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Some Graph Invariants of the Identity Graph of Non-cyclic Group $\mathbb{Z}_m \times \mathbb{Z}_n$

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Abstract

In this paper, we discuss some graph invariants such as connectivity, minimum degree, independence number, matching number, covering numbers and chromatic numbers of the identity graph of $\mathbb{Z}_m \times \mathbb{Z}_n$ where gcd $(m, n) \neq 1$.

Keyword: Identity graph, connectivity, independence number, matching number, covering numbers, chromatic numbers.

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1. Introduction

Algebraic graph theory portrays the relationship between algebraic structures and graphs. Kandasamy and Smarandache introduced the identity graphs of groups and semigroups and examined some special subgraphs [8]. Let $(G, \cdot)$ be a group. The identity graph $G_I(G)$ of G consists of the vertex set as the elements of G and the edge set is defined as follows: x is adjacent to y in $G_I(G)$ if $x \cdot y = e$ or any one of x or y is e, where e is the identity element of G.

In [7], N. Feyza Yalcin and Yakup Kirgil introduce the subset of self-inverse elements and the subset of mutual inverse elements of a finite cyclic group. Also, by using these subsets they determined the number of triangles and the number of edges in the identity graphs of finite

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cyclic groups. Furthermore, they computed Schutz, Gutman, first Zagreb, second Zagreb and Wiener indices for the identity graphs. In [1], Amina M.L derived some graph properties such as degree of vertex, clique, colouring, independent and dominating sets of the identity graph of the symmetric group of degree four (S_4).

K. Aruna Sakthi et. al in [5] investigated the various types of resolving set and its dimensions for the identity graph of finite groups. In [6], M. U. Romdhini et. al examined some graph matrices corresponding to the identity graph for group of integers modulo n; $\mathbb{Z}_n$ such as adjacency, Laplacian and signless Laplacian matrices. Also, they formulated the graph's characteristic polynomial, spectrum and energy.

In this paper we discuss the graph invariants such as number of edges, independence number, matching number, vertex covering number, edge covering number and colouring of the identity graph of the non-cyclic group $\mathbb{Z}_m \times \mathbb{Z}_n$, which is denoted by $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$.

2. Preliminaries

For basic definitions and results we refer [2], [3] and [4]. The vertex representing the identity element in $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ is the identity vertex. The edge in a triangle of $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ which is not adjacent to the identity vertex is the base edge. The vertices in the base edge is the base vertices. The removal of the identity vertex in the identity graph of any group disconnects the graph, hence the identity graph of any group is separable.

The girth of a simple graph G is the length of a smallest cycle and denoted by $gir(G)$. The diameter of a graph $G(V(G), E(G))$ denoted by $diam(G)$ is defined as $diam(G) = max\{d(u; v) : u, v \in V(G)\}$. The vertex connectivity of a graph G, denoted by $\kappa(G)$, is defined as the minimum number of vertices whose removal from G leaves the remaining graph disconnected. The edge connectivity of a graph G, denoted by $\lambda(G)$, is defined as the minimum number of edges whose removal from G leaves the remaining graph disconnected. The degree of a vertex is the number of edges adjacent to it. The minimum degree of a graph G is denoted by $\delta(G)$.

An independent vertex set of a graph G is a subset of the vertex set such that no two vertices in the subset are adjacent in G. An independent vertex set that cannot be enlarged to another independent vertex set by adding a vertex is called a maximal independent vertex set. The maximal independent vertex set with the largest possible number of vertices is called a maximum independent vertex set. The cardinality of a maximum independent vertex set is called the independence number of the graph and is denoted by β_0.

A matching in a graph is a subset of edges in which no two edges are adjacent. A maximal matching is a matching to which no edge in the graph can be added. A maximum matching is the maximal matching with the largest number of edges. The number of edges in a maximum matching is called the matching number of the graph and is denoted by β_1.

Let G be a graph, and let C be a subset of the vertex set of G. The set C is said to be a covering of G, if every edge has atleast one end in C. A covering C is said to be a minimum covering if there exist no covering C' in G such that $|C'|<|C|$. The number of elements in a minimum covering is called the vertex covering number of G and is denoted by α_0. The edge covering of G is a subset of the edge set that covers all the vertices of G, provided G has no isolated vertices. The number of elements in a minimum edge covering of G is called the edge covering number and is denoted by α_1.

A *k*-colouring of G is an assignment of *k* colours to the vertices of G in such a way that adjacent vertices receive different colours. If G has a *k*-colouring, then G is said to be *k*-colourable. The chromatic number of G, denoted by χ(G), is the smallest number *k* for which G is *k*-colourable. A *k*-edge colouring of G is an assignment of *k* colours to the edges of G so that any two adjacent edges must receive different colours. If G has a *k*-edge colouring, then G is said to be k-*edge* colourable. The edge chromatic number of G, denoted by χ'(G), is the minimum number *k* for which G is *k*-edge colourable. A total colouring is always assumed to be proper in the sense that no adjacent vertices, no adjacent edges, and no edge and its end-vertices are assigned the same colour. The total chromatic number χ''(G) of a graph G is the fewest colors needed in any total colouring of G.

Throughout this paper, *m, n* are positive integers.

Theorem 1. *[3] In any arbitrary (p; q) graph G, the sum of the degree of all vertices is equal to twice the number of edges. That is,* $\sum_{u\in G}\delta(u) = 2q$.

The set of all self-inverse elements in the group G is denoted by S(G). That is, $S(G) = \{a\epsilon G: a = a^{-1}\}$. Also $S^*(G) = S(G) - \{e\}$. The set of all mutual inverse elements in the group G is denoted by M(G). That is, $M(G) = \{a \in G : a \neq a^{-1}\}$.

The set of all pendant vertices in the identity graph of a group G is denoted by $S_I(G)$. That is, $S_I(G)$ is $S^*(G)$. The set of all base vertices in the identity graph of a group G is denoted by $M_I(G)$. That is, $M_I(G)$ is $M(G)$.

Theorem 2. *Let G be a finite group and v ϵ G. Then the degree of vertex v in* $G_I(G)$ *is,*

$$\delta(v) = \begin{cases} |G| - 1 & if v = e \\ 1 & if\ v \in S_I(G) \\ 2 & if\ v \in M_I(G) \end{cases}$$

Note:

1. Number of triangles in $G_I(G) = \frac{|M_I(G)|}{2}$
2. $|M_I(G)|$ is always even, since it contains pair of elements in G.
3. $|G| = |M_I(G)| + |S_I(G)| + 1$

3. Some Graph Invariants of $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$

In this section, we illustrate an example for the identity graph of a non-cyclic group $\mathbb{Z}_m \times \mathbb{Z}_n$, where gcd($m$, n) ≠ 1 and also find the connectivity, minimum degree, independence number, matching number, covering numbers and chromatic numbers of this identity graph.

Theorem 3: *For the identity graph of a non-cyclic group* $\mathbb{Z}_m \times \mathbb{Z}_n$*, we have*

(a) Number of triangles $= \begin{cases} \frac{mn-4}{2} & if\ mn\ is\ even \\ \frac{mn-1}{2} & if\ mn\ is\ odd \end{cases}$

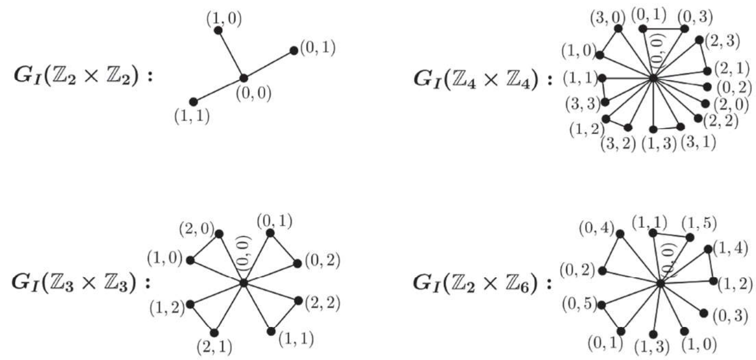
(b) Number of pendant vertices = $\begin{cases} 3 & if\ mn\ is\ even \\ 0 & if\ mn\ is\ odd \end{cases}$

Proof. When mn is even, the pendant vertices are those elements in $\mathbb{Z}_m$ x $\mathbb{Z}_n$ of the form $\left(0,\frac{n}{2}\right),\left(\frac{m}{2},0\right)$ *and* $(\frac{m}{2},\frac{n}{2})$. Thus, the number of pendant vertices are 3. If we remove the pendant vertices and the identity vertex from the vertex set of G_I ($\mathbb{Z}_m$ x $\mathbb{Z}_n$), we get the set of all base vertices. Also, the total number of triangles in G_I ($\mathbb{Z}_m$ x $\mathbb{Z}_n$) is half of the cardinality of the set of all base vertices. Thus, there are $\frac{mn-4}{2}$ triangles.

When *mn* is odd, there are no elements in $\mathbb{Z}_m$ x $\mathbb{Z}_n$ of the form ($(0,\frac{n}{2})$, $\left(\frac{m}{2},0\right)$ and $(\frac{m}{2},\frac{n}{2})$. Thus, there is no pendant vertex. If we remove the identity vertex from the vertex set of G_I ($\mathbb{Z}_m$ x $\mathbb{Z}_n$), we get the set of all base vertices. Also, the total number of triangles in G_I ($\mathbb{Z}_m$ x $\mathbb{Z}_n$) is half of the cardinality of the set of all base vertices. Thus, there are $\frac{mn-1}{2}$ triangles.

Illustration

The identity graph of the non-cyclic group $\mathbb{Z}_m$ x $\mathbb{Z}_n$, G_I ($\mathbb{Z}_m$ x $\mathbb{Z}_n$) is as follows:



Theorem 4. If *mn* ≠ 4, then $gir(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = 3$.

Proof. When *mn* ≠ 4, every $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ has atleast one triangle and it is the smallest cycle. So $gir(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = 3$.

Remark.

$$|S_I(\mathbb{Z}_m \times \mathbb{Z}_n)| = \begin{cases} 3, & if\ mn\ is\ even; \\ 0, & if\ mn\ is\ odd. \end{cases}$$

$$|M_I(\mathbb{Z}_m \times \mathbb{Z}_n| = \begin{cases} mn-4 & if\ mn\ is\ even \\ mn-1 & if\ mn\ is\ odd \end{cases}$$

Theorem 5. *The number of edges of $G_I(\mathbb{Z}_m X\ \mathbb{Z}_n)$ is,*

$$|E(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))| = \begin{cases} \dfrac{3(mn-2)}{2} & if\ mn\ is\ even \\ \dfrac{3\ (mn-1)}{2} & if\ mn\ is\ odd. \end{cases}$$

Proof. By Theorem 1, for the identity graph of a group $G_I(\mathbb{Z}_m \text{ x } \mathbb{Z}_n)$, the number of edges is, for *mn* is even,

$$2|E(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))|$$
$$= \deg(e) + \sum_{u \in M_I(\mathbb{Z}_m \times \mathbb{Z}_n)} \deg(u) + \sum_{v \epsilon S_I(\mathbb{Z}_m \times \mathbb{Z}_n)} \deg(v)$$

$= mn - 1 + 2|M_I(\mathbb{Z}_m \times \mathbb{Z}_n)| + |S_I(\mathbb{Z}_m \times \mathbb{Z}_n)|$ (by theorem 2)

$= \text{mn} - 1 + 2 \times (\text{mn} - 4) + 3$ (by theorem 3)

$= 3mn - 6$

$$|\text{E}(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))| = \frac{3(mn-2)}{2}$$

for *mn* is odd.

$$|E(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))|$$
$$= \deg(e) + \sum_{u \in M_I(\mathbb{Z}_m \times \mathbb{Z}_n)} \deg(u) + \sum_{v \epsilon S_I(\mathbb{Z}_m \times \mathbb{Z}_n)} \deg(v)$$

$= mn - 1 + 2|M_I(\mathbb{Z}_m \times \mathbb{Z}_n)| + |S_I(\mathbb{Z}_m \times \mathbb{Z}_n)|$ (by theorem 2)

$= mn - 1 + 2\ (mn - 1) + 0$ (by theorem 3)

$= 3\text{mn} - 3$

$$|E(G_I(\mathbb{Z}_m \times\ \mathbb{Z}_n))| = \frac{3(mn-1)}{2}$$

Theorem 6. *For all mn, diam* $(G_I(\mathbb{Z}_m X\, \mathbb{Z}_n))=2$

Theorem 7. *For the identity graph of the non-cyclic group* $\mathbb{Z}_m X\, \mathbb{Z}_n$, *we have*

(a) $\kappa\ (G_I(\mathbb{Z}_m X\, \mathbb{Z}_n)) = \lambda(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n)) = \delta(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n\ ))$ *if mn is even*

(b) $\kappa\ (G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n)) < \lambda(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n\ )) = \delta(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n\ ))$ *if mn is odd*

Proof: The removal of identity vertex disconnects the graph, so $\kappa\ (G_I(\mathbb{Z}_m X\, \mathbb{Z}_n)) = 1$.

Case (a): When mn is even, $G_I\,(\mathbb{Z}_m \text{ x } \mathbb{Z}_n)$ has three pedant vertices. So when we remove one of the pendant edges, the graph becomes disconnected. Thus $\lambda(G_I(G))=1$. Also the pendant vertex has degree 1, which is the minimum degree. Thus $\delta(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n\ ))=1$.

Case (b): When *mn* is odd, $G_I\,(\mathbb{Z}_m \text{ x } \mathbb{Z}_n)$ has only triangles. So $\lambda(G_I(\mathbb{Z}_m \text{ x } \mathbb{Z}_n) = 2$. Also the degree of the base vertices of triangle is 2, which becomes the minimum degree. Thus $\delta(G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n\ ))=2$.

Theorem 8: *The independence number of* $\boldsymbol{G_I\,(\mathbb{Z}_m \text{ x } \mathbb{Z}_n)}$ ***is,***

$$\beta_0 = \begin{cases} \frac{mn+2}{2} & if\ mn\ is\ even \\ \frac{mn-1}{2} & if mn\ is\ odd \end{cases}$$

Proof. When *mn* is even, $G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n)$ has $\frac{mn-4}{2}$ triangles and three pendant vertices. So, to form a maximum independent vertex set, we select one vertex from each triangle and all the pendant vertices. Thus $\beta_0 = \frac{mn+2}{2}$

When *mn* is odd, $G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n)$ has only $\frac{mn-1}{2}$ triangles. So to form a maximum independent vertex set, we select one vertex from each triangle. Thus $\beta_0 = \frac{mn-1}{2}$

Theorem 9. *The matching number of* $G_I\ (\mathbb{Z}_m\times\mathbb{Z}_n)$ *is,*

$$\beta_1 = \begin{cases} \frac{mn-2}{2} & if\ mn\ is\ even \\ \frac{mn-1}{2} & if\ mn\ is\ odd \end{cases}$$

Proof. When mn is even, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has $\frac{mn-4}{2}$ triangles and three pendant vertices. So to form a maximum matching, we select the base edge from each triangle and any one of the pendant edges. Thus
$\beta_1 = \frac{mn-2}{2}$

When mn is odd, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has only $\frac{mn-1}{2}$ triangles. So to form a maximum matching, we select the base edge from each triangle. Thus
$\beta_1 = \frac{mn-1}{2}$

Theorem 10. *The vertex covering number of* G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ *is,*

$$\alpha_0 = \begin{cases} \frac{mn-2}{2} & if\ mn\ is\ even \\ \frac{mn+1}{2} & if\ mn\ is\ odd \end{cases}$$

Proof. When mn is even, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has $\frac{mn-4}{2}$ triangles and three pendant vertices. So to form a minimum vertex covering, we select one base vertex from each triangle and the identity vertex. Thus
$\alpha_0 = \frac{mn-2}{2}$

When mn is odd, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has only $\frac{mn-1}{2}$ triangles. So to form a minimum vertex covering, we select one base vertex from each triangle and the identity vertex. Thus $\alpha_0 = \frac{mn+2}{2}$

Theorem 11. *The edge covering number of* G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ *is,*

$$\alpha_1 = \begin{cases} \frac{mn+2}{2} & if\ mn\ is\ even \\ \frac{mn+1}{2} & if\ mn\ is\ odd \end{cases}$$

Proof. When mn is even, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has $\frac{mn-4}{2}$ triangles and three pendant vertices. So to form a minimum edge covering, we select the base edge from each triangle and the pendant edge. Thus $\alpha_1 = \frac{mn+2}{2}$

When mn is odd, G_I $(\mathbb{Z}_m \times \mathbb{Z}_n)$ has only $\frac{mn-1}{2}$ triangles. So to form a minimum edge covering, we select the base edge from each triangle and any one edge which contains the identity vertex. Thus $\alpha_1 = \frac{mn+1}{2}$

Theorem 12.

$$\chi(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = \begin{cases} 2 & \text{if } mn = 4 \\ 3 & if\ mn \neq 4 \end{cases}$$

$$\chi'(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = \begin{cases} 3 & \text{if } mn = 4 \\ mn-1 & if\ mn \neq 4 \end{cases}$$

$$\chi''(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = \begin{cases} 5 & \text{if } mn = 4 \\ mn & if\ mn \neq 4 \end{cases}$$

Proof. When $mn = 4$, $G_I (\mathbb{Z}_m \times \mathbb{Z}_n)$ has three pendant vertices and the identity vertex. Give a colour, say, c_1 to the identity vertex. As the pendant vertices are only adjacent to the identity vertex, give them the same colour, say, c_2. Thus $\chi(G_I (\mathbb{Z}_m \times \mathbb{Z}_n))=2$. When $mn \neq 4$ and mn is even, $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains base vertices, identity vertex and pendant vertices. Give a colour, say, c_1 to the identity vertex. Give different colours, say, c_2 and c_3 to the base vertices if they are in the same triangle. As the pendant vertices are not adjacent to the base vertices, give the pendant vertices any one of the colour given to the base vertices, say, c_2. Thus $\chi(G_I (\mathbb{Z}_m \times \mathbb{Z}_n))=3$. When $mn \neq 4$ and mn is odd, $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains only base vertices and identity vertex. Give a colour, say, c_1 to the identity vertex. Give different colours, say, c_2 and c_3 to the base vertices if they are in the same triangle. Thus $\chi(G_I (\mathbb{Z}_m \times \mathbb{Z}_n))=3$.

When $mn = 4$, $G_I (\mathbb{Z}_m \times \mathbb{Z}_n)$ has only three adjacent edges, so give them different colours. Thus $\chi'(G_I (\mathbb{Z}_m \times \mathbb{Z}_n))=3$. When $mn \neq 4$ and mn is even, $G_I (\mathbb{Z}_m \times \mathbb{Z}_n)$ have base edges, the edges which is adjacent to the identity vertex and the pendant edges. As the base edges are not adjacent to each other, give them the same colour, say, c_1. Also since the base edges and pendant edge are not adjacent, give one of the pendant edge the same colour, that is, c_1. As the remaining edges are adjacent to each other, give them different colours other than c_1. Thus $\chi'(G_I (\mathbb{Z}_m \times \mathbb{Z}_n)) = mn - 1$. When $mn \neq 4$ and mn is odd, $G_I (\mathbb{Z}_m \times \mathbb{Z}_n)$ have only base edges and the edges which are adjacent to the identity vertex. As the base edges are not adjacent, give them the same colour, say, c_1 and since the remaining edges are adjacent to each other, give them different colours other than c_1. Thus $\chi'(G_I (\mathbb{Z}_m \times \mathbb{Z}_n))= mn - 1$.

When $mn = 4$, $G_I (\mathbb{Z}_m \times \mathbb{Z}_n)$ contains three pendant vertices, three edges and identity vertex. Give a colour, say, c_1 to the identity vertex. As these pendant vertices are only adjacent to the identity vertex, give them the same colour, say, c_2. Give different colours other than c_1 and c_2 to the pendant edges to make it total colourable. Thus $\chi''(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))$ = 5. When mn ≠ 4 and *mn* is even, $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains base vertices, base edges, pendant vertices, identity vertex and the edges which are adjacent to the identity vertex. Give a colour, say, c_1 to the identity vertex. Give different colours, say, c_2 and c_3 to the base vertices if they are in the same triangle. As the pendant vertices are only adjacent to the identity vertex, we can give them the colour, say, c_2. As the base edges are not adjacent to the identity vertex, we can give every base edges with the colour c_1. Now provide the edges other than base edge, of any one of the triangle with the colours c_2 and c_3. Next, we give different colours other than c_1, c_2 and c_3 to the remaining edges to make it total colourable. Thus $\chi''(G_I(\mathbb{Z}_m \times \mathbb{Z}_n)) = mn$. When $mn \neq 4$ and *mn* is odd, $G_I(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains base vertices, base edges, identity vertex and the edges which are adjacent to the identity vertex. Give a colour, say, c1 to the identity vertex. Give different colours, say, c_2 and c_3 to the base vertices if they are in the same triangle. As the base edges are not adjacent to the identity vertex, we can give every base edges with the colour c_1. Now provide the edges other than base edge, of any one of the triangle with the colours c_2 and c_3. Next, we give different colours other than c_1, c_2 and c_3 to the remaining edges to make it total colourable. Thus $\chi''(G_I(\mathbb{Z}_m \times \mathbb{Z}_n))=mn$.

Conclusion

In this paper, we have studied the structure and some graph invariants of the identity graph of a non-cyclic group $\mathbb{Z}_m \times \mathbb{Z}_n$. In future, we can also find the special identity graph of the non-cyclic group $\mathbb{Z}_m \times \mathbb{Z}_n$. We have already discussed about the identity graph of a dihedral group in the paper entitled "Some Graph Invariants of the Identity Graph of a Dihedral Group" which is accepted for publication in the journal of Calcutta Mathematical Society. Further study on the identity graph for a different non-abelian group can be done. Also we can check the special identity graphs of these groups.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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