{v}_\wp)\right]^2$. Then $E_{D^2}(\mathcal{G})\le\sqrt{\frac{\ell^2+(2Q+\eta)^2}{2}}$.

Proof. Let $\vartheta_1,\vartheta_2,\ldots,\vartheta_\ell$ be eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\alpha_x=\beta_x=p_x=q_x:=1$ and $\varsigma_x=\theta_x:=|\vartheta_x|$, $x=1,2,\ldots,\ell$, in lemma 5.5, then the inequality becomes

$$\sum_{x=1}^{\ell} 1 \sum_{x=1}^{\ell} 1 + \sum_{x=1}^{\ell} |\vartheta_x|^2 \sum_{x=1}^{\ell} |\vartheta_x|^2 \geq 2 \sum_{x=1}^{\ell} |\vartheta_x| \sum_{x=1}^{\ell} |\vartheta_x|$$

Since $E_{D^2}(\mathcal{G}) = \sum_{x=1}^{\ell} |\vartheta_x|$, $\sum_{x=1}^{\ell} 1 = \ell$ and $\sum_{x=1}^{\ell} |\vartheta_x|^2 = \sum_{x=1}^{\ell} {\vartheta_x}^2 = 2Q + \gamma_{D^2}(\mathcal{G}) = 2Q + \eta$, we get

$$\ell^2 + (2Q + \eta)^2 \geq 2(E_{D^2}(\mathcal{G}))^2$$

$$E_{D^2}(\mathcal{G}) \leq \sqrt{\frac{\ell^2 + (2Q + \eta)^2}{2}}$$

Theorem 5.11 Suppose that $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ is graph on ℓ vertices, $\gamma_{D^2}(\mathcal{G}) = \eta$ and $Q = \sum_{x<\wp} \left[D(\mathfrak{v}_x, \mathfrak{v}_\wp) - d(\mathfrak{v}_x, \mathfrak{v}_\wp)\right]^2$. Then $E_{D^2}(\mathcal{G}) \leq \frac{\ell + 2Q + \eta}{2}$.

Proof. Let $\vartheta_1, \vartheta_2, \ldots, \vartheta_\ell$ be eigen values of $A_{D^2}(\mathcal{G})$. If we substitute $\alpha_x = \beta_x = \theta_x = p_x = q_x := 1$ and $\varsigma_x := |\varepsilon_x|$, $x = 1,2,\ldots,\ell$, in lemma 5.5, then the inequality becomes

$$\sum_{x=1}^{\ell} 1 \sum_{x=1}^{\ell} 1 + \sum_{x=1}^{\ell} |\vartheta_x|^2 \sum_{x=1}^{\ell} 1 \geq 2 \sum_{x=1}^{\ell} |\vartheta_x| \sum_{x=1}^{\ell} 1$$

Since $E_{D^2}(\mathcal{G}) = \sum_{x=1}^{\ell} |\vartheta_x|$, $\sum_{x=1}^{\ell} 1 = \ell$ and $\sum_{x=1}^{\ell} |\vartheta_x|^2 = \sum_{x=1}^{\ell} {\vartheta_x}^2 = 2Q + \gamma_{D^2}(\mathcal{G}) = 2Q + \eta$, we get

$$\ell^2 + (2Q + \eta)\ell \geq 2(E_{D^2}(\mathcal{G}))\ell$$

$$E_{D^2}(\mathcal{G}) \leq \frac{\ell + 2Q + \eta}{2}$$

6. Application

Network Design & Optimization facilitates the strategic arrangement of resources to lessen the average or maximum distance to any node and involves the strategic allocation of resources within a network to meet certain objectives. The distance difference dominating matrix assists in assessing and enhancing the performance of a selected dominating set, particularly regarding distance-based balance. In scenarios such as sensor networks or server placements, the aim is to reduce the distance variance from any node to the nearest and furthest server or sensor. The distance difference dominating matrix aids in evaluating and improving such configurations.

The distance difference dominating energy serves as a method for examining and addressing issues related to control, influence, or coverage within a network depicted as a graph. It offers a matrix-centric approach for investigating dominating sets, which play a vital role in guaranteeing that every node in a network is directly connected. A low E_{D^2} implies that most nodes are nearly equidistant, indicating well-balanced coverage, which is desirable in critical systems like emergency service placement, wireless sensor networks, or logistics. Conversely, a high E_{D^2} reveals significant imbalance, where certain nodes are disproportionately far from other nodes, which could lead to inefficiencies or service delays.

In models of influence spreading, distance difference dominating matrix can clarify how rapidly or effectively influence can spread from dominators to other nodes. In chemical graph theory which is utilized to depict molecular structures, the domination concepts correspond to reactive sites and distances may indicate molecular characteristics. Hence the notion of distance difference dominating matrix has various practical application.

Conclusion

In this paper, the distance difference dominating energy $E_{D^2}(\mathcal{G})$ is determined which is a refined spectral measure that captures the structural efficiency of a graph's dominating set. It is derived from the eigen values of difference dominating matrix $A_{D^2}(\mathcal{G})$, which encodes the difference between the maximum and minimum distances between vertices. The computation of $E_{D^2}(\mathcal{G})$ for some familiar graphs is accomplished and properties of distance difference dominating eigen values are discussed. Finally, the boundaries of $E_{D^2}(\mathcal{G})$ are set forth. Further research extensions in $E_{D^2}(\mathcal{G})$ include examining the connections between $E_{D^2}(\mathcal{G})$ and various dominating energy concepts, investigating the spectrum (eigenvalues) of the distance difference dominating matrix, and developing efficient algorithms to calculate $E_{D^2}(\mathcal{G})$ for large graphs.

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Conflict of Interest

The authors hereby declare no potential conflicts of interest with respect to the research, funding, authorship, and/or publication of this article