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Effect of Non-Uniform Temperature Gradient on Rayleigh–Bénard – Marangoni – Magnetoconvection in a Micropolar Fluid with Maxwell – Cattaneo Law

R V Kiran* and Kalyani Attluri†

Abstract

The effect of non-uniform temperature gradient on the onset of Rayleigh-Bénard-Marangoni- Magneto-convection in a Micropolar fluid with Maxwell-Cattaneo law is studied using the Galerkin technique. The eigen value is obtained for rigid-free velocity boundary combination with isothermal and adiabatic condition on the spin-vanishing boundaries. A linear stability analysis is performed. The influence of various parameters on the onset of convection has been analyzed. One linear and five non-linear temperature profiles are considered and their comparative influence on onset is discussed. The classical approach predicts an infinite speed for the propagation of heat. The present non-classical theory involves a wave type heat transport (Second Sound) and does not suffer from the physically unacceptable drawback of infinite heat propagation speed.

Keywords: Rayleigh-Bénard-Marangoni-Magneto-convection, Maxwell-Cattaneo Law, Micropolar fluid

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* Christ Junior College, Hosur Road, Bangalore 560 029, India; *kiran.rv@cjc.christcollege.edu*

† Department of Mathematics, Christ University, Hosur Road, Bangalore 560 029, India; *kalyani.attluri@maths.christuniversity.in*

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1. Introduction

The instability of Rayleigh-Bénard convection is due to the effect of thermal buoyancy. Theoretical studies of the onset of convection in classical viscous fluids with non-uniform heating have been made by Currie [1] with isothermal boundaries and by Nield [2] with adiabatic boundaries and showed that in the case of piecewise linear temperature profile the onset of convection could occur at a smaller Rayleigh number than of uniform heating or cooling. The non-uniform temperature gradient finds its origin in the transient heating or cooling at the boundaries and as a result the basic temperature profile depends explicitly on position. This has to be determined by solving the coupled momentum and energy equations. This coupling makes the problem very complicated. In the present study, therefore, we adopt a series of temperature profiles based on a simplification in the form of a quasi – static approximation (Currie [1], Lebon and Cloot [3]) that consists of freezing the temperature distribution at a given instant of time. In this method, we assume that the perturbation grows much faster than the initial state and hence freeze the initial state into some spatial distribution. This hypothesis is sufficient for our purpose because we are interested only in finding the conditions for the onset of convection. Even with these simplifications, the solutions to the variable-coefficients stability equation pose a problem because the temperature gradient varies with depth.

Marangoni convection resulting from the local variation of surface tension due to a non-uniform temperature distribution is an interesting fluid mechanical problem. It has many important applications in a number of engineering problems, such as energy storage in molten salts, crystal growth from a melt in microgravity conditions, and paints, colloids and detergents in chemical engineering. The first theoretical study on steady Marangoni convection in a planar fluid layer with a non-deformable free upper surface was made by Pearson [4]. Takashima [5, 6], Smith [7], Perez-Garcia and Carneiro [8] subsequently extended the stability analysis of Pearson [1] studied the effect of free surface deformation on the stationary/oscillatory Marangoni convection. Later, many authors Rudraiah [9], Maekawa and Tanasawa [10] and studied the onset of convection in a horizontal layer of Newtonian fluid driven by both surface tension variations and buoyancy force by considering the non-deformable surfaces. The corresponding problem with deformable boundaries is studied by Sarma [11, 12], Wilson [13, 14], Hashim and Wilson [15] and Rudraiah and Siddheshwar [16].

Convection in Micropolar fluid has been the subject of intensive study because of the remarkable physical properties of the fluid as well as its practical applications (see Power [17], Lukaszewicz [18] and Eringen [19]). Rayleigh-Bénard convection in Micropolar fluid has been studied by many authors [20-25]. The main results from all these studies are that for heating from below stationary convection is the preferred mode. The effect of non-uniform temperature gradient on the onset of Rayleigh-Bénard/Marangoni convection in a Micropolar fluid is investigated by Siddheshwar and Pranesh [26, 27]. The effect of non-uniform temperature gradients on the onset of Rayleigh-Bénard-Marangoni-Electro-convection in a micropolar fluid is investigated by Pranesh and Riya Baby [28] and more, recently Pranesh and Joseph [29]. All the above reported works are with classical Fourier heat flux law.

A well known consequence of Classical Fourier heat conduction law is that heat perturbations propagate with an infinite velocity. This drawback of the classical law motivated Maxwell [30], Cattaneo [31], Lindsay and Stranghan [32], Straughan and Franchi [33] and Pranesh and Kiran [34, 35, 36, 37], to adopt a non-classical heat flux Maxwell-Cattaneo law in studying Rayleigh-Bénard / Marangoni convection to get rid of this unphysical results. This Maxwell-Cattaneo equation contains an extra inertial term with respect to the Fourier law

$$\tau \frac{d\vec{Q}}{dt} + \vec{Q} = -\kappa \nabla T$$

where, $\vec{Q}$ is the heat flux, τ is a relaxation time and κ is the heat conductivity. This heat conductivity equation and the conservation of energy equation introduce the hyperbolic equation, which describes heat propagation with finite speed. Puri and Jordan [38, 39], Puri and Kythe [40, 41] and Straughan [42] have studied other fluid mechanics problems by employing the Maxwell-Cattaneo heat flux law. The above mentioned works are with linear temperature gradient.

The objective of this paper is to replace the classical parabolic heat equation by non-classical Maxwell-Cattaneo Law and study the effect of non-uniform basic temperature gradients on Rayleigh-Bénard-Marangoni-Magneto convection in Micropolar fluids.

2. Mathematical Formulation

Consider an infinite horizontal layer of a Boussinesquian, electrically conducting fluid, with non-magnetic suspended particle of depth 'd' permeated by an externally applied uniform magnetic field normal to the fluid layer. Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards. Let ΔT be the temperature difference between the upper and lower boundaries. (See Figure (1)).

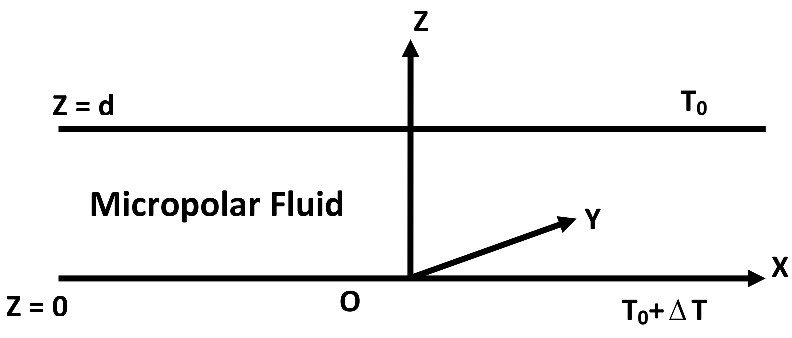

Fig 1. Schematic diagram of the Rayleigh-Bénard situation for a Micropolar fluid.

The governing equations for the Rayleigh-Bénard situation in a Boussinesquian fluid with suspended particles are

Continuity equation:

$$\nabla.\vec{q} = 0, \quad (1)$$

Conservation of linear momentum:

$$\rho_o\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g\hat{k} + (2\zeta + \eta)\nabla^2\vec{q} + \zeta\nabla \times \vec{\omega} + \mu_m(\vec{H}.\nabla)\vec{H}, \quad (2)$$

Conservation of angular momentum:

$$\rho_o I\left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + (\eta'\nabla^2\vec{\omega}) + \zeta(\nabla \times \vec{q} - 2\vec{\omega}), \quad (3)$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + \left(\vec{q} - \frac{\beta}{\rho_o C_v}\nabla \times \vec{\omega}\right).\nabla T = -\nabla.\vec{Q}, \quad (4)$$

Maxwell – Cattaneo heat flux law:

$$\tau\left[\dot{\vec{Q}} + \vec{\omega}_1 \times \vec{Q}\right] = -\vec{Q} - \kappa\nabla T, \quad (5)$$

Magnetic Induction equation:

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = (\vec{H}.\nabla)\vec{q} + \gamma_m\nabla^2\vec{H}, \quad (6)$$

Equation of state:

$$\rho = \rho_o[1 - \alpha(T - T_o)]. \quad (7)$$

where, $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, P is the hydromagnetic pressure, ρ is the density, ρ_o is the density of the fluid at reference temperature $T = T_o$, $\gamma_m = \frac{1}{\mu_m \sigma}$, μ_m is magnetic permeability, g is the acceleration due to gravity, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are the bulk and shear spin viscosity coefficient, β is the Micropolar heat conduction coefficient, C_v is the specific heat, κ is the thermal conductivity, α is the co – efficient of thermal expansion, $\vec{\omega}_1 = \frac{1}{2}\nabla \times \vec{q}$, $\vec{Q}$ is the heat flux vector and τ is the constant relaxation time.

3. Basic State

The basic state of the fluid being quiescent is described by

$$\left.\begin{array}{l} \vec{q}_b = 0,\ \vec{\omega}_b = 0,\ P = P_b(z),\ \rho = \rho_b(z),\ \vec{\omega} = \vec{\omega}_b(z), \\ \vec{H} = H_0\hat{k},\ \vec{Q} = \left(0, 0, Q_b(z)\right),\ \frac{dT_b}{dz} = \frac{-\Delta T}{d} f(z). \end{array}\right\} \tag{8}$$

The monotonic, non-dimensional basic temperature gradient f(z)which is non-negative satisfies the condition $\int_0^1 f(z)dz = 1$. We have considered various steady state temperature gradients in this paper and these are defined below.

Table (1): Basic-State Temperature Gradients

Model	Basic temperature gradients	f(z)
	Linear	1
	Heating from below	$\begin{cases} \varepsilon^{-1} & 0 \le z < \varepsilon \\ 0 & \varepsilon < z \le 1 \end{cases}$
	Cooling from above	$\begin{cases} 0 & 0 \le z < 1-\varepsilon \\ \varepsilon^{-1} & 1-\varepsilon < z \le 1 \end{cases}$
	Step function	$\delta(z-\varepsilon)$
	Inverted Parabolic	$2(1-z)$
	Parabolic	$2z$

Equations (2), (4), (5) and (7) in the basic state specified by equation (8) respectively become

$$\left.\frac{dP_b}{dz} = -\rho_o g\hat{k},\ \frac{d\vec{Q}_b}{dz} = 0,\ \vec{Q}_b = -\kappa\frac{dT_b}{dz},\ \rho_b = \rho_o[1-\alpha(T_b - T_o)],\ \frac{d^2T_b}{dz^2} = 0.\right\} \tag{9}$$

4. Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\left.\begin{aligned}&\vec{q} = \vec{q}_b + \vec{q}',\ \vec{\omega} = \vec{\omega}_b + \vec{\omega}',\ P = P_b + P',\ \vec{Q} = \vec{Q}_b + \vec{Q}',\\ &\rho = \rho_b + \rho',\ T = T_b + T',\ \vec{H} = \vec{H}_0 + \vec{H}'.\end{aligned}\right\} \tag{10}$$

The primes indicate that the quantities are infinitesimal perturbations and subscript 'b' indicates basic state value.

Substituting equation (10) into equations (1) – (7) and using the basic state (9), we get linearised equation governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q}' = 0, \tag{11}$$

$$\rho_o\left[\frac{\partial \vec{q}'}{\partial t}\right] = -\nabla P' - \rho' g\hat{k} + (2\zeta + \eta)\nabla^2\vec{q}' + (\zeta\nabla\times\vec{\omega}') + \mu_m(H_0\hat{k}.\nabla)\vec{H}', \tag{12}$$

$$\rho_o I\left[\frac{\partial \vec{\omega}'}{\partial t}\right] = (\lambda' + \eta')\nabla(\nabla\vec{\omega}') + (\eta'\nabla^2\vec{\omega}') + \zeta(\nabla\times\vec{q}' - 2\vec{\omega}'), \tag{13}$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d} f(z)\left[\vec{q}' - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}'\right] - \nabla.\vec{Q}', \tag{14}$$

$$\left[1+\tau\frac{\partial}{\partial t}\right]\vec{Q}' = -\frac{1}{2}\chi_1\frac{\Delta T}{d}\left(\frac{\partial\vec{q}'}{\partial z}-\nabla W'\right)-\kappa\,\nabla T', \tag{15}$$

$$\frac{\partial\vec{H}'}{\partial t} = \gamma_m\nabla^2\vec{H}' + H_0\frac{\partial\vec{q}'}{\partial z}, \tag{16}$$

$$\rho' = -\alpha\rho_o T'. \tag{17}$$

where, $\chi_1 = \tau\kappa$

Operating divergence on the equation (15) and substituting in equation (14), on using equation (11), we get

$$\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial T'}{\partial t} = \left(1+\tau\frac{\partial}{\partial t}\right)\frac{\Delta T}{d}f(z)\left(W'-\frac{\beta}{\rho_0 C_v}\Omega_z\right)+\kappa\nabla^2 T'-\frac{1}{2}\chi_1\frac{\Delta T}{d}f(z)\left(\nabla^2 W'\right), \tag{18}$$

where, $\Omega = \nabla\times\vec{\omega}'$

The perturbation equation (12), (13), (16) and (18) are non – dimensionalised using the following definition:

$$\left.\begin{aligned}&(x^*,y^*,z^*) = \frac{(x,y,z)}{d},\ W^* = \frac{W'}{\left(\kappa/d\right)},\ \vec{\omega}^* = \frac{\omega'}{\left(\kappa/d^2\right)},\ t^* = \frac{t}{\left(d^2/\kappa\right)},\\ &T^* = \frac{T'}{\Delta T},\ \Omega^* = \frac{\Omega_z}{\left(\kappa/d^3\right)},\ H^* = \frac{H'}{H_0}.\end{aligned}\right\} \tag{19}$$

Using equation (17) in (12), operating curl twice on the resulting equation, operating curl once on equation (13) and non-dimensionalising the two resulting equation and also equations (16) and (18), we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R\nabla_1^2 T + (1+N_1)\nabla^4 W + N_1\nabla^2\Omega_z + Q\frac{Pr}{Pm}\nabla^2\left(\frac{\partial H_z}{\partial z}\right), \tag{20}$$

$$\frac{N_2}{Pr}\frac{\partial}{\partial t}(\Omega_z) = N_3\nabla^2\Omega_z - N_1\nabla^2 W - 2N_1\Omega_z, \tag{21}$$

$$\frac{\partial H_z}{\partial t} = \frac{\partial W}{\partial z} + \frac{Pr}{Pm}\nabla^2 H_z, \tag{22}$$

$$\left(1+2C\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} = \left(1+2C\frac{\partial}{\partial t}\right)f(z)W - \left(1+2C\frac{\partial}{\partial t}\right)N_5\, f(z)\Omega_z + \nabla^2 T - C\,f(z)\nabla^2 W, \tag{23}$$

where the asterisks have been dropped for simplicity and the non-dimensional parameters N_1, N_3, N_5, R, Q, Pr, Pm and C are as defined as

$N_1 = \frac{\zeta}{\zeta+\eta}$ (Coupling Parameter),

$N_3 = \frac{\eta'}{(\zeta+\eta)d^2}$ (Couple Stress Parameter),

$N_5 = \frac{\beta}{\rho_o C_v d^2}$ (Micropolar Heat Conduction Parameter),

$R = \frac{\rho_o \alpha g \Delta T d^3}{\kappa(\zeta+\eta)}$ (Rayleigh number),

$Q = \frac{\mu_m \vec{H}_o d^2}{\gamma_m(\zeta+\eta)}$ (Chandrasekhar Number),

$Pr = \frac{\zeta+\eta}{\rho_o \kappa}$ (Prandtl Number),

$Pm = \frac{\zeta+\eta}{\rho_o \gamma_m}$ (Magnetic Prandtl Number),

$C = \frac{\tau\kappa}{2d^2}$ (Cattaneo Number).

The infinitesimal perturbation W, Ω_z, H_z and T are assumed to be periodic waves and hence these permit a normal mode solution in the form

$$\begin{bmatrix} W \\ \Omega_z \\ H_z \\ T \end{bmatrix} = \begin{bmatrix} W(z)e^{i(lx+my)} \\ G(z)e^{i(lx+my)} \\ H_z(z)e^{i(lx+my)} \\ T(z)e^{i(lx+my)} \end{bmatrix} \tag{24}$$

where, l and m are horizontal components of the wave number $\vec{a}$,

Substituting equation (24) into equations (20)-(23), we get

$$(1+N_1)(D^2-a^2)^2 W + N_1(D^2-a^2)G - Ra^2T + Q\frac{Pr}{Pm}(D^2-a^2)(DH_z) = 0, \tag{25}$$

$$2N_1G - N_3(D^2-a^2)G + N_1(D^2-a^2)W = 0, \tag{26}$$

$$DW + \frac{Pr}{Pm}(D^2-a^2)H_z = 0, \tag{27}$$

$$f(z)(W - N_5G) + (D^2-a^2)T - C\,f(z)(D^2-a^2)W = 0. \tag{28}$$

where, $D = \frac{d}{dz}$

Eliminating H_z between equations (25) and (27), we get

$$(1+N_1)(D^2-a^2)^2 W + N_1(D^2-a^2)G - Ra^2T - QD^2W = 0, \tag{29}$$

The equations (26),(28) and (29) are solved subject to the following boundary conditions

$$\left.\begin{array}{ll} W = DW = T = G = 0 & \text{at} \quad z = 0 \\ W = D^2W + a^2MT = DT = G = 0 & \text{at} \quad z = 1 \end{array}\right\} \tag{30}$$

where: M is the Marangoni number. The condition on G is the spin-vanishing boundary condition.

We now apply the single term Galerkin expansion to find the critical eigen value M_c and the equations (26), (28) and (29) that gives general results on the eigen value of the problem for various basic temperature gradients using simple, polynomial, trial functions for the lowest eigen value. Now we multiplying equation (29) by W, equation (26) by G and equation (28) by T, integrating the resulting equation by parts with respect to z from 0 to 1, using boundary condition (30) and taking $W = AW_1$, $G = BG_1$ and $T = CT_1$ in which A, B and C are constants with W_1, G_1 and T_1 are trial functions. This procedure yields the following equation for the Marangoni number M:

$$M = \frac{\left[\dfrac{Ra^2 \langle W_1 T_1 \rangle \, Y_4}{\langle T_1 (D^2 - a^2) T_1 \rangle} - \left[Y_1 . Y_2 + N_1^2 . Y_3 \right] \right] \langle T_1 (D^2 - a^2) T_1 \rangle}{(1 + N_1) a^2 DW_1(1) T_1(1) \, Y_4} \tag{31}$$

Where

$$Y_1 = (1 + N_1) \left[\langle (D^2 W_1)^2 \rangle - 2a^2 \langle W_1 D^2 W_1 \rangle + a^4 \langle W_1^2 \rangle \right] - Q \langle W_1 D^2 W_1 \rangle,$$

$$Y_2 = N_3 \langle G_1 (D^2 - a^2) G_1 \rangle - 2N_1 \langle G_1^2 \rangle,$$

$$Y_3 = \langle G_1 (D^2 - a^2) W_1 \rangle \langle W_1 (D^2 - a^2) G_1 \rangle,$$

$$Y_4 = N_1 N_5 \langle G_1 (D^2 - a^2) W_1 \rangle \langle f(z) T_1 G_1 \rangle + Y_2 \left[C . \langle T_1 f(z) (D^2 - a^2) W_1 \rangle - \langle f(z) T_1 W_1 \rangle \right]$$

In the equation (31), $\langle - - - \rangle$ denotes integration with respect to z between $z = 0$ and $z = 1$.

The value of critical Marangoni number depends on the boundaries. In this paper we consider the rigid-free isothermal/adiabatic, no-spin boundary combinations. The trial functions satisfying the boundary conditions are:

Boundary conditions	**Rigid–free**
W_1	$z^2(1-z^2)$
G_1	$z(1-z)$
Isothermal T_1	$z(2-z)$
Adiabatic T_1	1

5. Results and Discussion

In this paper, we study the classical Rayleigh-Bénard-Marangoni-Magneto convection in micropolar fluids in presence of non-uniform temperature gradients by replacing the classical Fourier heat flux law by a non-classical Maxwell-Cattaneo heat flux law. Keeping in mind the laboratory and geophysical problem, the rigid-free boundary shave been investigated with isothermal/adiabatic and no-spin condition.

One uniform and five non-uniform basic temperature gradients are chosen for study. On the basis of this following grouping of non-uniform temperature profile can be made for rigid-free boundary.

Group 1	**Group 2**	**Group 3**
Linear (M_{C1}) Inverted parabolic (M_{C5}) Parabolic (M_{C6})	Piecewise linear heating from below (M_{C2}) Piecewise linear cooling from above (M_{C3})	Step function (M_{C4})

In the case of rigid-free boundaries (non-symmetric boundary combinations) we find that,

$M_{C3} < M_{C4} < M_{C6} < M_{C2} < M_{C1} < M_{C5}$

i.e., for non-symmetric boundaries we find that the cooling from above is the most destabilizing basic temperature and inverted parabolic is the most stabilizing basic temperature distribution. Figures (2)-(9) are the plot of critical Marangoni number M_C versus Cattaneo number C, for different values of coupling parameter N_1, couples stress parameter N_3, Micropolar heat condition parameter N_5 and Chandrasekhar number Q and for different basic temperature gradient for rigid-free boundaries respectively. From these figures following observation are made:

1. As C increases M_C decreases. C is the scaled relaxation time and it accelerates the onset of convection. Also it is observed that C which represents second sound which has a destabilizing influence. Increase in Cattaneo number leads to narrowing of the convection cells and thus lowering of the critical Marangoni number. It is also observed from the figures that influence of Cattaneo number is dominant for small values because the convection cells have fixed aspect ratio.
2. The increase in N_1 increases M_C. Increase in N_1 indicates the increase in the concentration of microelements. These elements consume the greater part of the energy in forming the gyrational velocities and as a result the onset of convection is delayed. From these we conclude that increases N_1 is to stabilize the system.
3. As N_3 increases M_C decreases, because when N_3 increases the couple stress of the fluid increases, which causes the microroation to decrease. Therefore, increase in N_3 destabilizes the system.
4. When N_5 increases the heat induced in to the fluid due to these microelements also increases, thus reducing the heat transfers from bottom to top. The decrease in heat transfer is responsible for delaying onset of instability. Therefore, increase in N_5 increase in M_C and thereby stabilizes the system.
5. As Q increases the M_C also increases. From this we conclude that Q has stabilizing effect on the system.

6. Conclusion

Following conclusions are drawn from the problem:

1) Cattaneo number which represents scaled relaxation time destabilizes the system.
2) The cooling form above basic temperature profile is most destabilizing temperature profile. The inverted temperature profile is the most stabilizing temperature profile.

3) By creating conditions for appropriate basic temperature gradients we can also make a prior decision on advancing or delaying convection.
4) By adjusting the Chandrasekhar number Q we can control the convection.
5) Rayleigh-Bénard-Marangoni convection in Newtonian fluids may be delayed by adding micron sized suspended particles.
6) The non-classical Maxwell-Cattaneo heat flux law involves a hyperbolic type heat transport equation that predicts finite speeds of heat wave propagation. Hence it does not suffer from the physically unacceptable drawback of infinite heat propagation speed predicted by the parabolic heat equation. The classical Fourier flux law overpredicts the critical Marangoni number compared to that predicted by the non-classical law.

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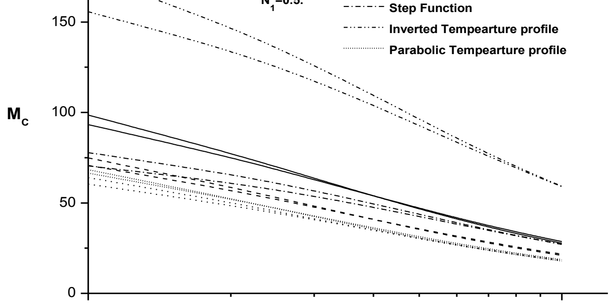

Fig 2: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free isothermal no-spin boundary condition for different values of coupling parameter N_1 and different non-uniform basic temperature gradients.

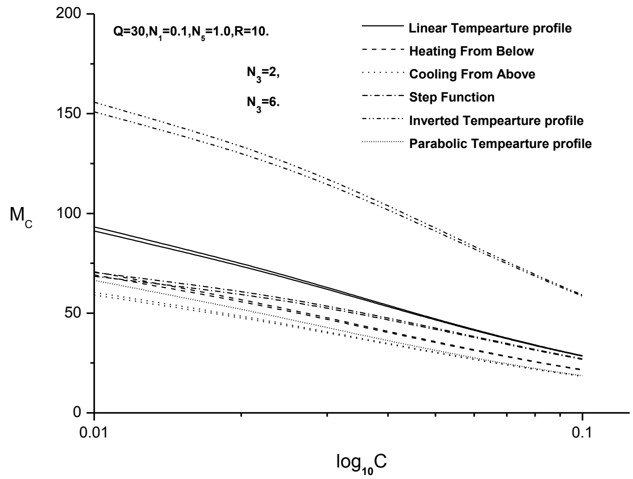

Fig 3**:** Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free isothermal no-spin boundary condition for different values of couple stress parameter N_3 and different non-uniform basic temperature gradients.

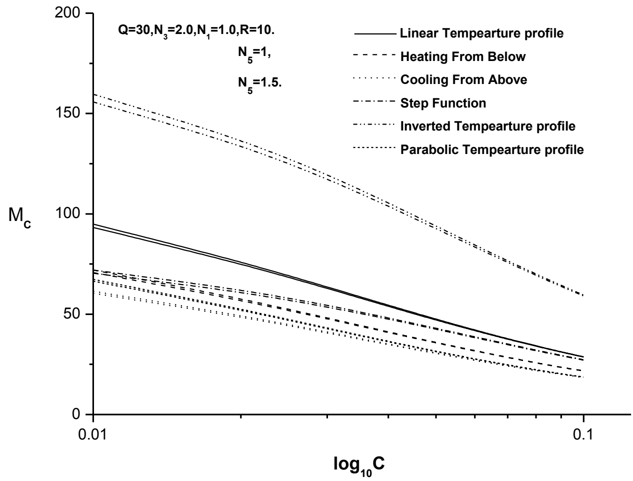

Fig 4: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free isothermal no-spin boundary condition for different values of micropolar heat conduction parameter N_5 and different non-uniform basic temperature gradients.

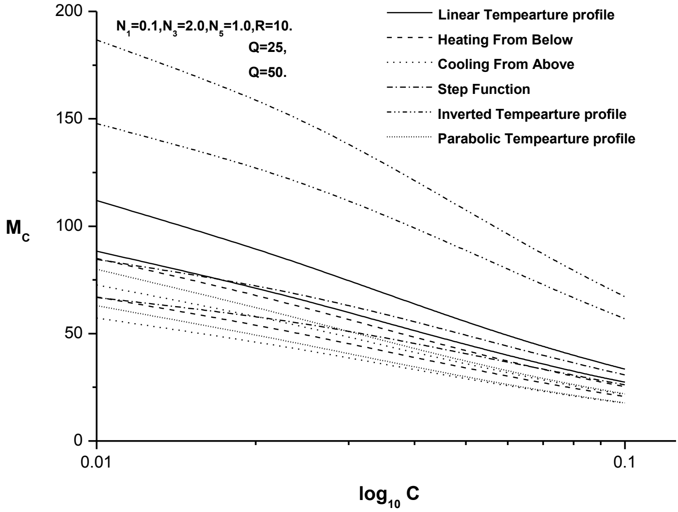

Fig 5: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free isothermal no-spin boundary condition for different values of Chandrasekhar number Q and different non-uniform basic temperature gradients.

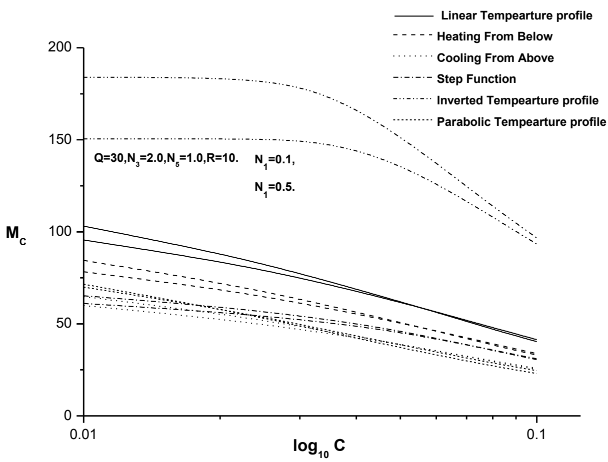

Fig 6: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free adiabatic no-spin boundary condition for different values of coupling parameter N_1 and different non-uniform basic temperature gradients.

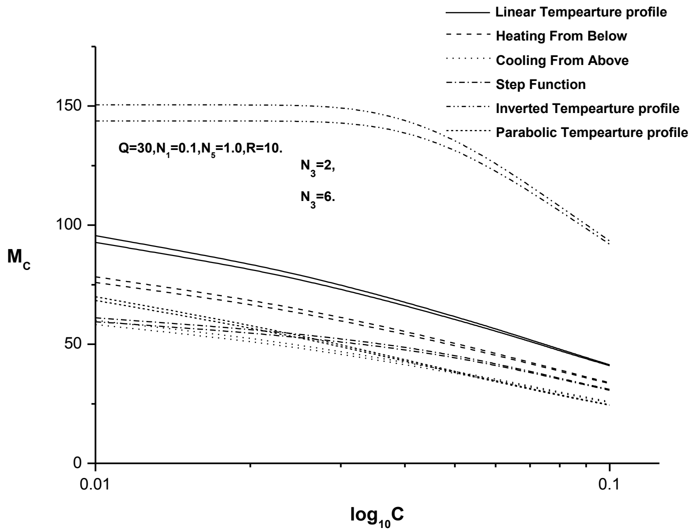

Fig 7: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free adiabatic no-spin boundary condition for different values of couple stress parameter N_3 and different non-uniform basic temperature gradients.

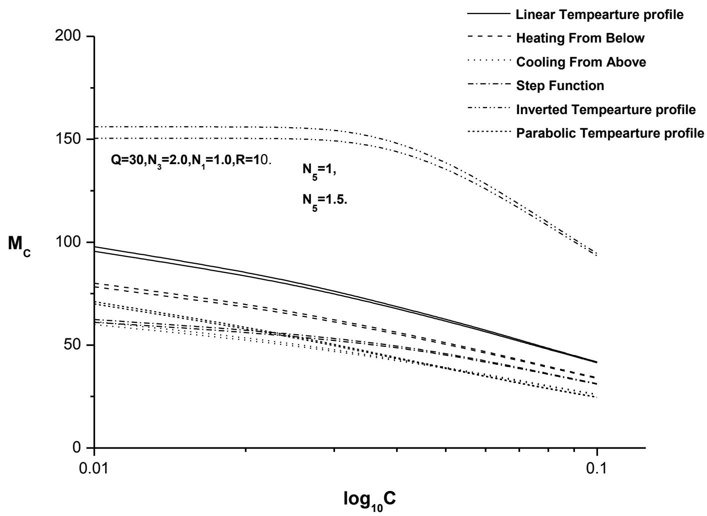

Fig 8: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free adiabatic no-spin boundary condition for different values of micropolar heat conduction parameter N_5 and different non-uniform basic temperature gradients.

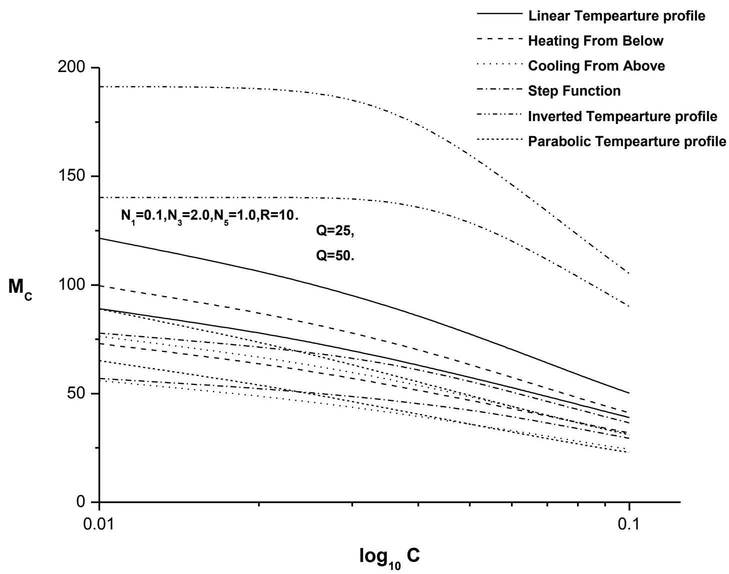

Fig 9: Plot of critical Marangoni number M_C verses Cattaneo number C with respect to rigid-free adiabatic no-spin boundary condition for different values of Chandrasekhar number Q and different non-uniform basic temperature gradients.