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Optimizing a Single-Vendor Multi-Purchaser for Multi-Item Fuzzy Inventory System along Lead Time with Carbon Emission Cost

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Abstract

Multi-item investigation with a multi-purchaser inventory system exposes remarkable perceptions of improved demand enhancement in overall income and manufacturing time proficiency. Similarly, lower transporting costs for multiple items positively influence minimum integrated total cost by lead time suitability. Owing to the imprecision of several factors, the objective seems to be inaccurate. As the development of fuzzy objective is uncertain, a model is formulated to suit assured problems and doubtful earnings with some indecision. The model is solved by means of graded mean integration technique with the addition of Kuhn-Tucker method when the fuzzy equivalent of the problem remains available. An algorithm is established to attain each item's optimal order quantity for each purchaser and the minimum integrated total cost for a whole inventory system. The evaluation of a fuzzy multi-item, multi-purchaser inventory system through crisp multi-item, multi-purchaser inventory system is completed utilizing mathematical illustrations. Lastly, the graphical demonstration establishes the suggested system.

Keywords: Fuzzy multi-item with multi-purchaser, Graded mean integration technique, Minimum integrated total cost, Optimal order quantity, Kuhn-Tucker conditions.

1. Introduction

The dual-level single vendor multi-purchaser inventory system for multi-item is a thought-provoking zone and is suitable for several tangible

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circumstances in the supply chain management. Learning in a dual-level inventory system has been stretched to the multi-purchaser procedure. The organisational procedure and synchronization in creation stream are significant in the study of inventory by multiple purchasers.

Many investigation shortcomings have been recognized according to the prevailing literature explanation. These unique shortcomings lie in the inadequate consideration of supply sequence concerning more than a single purchaser. Contemporary literature frequently focuses on seller-purchaser inventory types containing single-seller and single-purchaser. Kim and Sarkar [11] described a combined refill issue for compound several-level eminence enhancement by combining the well-regulated lead time. Taleizadeh et al. [19] dealt with a multiple purchasers, multiple sellers and multi-item stream series in which every purchaser and every seller is bound by stockroom restrictions in collection of things.

Pan and Yang [16] considered delivering a least total cost and lesser lead time compared to previous inventory problems. Evaluation of choice on single-seller multi-retailer was testified in Hen and Sarker [9] in which the subdivision group optimization and significant works are established to resolve the prototype. Uthayakumar and Ganesh Kumar [22] handled a single-seller multi-customer stream chain system with several goods. The requirement of this stream sequence for every object is stochastic adaptable. Chavarroa et.al. [3] considered a two-level supply chain in which one storeroom delivers a single item to N vendors, using integer-ratio procedures. This investigation deliberates purchaser demands as a normal density function. A set of 240 random cases was produced and used in assessing deterministic and stochastic result methodologies. Esmaeili and Nasrabadi [7] presented an inventory model in the inflation for deteriorating objects in a single dealer and multi-vendor supply chain. The dealer offers the vendors trade credit. Dual situations considered the variations in inflation proportion as a discrete-time Markov Chain and without allowing Markovian situations. The correlation between the dealer and the vendors is demonstrated as a Stackelberg – game.

Giri et.al. [8] established a two-level supply chain that is collected from a single producer and various sellers. The decentralized model is solved through the Stackelberg gaming approach. Barman and Mahata [2] developed a combined two-echelon supply chain inventory system with a single producer and multi-vendors in which every vendor's demand is reliant on vending charge of the item. The industrialized system is demonstrated through the support of numerical examples utilizing a stochastic search genetic algorithm. Demizu et. al. [5] investigated the optimal inventory range indication to decrease lost chances and flawed inventory, which is a vital problem from an income enhancement angle. In addition, the results validate multi-item and multi-store supply chains. Utama et. al. [21] investigation addressed the single-vendor multi-buyer model by integrating manifold raw resources and quality deprivation to maximize combined total revenue. The model employed Whale Optimization Algorithm for optimization, with experimental statistics resulting from an agri-nutrition business site learning in Indonesia.

The ordering cost decreases inventory system by manageable principal period and a facility range restriction stayed established in Annadurai [1]. Vithyadevi and Annadurai [24] considered a combined inventory typical with ordering charge decrease reliant on lead time happening in fuzzy situation by hiring trapezoidal fuzzy quantity. Vithyadevi and Annadurai [25] developed a two-level production system under fuzzy parameters and decision variables by implementing a pentagonal fuzzy quantity. Multiple item inventory representation through stock-related claim is established in a fuzzy atmosphere. Articles are getting worse at a stable rate and retailed through various exits in the town under an individual organization presented by Maiti [12]. Malleeswaran and Uthayakumar [14] considered a combined seller–purchaser supply sequence type on behalf of backorder amount deduction, cost-related demand including provision level restrictions and carbon discharge rate.

The lead time generally contains the subsequent modules: dealer lead time, order planning, transport time, order shipment, and arrangement period. Dey et al. [6] established a combined inventory typical including distinct structure price decline, flexible protection factors, and vending cost-dependent demand. Malik and Sarkar [13] measured multi-item unremitting assessment inventory system and indeterminate request, eminence enhancement, structure rate drop in addition to disparity resistor in principal period. Tiwari et al. [20] examined ecological inventory organization with worsening and defective feature matters allowing for carbon discharge. Kamble [10] deliberated the perception of pentagonal fuzzy numbers. Canonical pentagonal fuzzy numbers are measured via inner calculation processes through consuming alpha-cut processes. Fuzzy model for declining inventory articles using time changing demand and shortages in entirely backlogged conditions was framed by Nagar and Surana [15].

The current situation stagnates due to the absence of maximum consideration of the scenario of multiple purchasers entangled in the progression of the transaction. Additionally, early investigations often missed the position of controlling several items in the manufacturing process and allowing multiple retailers to have access to these supplies. Furthermore, additional shortcomings lie in fuzzification processes when elevating vendor-purchaser models, whereas earlier studies utilized crisp methodologies that did not produce optimal results. As a result, this study examines the possibility of overloading the processes involved in single-vendor multi-purchaser optimizations through the use of fuzzification process such as fuzzy numbers. This study aims to improve a single-vendor multi-purchaser type containing multiple items. To maximize the firm's entire income, this study also examines the use of fuzzification to the single-vendor with multi-purchaser inventory type.

Taha [18] provided the Khun-Tucker technique used to resolve indecision issues by stating in operations research. Vijayashree and Uthayakumar [23] concentrated on the decreased combined whole price through assuming logarithmic and linear ordering charges the reduction of which is reliant on lead time. Fuzzy set concept presented by Zimmerman [27] concentrated on ambiguous groups in operational research. Chen [4] deliberated arithmetic processes in fuzzy numbers through utility code. Setiawan et al. [17] investigated further accurate reasons such as arbitrary mandate, multiple products and multiple purchasers. In certainty, there is no assurance for factory made goods to be flawless. Yadav et al. [26] presented a stream sequence type comprising a single seller and single producer which is inspected for commercial feasibility. A defective multiple level business procedure is measured at this point through a probabilistic worsening article.

The paper is structured as: In Section 2, the notations, assumptions are implemented. Section 3 deliberates a crisp mathematical system with algorithm for the purpose of optimizing the integrated total cost for the system and optimal order for each item. Similarly, graded mean technique, designed fuzzy inventory system and an algorithm framed towards determining optimum solution for the system. In Section 4, arithmetical illustrations and then graphical demonstrations are obtainable towards initiating a crisp and then a fuzzy multi-item, multi-purchaser inventory model. Section 5 demonstrates a comparative evaluation. Section 6 provides the conclusion.

2. Notations and Assumptions

The succeeding notations are presented in this inventory system.

2.1. Notations

For the purchasers q = 1,2,3....., V and items i = 1,2,3,.....,U are used to build the system.

Q_{iq} − Order quantity for $i-$ th item of the $q-$ th purchaser,

L_{iq} − Lead time span for i −th item of the $q-$th purchaser,

A_{iq} − Ordering cost for $i-$th item of $q-$th purchaser per order,

m_{iq}− Lots quantity for $i-$th item, its manufactured goods are supplied from the vendor to the in $q-$ th purchaser in single phase,

$D_{iq}-$ Average demand for $i-$th item of per unit time on the $q-$th purchaser,

$P_{iq}-$ Manufacturing rate for $i-$ th item in $q-$ th purchaser of the seller $P_{iq} > D_{iq}$,

$S_{iq}-$Vendor's setup cost for $i-$th item of $q-$th purchaser per arrangement,

$C_{viq}-$ Production cost for $i-$th item of $q-$th purchaser funded through vendor $C_{viq} < C_{biq}$,

$C_{biq}-$Purchasing cost for $i-$th item of $q-$th purchaser funded by the purchaser,

$r_{iq}-$Yearly inventory holding cost for $i-$th item for $q-$th purchaser in which each dollar is capitalized in stocks,

$R_{iq}-$ Reorder point for $i-$th item of the $q-$th purchaser,

$VEC_{viq}-$Vendor's flexible carbon emission cost for $i-$th item of $q-$th purchaser,

$FEC_{viq}-$ Vendor's stable carbon emission cost for $i-$th item of $q-$th purchaser,

$FTC_{viq}-$Vendor's stable transportation cost for $i-$th item of $q-$th purchaser,

$VTC_{viq}-$Vendor's flexible transportation cost for $i-$th item of $q-$th purchaser,

$ITCMIB-$ Integrated total cost of the whole crisp inventory system,

$I\tilde{T}CMIB-$Integrated total cost of the whole fuzzy inventory system.

2.2. Assumptions

1. The coordination comprises single-vendor with multiple purchasers aimed at a multi-item inventory system.
2. The demand between the purchasers is self-determining over time.
3. The $q-$th purchaser for $i-$th item orders a lot of size Q_{iq} and the vendor manufactures $m_{iq}Q_{iq}$ with a limited manufacture ratio $P_{iq}(P_{iq} > D_{iq})$. At a single setup, the quantity Q_{iq} is transported to the purchaser over m_{iq} times. The vendor sustains a set up cost S_{iq} for each manufacture run and the purchaser sustains an ordering cost A_{iq} for every order of quantity Q_{iq}.
4. The demand of an $i-$th item for $q-$th purchaser X_{iq}, throughout lead time L_{iq} follows a normal distribution with mean $\mu_{iq}L_{iq}$ and standard deviation $\sigma_{iq}\sqrt{L_{iq}}$.

5. The inventory is continuously noticed. Upon reaching the reorder point $R_{iq,}$ the purchaser needs to order the items.

6. The reorder point equals the summation of the expected demand for the period and safety stock. The reorder point R_{iq} = expected demand for the period of lead time for $i-$th item along $q-$th purchaser + safety stock, $R_{iq} = D_{iq} L_{iq} + k_{iq}\sigma_{iq}\sqrt{L_{iq}}$ where k_{iq} is the safety factor.

7. The lead time L_{iq} for all items is similar and it involves n_{iq} mutually independent modules. The $z-$th module has a normal duration b_{iqz}, least period a_{iqz}, and crashing cost per unit time c_{iqz}. For suitability, c_{iqz} is organized as $c_{iq1} < c_{iq2} < c_{iq3} < \ldots < c_{iqn.}$

8. The modules of lead time are crashing cost that is unique at the beginning of a period. Since, the principal module for the situation takes the least unit crashing cost, the subsequent modules follow the same.

9. Let $L_{iq0} = \sum_{j=1}^{n} b_{iqz}$ and L_{iqz} be the span of lead time using modules 1,2,3,....z_{ip} crashed to their least period, then L_{iqz} can be expressed as $L_{iqz} = L_{iq0} - \sum_{z=1}^{n_{iq}} (b_{iqz} - a_{iqz}), z = 1,2,3,\ldots,n_{iq}$; and the lead time crashing cost per cycle $R(L_{iqz})$ is given as
$$R(L_{iq}) = c_{iqz}(L_{iq(z\text{-}1)} - L_{iq}) + \sum_{j=1}^{z-1} c_{iqj}(L_{iq(j-1)} - L_{iq}), \quad L_{iq} \in [L_{iqz}, L_{iq(z-1)}]$$
Furthermore, the span of lead time is equivalent to whole transport rotations, and the lead time crashing cost arises in every transport rotation. The association is among crashing cost and lead time.

10. The decrease of lead time L_{iqz} attends condensed ordering cost A_{iq} and A_{iq} is resolutely the concave function of L_{iqz} i.e., $A_{iq}'(L_{iqz})>0$ and $A_{iq}''(L_{iqz})<0$.

11. If additional charges remain sustained through the vendor, it is mandatory to fully shift towards the purchaser after reduced lead time.

3. Mathematical system

A single-vendor with multi-purchaser for multi-item along lead time with carbon emission cost inventory system is designed in crisp and fuzzy scenario.

3.1. Crisp Multi-item with Multi-purchaser Inventory System

Integrated total cost of multi-item with multi-purchaser *(ITCMIB)*

Integrated total cost per unit time is derived for multi-item with multi-purchaser designed here and summated for succeeding components.

Ordering cost for $i-$th item of $q-$th purchaser per unit time $=\frac{A_{iq}}{Q_{iq}/D_{iq}}=\frac{A_{iq}D_{iq}}{Q_{iq}}$, (1)

Buyer's holding cost for $i-$th item of $q-$th purchaser per unit time

$$=\left(\frac{Q_{iq}}{2}+k_{iq}\sigma_{iq}\sqrt{L_{iq}}\right)r_{iq}C_{biq},\tag{2}$$

Lead time crashing cost for $i-$th item of $q-$th purchaser per unit time

$$=\left(\frac{D_{iq}}{Q_{iq}}\right)R(L_{iq}),\tag{3}$$

Vendor setup cost for $i-$th item of $q-$th purchaser per year

$$=\left(\frac{D_{iq}}{m_{iq}Q_{iq}}\right)S_{iq},\tag{4}$$

Vendor's average inventory cost for $i-$th item of $q-$th purchaser

$$=\left\{\left[m_{iq}Q_{iq}\left(\frac{Q_{iq}}{P_{iq}}+(m_{iq}-1)\frac{Q_{iq}}{D_{iq}}\right)-\frac{m_{iq}{}^2Q_{iq}{}^2}{2P_{iq}}\right]-\left[\frac{Q_{iq}{}^2}{D_{iq}}(1+2+......+(m_{iq}-1))\right]\right\}\frac{D_{iq}}{m_{iq}Q_{iq}},$$

$$=\frac{Q_{iq}}{2}\left[m_{iq}\left(1-\frac{D_{iq}}{P_{iq}}\right)-1+\frac{2D_{iq}}{P_{iq}}\right].$$

So the vendor's holding cost for $i-$th item of $q-$th purchaser per unit time

$$=\frac{Q_{iq}}{2}\left[m_{iq}\left(1-\frac{D_{iq}}{P_{iq}}\right)-1+\frac{2D_{iq}}{P_{iq}}\right]r_{iq}C_{viq}.\tag{5}$$

Vendor annual transportation cost for $i-$th item of $q-$th purchaser

$=m_{iq}(FTC_{viq}+VTC_{viq})$, (6)

Annual carbon emission cost for $i-$th item of $q-$th purchaser

$=m_{iq}FEC_{viq}+Q_iVEC_{viq}$. (7)

Based on the assumptions (1) to (11) defined above, the integrated total cost per unit time for $i-$th item of $q-$th purchaser is a collection of above mentioned costs expressed as

$$ITCMIB_{iq}(Q_{iq},L_{iq},m_{iq})=\left[\frac{D_{iq}}{Q_{iq}}\left(A_{iq}+\frac{S_{iq}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq}r_{iq}C_{viq}}{2}\left(\frac{m_{iq}D_{iq}}{P_{iq}}+1\right)+\frac{Q_{iq}r_{iq}}{2}\left(\left(m_{iq}+\frac{2D_{iq}}{P_{iq}}\right)\right.\right.$$
$$\left.\left.C_{viq}+C_{biq}\right)+r_{iq}C_{biq}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq}VEC_{viq}+m_{iq}(FTC_{viq}+VTC_{viq}+FEC_{viq})\right]. \quad (8)$$

The crisp integrated total cost of $q-$th purchaser for all U items is represented by $ITCMIB_q$, that is $ITCMIB_q=\sum_{i=1}^{U}ITCMIB_{iq}\ (Q_i, L_i, m_i)$ for q = 1,2,...., V and the crisp integrated total cost of the whole system is

$$ITCMIB=\sum_{q=1}^{V}ITCMIB_q=\sum_{q=1}^{V}\sum_{i=1}^{U}ITCMIB_{iq}(Q_{iq},L_{iq},m_{iq})$$

$$=\sum_{q=1}^{V}\sum_{i=1}^{U}\left[\frac{D_{iq}}{Q_{iq}}\left(A_{iq}+\frac{S_{iq}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq}r_{iq}C_{viq}}{2}\left(\frac{m_{iq}D_{iq}}{P_{iq}}+1\right)+\frac{Q_{iq}r_{iq}}{2}\left(\left(m_{iq}+\frac{2D_{iq}}{\tilde{P}_{iq}}\right)C_{viq}+C_{biq}\right)\right.$$
$$\left.+r_{iq}C_{biq}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq}VEC_{viq}+m_{iq}(FTC_{viq}+VTC_{viq}+FEC_{viq})\right]. \quad (9)$$

With the specific rate of m_{iq}, L_{iq} and the integrated total cost for $i-$th item of $q-$th purchaser is $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$, then optimal order quantity Q_{iq} obtained while integrated total cost $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$ is minimum. In order to obtain minimization of $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$ the partial derivative of $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$with Q_{iq} is found and equated to zero. Then,

$$-\frac{D_{iq}}{Q_{iq}^{\ 2}}\left(A_{iq}+\frac{S_{iq}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq}C_{viq}}{2}\left(\frac{m_{iq}D_{iq}}{P_{iq}}+1\right)+\frac{r_{iq}}{2}\left(\left(m_{iq}+\frac{2D_{iq}}{P_{iq}}\right)C_{viq}+C_{biq}\right)+VEC_{viq}=0. \quad (10)$$

For a static m_{iq} and L_{iq} the integrated total cost for $i-$th item of $q-$th purchaser $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$, is positive definite on the point Q_{iq} While inspecting the sufficient conditions to get minimum value of $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$, second order partial derivatives of $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$ with respect to Q_{iq} are used to obtain

$$\frac{\partial^2 ITCMIB_{iq}(Q_{iq},L_{iq},m_{iq})}{\partial Q_{iq}^{\ 2}}=\frac{2D_{iq}}{Q_{iq}^{\ 3}}\left(A_{iq}+\frac{S_{iq}}{m_{iq}}+R(L_{iq})\right)>0. \quad (11)$$

Therefore, $ITCMIB_{iq}\ (Q_{iq}, L_{iq}, m_{iq})$ is convex in Q_{iq} for a static m_{iq} and L_{iq}. As a result, optimal derivatives $Q_{iq}{}^*$ decrease so that a local minimum is obtained. Hence, the optimal order quantity $Q_{iq}{}^*$ found by the above equation (10) is

$$Q_{iq}^{*} = Q_{iq} = \sqrt{\frac{2D_{iq}\left(A_{iq} + \frac{S_{iq}}{m_{iq}} + R(L_{iq})\right)}{r_{iq}\left(\left(m_{iq}\left(1 - \frac{D_{iq}}{P_{iq}}\right) - 1 + \frac{2D_{iq}}{P_{iq}}\right)C_{viq} + C_{biq}\right) + 2VEC_{viq}}}. \tag{12}$$

3.1.1. Algorithm for Crisp Inventory Systems

The subsequent algorithm is used for the determination of each item's optimal order quantity for every purchaser and then the minimum integrated total cost for crisp system is found.

Algorithm: 1

Step 1: Fix the iteration p = 1.

Step 2: Each L_{iqw}, m_{iqw}, w = 0,1,2...t_{iq} determine Q_{iq} for all i = 1,2,3...U, q = 1,2,3...V, and find the corresponding $ITCMIB_{iq}\,(Q_{iq}, L_{iqw}, m_{iqw})$.

Step 3: Set $ITCMIB_{q}\,(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}) = \underset{w=0,1,2,3,\ldots,t_{ij}}{Min}\ ITCMIB_{q}\,(Q_{iq}, L_{iqw}, m_{iqw})$ and $(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)})$ is the optimal solution of fixed p.

Step 4: Calculate $ITCMIB(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}) = \sum_{q=1}^{V} ITCMIB_{q}\,(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)})$ as the integrated total cost for stable assessment of p.

Step 5: Replace $p + 1$ instead of p and repeat steps 2 to 4 to find $ITCMIB\,(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}, p)$.

Step 6: If $ITCMIB\,(Q_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}, p) \leq ITCMIB\,(Q_{iq}^{(p-1)}, L_{iq}^{(p-1)}, m_{iq}^{(p-1)}, p-1)$ then go to step 5, else go to step 7.

Step 7: Set $(Q_{iq}^{*}, L_{iq}^{*}, m_{iq}^{*}, p^{*}) = (Q_{iq}^{(p-1)}, L_{iq}^{(p-1)}, m_{iq}^{(p-1)}, p-1)$ and $(Q_{iq}^{*}, L_{iq}^{*}, m_{iq}^{*}, p^{*})$ is the optimum result of the crisp model.

3.2. Fuzzy Multi-item with Multi-purchaser Inventory System

The fuzzy-integrated total cost for multi-item with multi-purchaser in a two-level inventory system constructed on the graded mean integration technique is given below:

3.2.1. Pentagonal Fuzzy Number by Graded Mean Integration Technique (Nagar and Surana [15])

The graded mean integration technique for ã is defined by $\tilde{a} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$ as a pentagonal fuzzy number. Then, defuzzification is

$$P(\tilde{\alpha}) = \frac{1}{2}\frac{\int_0^1 \frac{h}{2}[\alpha_1 + \alpha_2 + (\alpha_3 - \alpha_1)h + \alpha_4 + \alpha_5 - (\alpha_5 - \alpha_3)h]dh}{\int_0^1 hdh},$$

$$P(\tilde{\alpha}) = \frac{1}{12}(\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5). \quad (13)$$

3.2.2. Integrated Total Cost for Fuzzy Multi-item with Multi-purchaser Inventory System

All over this paper, succeeding decision variable and parameters are beneficial to shorten the action of fuzzy quantities. Take $\tilde{D}_{iq}, \tilde{A}_{iq}, \tilde{S}_{iq}, \tilde{r}_{iq}, \tilde{P}_{iq}, \tilde{C}_{viq}, \tilde{C}_{biq}, V\tilde{E}C_{biq}$, and $V\tilde{T}C_{biq}$ are fuzzy parameters. Currently, fuzzy multi-item with multi-purchaser inventory system brings together fuzzy order quantity $\tilde{Q}_{iq}$ to be a pentagonal fuzzy number $\tilde{Q}_{iq} = (Q_{iq1}, Q_{iq2}, Q_{iq3}, Q_{iq4}, Q_{iq5})$ with constraint $0<Q_{iq1}\leq Q_{iq2}\leq Q_{iq3}\leq Q_{iq4}\leq Q_{iq5}$)

The fuzzy integrated total cost for multi-item of $q-$th purchaser (Chen [4]) is

$$\begin{aligned} I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq}, L_{iq}, m_{iq}) = {} & (\tilde{D}_{iq} \oslash \tilde{Q}_{iq}) \otimes (\tilde{A}_{iq} \oplus (\tilde{S}_{iq} \oslash m_{iq}) \oplus R(L_{iq}))] \ominus [[(\tilde{Q}_{iq} \otimes \tilde{r}_{iq} \otimes \tilde{C}_{viq}) \oslash 2] \otimes [(m_{iq} \otimes \tilde{D}_{iq} \oslash \tilde{P}_{iq}) + 1] \\ & \oplus [[(\tilde{Q}_{iq} \otimes \tilde{r}_{iq}) \oslash 2] \otimes [[m_{iq} \oplus ((2 \otimes \tilde{D}_{iq}) \oslash \tilde{P}_{iq})] \otimes \tilde{C}_{viq} \oplus \tilde{C}_{biq}]] \oplus [\tilde{r}_{iq} \otimes \tilde{C}_{biq} \otimes k_{iq} \otimes \sigma_{iq} \otimes \sqrt{L_{iq}}] \\ & + m_{iq} \otimes (FTC_{viq} + V\tilde{T}C_{viq}) + (m_{iq} \otimes FEC_{viq} + \tilde{Q}_{iq} \otimes V\tilde{E}C_{viq}). \end{aligned} \quad (14)$$

where $\oslash$, $\oplus$, $\ominus$ and $\otimes$ are the fuzzy arithmetical operatives under function principle.

Assume $\tilde{D}_{iq} = (D_{iq1}, D_{iq2}, D_{iq3}, D_{iq4}, D_{iq5})$, $\tilde{A}_{iq} = (A_{iq1}, A_{iq2}, A_{iq3}, A_{iq4}, A_{iq5})$, $\tilde{r}_{iq} = (r_{iq1}, r_{iq2}, r_{iq3}, r_{iq4}, r_{iq5})$, $\tilde{S}_{iq} = (S_{iq1}, S_{iq2}, S_{iq3}, S_{iq4}, S_{iq5})$, $\tilde{P}_{iq} = (P_{iq1}, P_{iq2}, P_{iq3}, P_{iq4}, P_{iq5})$, $\tilde{C}_{vqi} = (C_{viq1}, C_{viq2}, C_{viq3}, C_{viq4}, C_{viq5})$, $\tilde{C}_{biq} = (C_{biq1}, C_{biq2}, C_{biq3}, C_{biq4}, C_{biq5})$, $V\tilde{E}C_{biq} = (VEC_{biq1}, VEC_{biq2}, VEC_{biq3}, VEC_{biq4}, VEC_{biq5})$, and $V\tilde{T}C_{biq} = (VTC_{biq1}, VTC_{biq2}, VTC_{biq3}, VTC_{biq4}, VTC_{biq5})$ are positive

pentagonal fuzzy numbers. Then, the optimal order quantity for each item of every purchaser of equation (14) is obtained as follows.

Fuzzy integrated total cost for each item of every purchaser $I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq}, L_{iq}, m_{iq})$ is given in equation (14). Then,

$$
\begin{aligned}
I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq},L_{iq},m_{iq}) = &\left(\frac{D_{iq1}}{Q_{iq5}}\left(A_{iq1}+\frac{S_{iq1}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq5}r_{iq5}C_{viq5}}{2}\left(\frac{m_{iq}D_{iq5}}{P_{iq1}}+1\right)+\frac{Q_{iq1}r_{iq1}}{2}\left(\left(m_{iq}+\frac{2D_{iq1}}{P_{iq5}}\right)C_{viq1}+C_{biq1}\right)\right.\\
&\left.+r_{iq1}C_{biq1}k_{iq}\sigma_{iq}\sqrt{L_{iq1}}+Q_{iq1}VEC_{viq1}+m_{iq}(FTC_{viq}+VTC_{viq1}+FEC_{viq})\right),\\
&\left(\frac{D_{iq2}}{Q_{iq4}}\left(A_{iq2}+\frac{S_{iq2}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq4}r_{iq4}C_{viq4}}{2}\left(\frac{m_{iq}D_{iq4}}{P_{iq2}}+1\right)+\frac{Q_{iq2}r_{iq2}}{2}\left(\left(m_{iq}+\frac{2D_{iq2}}{P_{iq4}}\right)C_{viq2}+C_{biq2}\right)\right.\\
&\left.+r_{iq2}C_{biq2}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq}VEC_{viq2}+m_{iq}(FTC_{viq}+VTC_{viq2}+FEC_{viq})\right),\\
&\left(\frac{D_{iq3}}{Q_{iq3}}\left(A_{iq3}+\frac{S_{iq3}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq3}r_{iq3}C_{viq3}}{2}\left(\frac{m_{iq}D_{iq3}}{P_{iq3}}+1\right)+\frac{Q_{iq3}r_{iq3}}{2}\left(\left(m_{iq}+\frac{2D_{iq3}}{P_{iq3}}\right)C_{viq3}+C_{biq3}\right)\right.\\
&\left.+r_{iq3}C_{biq3}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{i3}VEC_{viq3}+m_{iq}(FTC_{vi}+VTC_{viq3}+FEC_{viq})\right),\\
&\left(\frac{D_{iq4}}{Q_{iq2}}\left(A_{iq4}+\frac{S_{iq4}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq2}r_{iq2}C_{viq2}}{2}\left(\frac{m_{iq}D_{iq2}}{P_{iq4}}+1\right)+\frac{Q_{iq4}r_{iq4}}{2}\left(\left(m_{iq}+\frac{2D_{iq4}}{P_{iq2}}\right)C_{viq4}+C_{biq4}\right)\right.\\
&\left.+r_{iq4}C_{biq4}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq4}VEC_{viq4}+m_{iq}(FTC_{viq}+VTC_{viq4}+FEC_{viq})\right),\\
&\left(\frac{D_{iq5}}{Q_{iq1}}\left(A_{iq5}+\frac{S_{iq5}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq1}r_{iq1}C_{viq1}}{2}\left(\frac{m_{iq}D_{iq1}}{P_{iq5}}+1\right)+\frac{Q_{iq5}r_{iq5}}{2}\left(\left(m_{iq}+\frac{2D_{iq5}}{P_{iq1}}\right)C_{viq5}+C_{biq5}\right)\right.\\
&\left.+r_{iq5}C_{biq5}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq5}VEC_{viq5}+m_{iq}(FTC_{viq}+VTC_{viq5}+FEC_{viq})\right).
\end{aligned}
\tag{15}
$$

Also, the Graded mean integration representation of $I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq},L_{iq},m_{iq})$ is obtained by equation (13) as

$$
\begin{aligned}
P(I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq},L_{iq},m_{iq})) = &\left[\frac{1}{12}\left(\frac{D_{iq1}}{Q_{iq5}}\left(A_{iq1}+\frac{S_{iq1}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq5}r_{iq5}C_{viq5}}{2}\left(\frac{m_{iq}D_{iq5}}{P_{iq1}}+1\right)+\frac{Q_{iq1}r_{iq1}}{2}\left(\left(m_{iq}+\frac{2D_{iq1}}{P_{iq5}}\right)C_{viq1}+C_{biq1}\right)\right.\right.\\
&\left.+r_{iq1}C_{biq1}k_{iq}\sigma_{iq}\sqrt{L_{iq1}}+Q_{iq1}VEC_{viq1}+m_{iq}(FTC_{viq}+VTC_{viq1}+FEC_{viq})\right)\\
&+\frac{3}{12}\left(\frac{D_{iq2}}{Q_{iq4}}\left(A_{iq2}+\frac{S_{iq2}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq4}r_{iq4}C_{viq4}}{2}\left(\frac{m_{iq}D_{iq4}}{P_{iq2}}+1\right)+\frac{Q_{iq2}r_{iq2}}{2}\left(\left(m_{iq}+\frac{2D_{iq2}}{P_{iq4}}\right)C_{viq2}+C_{biq2}\right)\right.\\
&\left.+r_{iq2}C_{biq2}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq}VEC_{viq2}+m_{iq}(FTC_{viq}+VTC_{viq2}+FEC_{viq})\right)\\
&+\frac{4}{12}\left(\frac{D_{iq3}}{Q_{iq3}}\left(A_{iq3}+\frac{S_{iq3}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq3}r_{iq3}C_{viq3}}{2}\left(\frac{m_{iq}D_{iq3}}{P_{iq3}}+1\right)+\frac{Q_{iq3}r_{iq3}}{2}\left(\left(m_{iq}+\frac{2D_{iq3}}{P_{iq3}}\right)C_{viq3}+C_{biq3}\right)\right.\\
&\left.+r_{iq3}C_{biq3}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{i3}VEC_{vi3}+m_{iq}(FTC_{viq}+VTC_{viq3}+FEC_{viq})\right)\\
&+\frac{3}{12}\left(\frac{D_{iq4}}{Q_{iq2}}\left(A_{iq4}+\frac{S_{iq4}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq2}r_{iq2}C_{viq2}}{2}\left(\frac{m_{iq}D_{iq2}}{P_{iq4}}+1\right)+\frac{Q_{iq4}r_{iq4}}{2}\left(\left(m_{iq}+\frac{2D_{iq4}}{P_{iq2}}\right)C_{viq4}+C_{biq4}\right)\right.\\
&\left.+r_{iq4}C_{biq4}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq4}VEC_{viq4}+m_{iq}(FTC_{viq}+VTC_{viq4}+FEC_{viq})\right)\\
&+\frac{1}{12}\left(\frac{D_{iq5}}{Q_{iq1}}\left(A_{iq5}+\frac{S_{iq5}}{m_{iq}}+R(L_{iq})\right)-\frac{Q_{iq1}r_{iq1}C_{viq1}}{2}\left(\frac{m_{iq}D_{iq1}}{P_{iq5}}+1\right)+\frac{Q_{iq5}r_{iq5}}{2}\left(\left(m_{iq}+\frac{2D_{iq5}}{P_{iq1}}\right)C_{viq5}+C_{biq5}\right)\right.\\
&\left.\left.+r_{iq5}C_{biq5}k_{iq}\sigma_{iq}\sqrt{L_{iq}}+Q_{iq5}VEC_{viq5}+m_{iq}(FTC_{viq}+VTC_{viq5}+FEC_{viq})\right)\right].
\end{aligned}
\tag{16}
$$

The integrated total cost of $q-$th purchaser for all U items is represented by $I\tilde{T}CMIB_q$, that is $I\tilde{T}CMIB_q=\sum_{i=1}^{U}P(I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq},L_{iq},m_{iq}))$ for q = 1, 2,....,V and the fuzzy integrated total cost of the whole system is

$$I\tilde{T}CMIB = \sum_{q=1}^{V} I\tilde{T}CMIB_q = \sum_{q=1}^{V}\sum_{i=1}^{U} P(I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq}, L_{iq}, m_{iq})), \tag{17}$$

where $0<Q_{iq1}\leq Q_{iq2}\leq Q_{iq3}\leq Q_{iq4}\leq Q_{iq5}$. We esteem $P(I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq}, L_{iq}, m_{iq}))$ in this way determine for the integrated total cost per unit as the fuzzy situation. Exchange the inequality condition $0<Q_{iq1}\leq Q_{iq2}\leq Q_{iq3}\leq Q_{iq4}\leq Q_{iq5}$ with $Q_{iq2}-Q_{iq1}\geq 0$, $Q_{iq3}-Q_{iq2}\geq 0$, $Q_{iq4}-Q_{iq3}\geq 0$, $Q_{iq5}-Q_{iq4}\geq 0$, and $Q_{iq1}>0$, equation (16) will remain the same.

In the resulting steps, addition of Kuhn-Tucker process (Taha [18]) is used to get Q_{iq1}, Q_{iq2}, Q_{iq3}, Q_{iq4}, and Q_{iq5}, to minimize $P(I\tilde{T}CMIB_{iq}(\tilde{Q}_{iq}, L_{iq}, m_{iq}))$ in equation (16).

An optimal solution of $P(I\tilde{T}CMIB_{iq})$ is found by applying the Kuhn-Tucker process subject to five inequalities as imposed conditions. The conditions are as follows $\lambda_{iq}\geq 0$, $\nabla P(I\tilde{T}CMIB_{iq}) - \lambda_{iq}\nabla \mathrm{E}(\tilde{Q}_{iq}, L_{iq}, m_{iq})] = 0, \lambda_{iq}\mathrm{E}[(\tilde{Q}_{iq}, L_{iq}, m_{iq})] = 0, \mathrm{E}[(\tilde{Q}_{iq}, L_{iq}, m_{iq})] \leq 0$

These conditions shorten as

$$\nabla P(I\tilde{T}CMIB_{iq}) - \lambda_{iq1}(Q_{iq1}-Q_{iq2}) - \lambda_{iq2}(Q_{iq2}-Q_{iq3}) - \lambda_{iq3}(Q_{iq3}-Q_{iq4}) - \lambda_{iq4}(Q_{iq4}-Q_{iq5}) - \lambda_{iq5}(-Q_{iq1}) = 0, \tag{18}$$

$$\frac{1}{12}\left[-\frac{D_{iq5}}{Q_{iq1}^{\;2}}\left(A_{iq5}+\frac{S_{iq5}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq1}C_{viq1}}{2}\left(\frac{m_{iq}D_{iq1}}{P_{iq5}}+1\right)+\frac{r_{iq1}}{2}\left(\left(m_{iq}+\frac{2D_{iq1}}{P_{iq5}}\right)C_{viq1}+C_{biq1}\right)+VEC_{viq1}\right]-\lambda_{iq1}+\lambda_{iq5}=0, \tag{19}$$

$$\frac{3}{12}\left[-\frac{D_{iq4}}{Q_{iq2}^{\;2}}\left(A_{iq4}+\frac{S_{iq4}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq2}C_{viq2}}{2}\left(\frac{m_{iq}D_{iq2}}{P_{iq4}}+1\right)+\frac{r_{iq2}}{2}\left(\left(m_{iq}+\frac{2D_{iq2}}{P_{iq4}}\right)C_{viq2}+C_{biq2}\right)+VEC_{viq2}\right]-\lambda_{iq2}+\lambda_{iq1}=0, \tag{20}$$

$$\frac{4}{12}\left[-\frac{D_{iq3}}{Q_{iq3}^{\;2}}\left(A_{iq3}+\frac{S_{iq3}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq3}C_{viq3}}{2}\left(\frac{m_{iq}D_{iq3}}{P_{iq3}}+1\right)+\frac{r_{iq3}}{2}\left(\left(m_{iq}+\frac{2D_{iq3}}{P_{iq3}}\right)C_{viq3}+C_{biq3}\right)+VEC_{viq3}\right]-\lambda_{iq3}+\lambda_{iq2}=0, \tag{21}$$

$$\frac{3}{12}\left[-\frac{D_{iq2}}{Q_{iq4}^{\;2}}\left(A_{iq2}+\frac{S_{iq2}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq4}C_{viq4}}{2}\left(\frac{m_{iq}D_{iq4}}{P_{iq2}}+1\right)+\frac{r_{iq4}}{2}\left(\left(m_{iq}+\frac{2D_{iq4}}{P_{iq2}}\right)C_{viq4}+C_{biq4}\right)+VEC_{viq4}\right]-\lambda_{iq4}+\lambda_{iq3}=0, \tag{22}$$

$$\frac{1}{12}\left[-\frac{D_{iq1}}{Q_{iq5}^{\;2}}\left(A_{iq1}+\frac{S_{iq1}}{m_{iq}}+R(L_{iq})\right)-\frac{r_{iq5}C_{viq5}}{2}\left(\frac{m_{iq}D_{iq5}}{P_{iq1}}+1\right)+\frac{r_{iq5}}{2}\left(\left(m_{iq}+\frac{2D_{iq5}}{P_{iq1}}\right)C_{viq5}+C_{biq5}\right)+VEC_{viq5}\right]+\lambda_{iq4}=0, \tag{23}$$

$$Q_{iqj} - Q_{iq(j+1)} \leq 0, \qquad j = 1, 2, 3, 4, \tag{24}$$

$$-Q_{iq1} < 0, \tag{25}$$

$$\lambda_{iqj}(Q_{iqj} - Q_{iq(j+1)}) = 0, \qquad j = 1, 2, 3, 4, \tag{26}$$

$$\lambda_{iq5}(-Q_{iq1}) = 0, \tag{27}$$

$$Q_{iqj} \geq 0, \qquad j = 1,2,3,4,5, \quad i = 1,2,.....,5 \text{ and } \lambda_{iqj} \geq 0. \tag{28}$$

As Q_{iq1} >0, and $\lambda_{iq5}Q_{iq1} = 0$, $\lambda_{iq5} = 0$. If $\lambda_{iq1} = \lambda_{iq2} = \lambda_{iq3} = \lambda_{iq4} = 0$, then $Q_{iq5}<Q_{iq4}<Q_{iq3} < Q_{iq2}<Q_{iq1}$, that cannot satisfy the constraints $0<Q_{iq1} \leq Q_{iq2} \leq Q_{iq3} \leq Q_{iq4} \leq Q_{iq5}$. Therefore, $Q_{ia1} = Q_{iq2}$, $Q_{iq2} = Q_{iq3}$, $Q_{iq3} = Q_{iq4}$, $Q_{iq4} = Q_{iq5}$, that is $Q_{iq1} = Q_{iq2} = Q_{iq3} = Q_{iq4} = Q_{iq5} = \tilde{Q}^*_{iq}$. Hence, from equations (19)–(23), the fuzzy i – th item for q – purchaser optimal order quantity $\tilde{Q}^*_{iq}$ is obtained as follows

$$\tilde{Q}_{iq}^* = \sqrt{\frac{2D_{iq5}\left(A_{iq5}+\frac{S_{iq5}}{m_{iq}}+R(L_{iq})\right)+6D_{iq4}\left(A_{iq4}+\frac{S_{iq4}}{m_{iq}}+R(L_{iq})\right)+8D_{iq3}\left(A_{iq3}+\frac{S_{iq3}}{m_{iq}}+R(L_{iq})\right)+6D_{iq2}\left(A_{iq2}+\frac{S_{iq2}}{m_{iq}}+R(L_{iq})\right)+2D_{iq1}\left(A_{iq1}+\frac{S_{iq1}}{m_{iq}}+R(L_{iq})\right)}{\begin{array}{l}\left[r_{iq1}\left(\left(m_{iq}\left(1-\frac{D_{iq1}}{P_{iq5}}\right)-1+\frac{2D_{iq1}}{P_{iq5}}\right)C_{viq1}+C_{biq1}\right)+2VEC_{viq1}\right]+3\left[r_{iq2}\left(\left(m_{iq}\left(1-\frac{D_{iq2}}{P_{iq4}}\right)-1+\frac{2D_{iq2}}{P_{iq4}}\right)C_{viq2}+C_{biq2}\right)+2VEC_{viq2}\right]\\ 4\left[r_{iq3}\left(\left(m_{iq}\left(1-\frac{D_{iq3}}{P_{iq3}}\right)-1+\frac{2D_{iq3}}{P_{iq3}}\right)C_{viq3}+C_{biq3}\right)+2VEC_{viq3}\right]+3\left[r_{iq4}\left(\left(m_{iq}\left(1-\frac{D_{iq4}}{P_{iq2}}\right)-1+\frac{2D_{iq4}}{P_{iq2}}\right)C_{viq4}+C_{biq4}\right)+2VEC_{viq4}\right]\\ +\left[r_{iq5}\left(\left(m_{iq}\left(1-\frac{D_{iq5}}{P_{iq1}}\right)-1+\frac{2D_{iq5}}{P_{iq1}}\right)C_{viq5}+C_{biq5}\right)+2VEC_{viq5}\right]\end{array}}}. \tag{29}$$

The optimum fuzzy integrated total cost for each item $P(I\tilde{T}CMIB_{iq})$ is obtained by direct substitution of equation (29) into equation (16).

3.2.3. Algorithm for Fuzzy Inventory System

Multi-item multi-purchaser's order quantities are calculated using the subsequent algorithm to determine each item's optimal order quantity for each purchaser. Then, the minimum integrated total cost for fuzzy system is found.

Algorithm: 2

Step 1: Fix the iteration p = 1.

Step 2: Each L_{iqw}, m_{iqw}, $w = 0,1,2,...t_{iq}$ determine $\tilde{Q}_{iq}$ for all i=1,2,3,...U, q=1,2,3,...V and find the corresponding $I\tilde{T}CMIB_q\,(\tilde{Q}_{iq}, L_{iqw}, m_{iqw})$

Step 3: Set $I\tilde{T}CMIB_q\,(\tilde{Q}_{iq}{}^{(p)}, L_{iqw}{}^{(p)}, m_{iqw}{}^{(p)}) = \underset{w=0,1,2,3,...,t_{ij}}{Min}\; I\tilde{T}CMIB_q\,(\tilde{Q}_{iq}, L_{iqw}, m_{iqw})$ and $(\tilde{Q}_{iq}{}^{(p)}, L_{iq}{}^{(p)}, m_{iq}{}^{(p)})$ as the optimal solution of fixed p.

Step 4: Calculate as the $I\tilde{T}CMIB(\tilde{Q}_{iq}{}^{(p)}, L_{iq}{}^{(p)}, m_{iq}{}^{(p)}) = \sum_{q=1}^{V} I\tilde{T}CMIB_q\,(\tilde{Q}_{iq}{}^{(p)}, L_{iqw}{}^{(p)}, m_{iqw}{}^{(p)})$ integrated total cost for stable assessment of p.

Step 5: Replace p+1 instead of p and repeat steps 2 to 4 to obtain

$I\tilde{T}CMIB(\tilde{Q}_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}, p)$.

Step 6: If $I\tilde{T}CMIB(\tilde{Q}_{iq}^{(p)}, L_{iq}^{(p)}, m_{iq}^{(p)}, p) \le I\tilde{T}CMIB(\tilde{Q}_{iq}^{(p-1)}, L_{iq}^{(p-1)}, m_{iq}^{(p-1)}, p-1)$ then go to step 5, else go to step 7.

Step 7: Set $(\tilde{Q}_{iq}^{*}, L_{iq}^{*}, m_{iq}^{*}, p^{*}) = (\tilde{Q}_{iq}^{(p-1)}, L_{iq}^{(p-1)}, m_{iq}^{(p-1)}, p-1)$ and $(\tilde{Q}_{iq}^{*}, L_{iq}^{*}, m_{iq}^{*}, p^{*})$ is the optimum result of the fuzzy system.

4. Numerical Examples

Numerical cases are specified to exhibit the above outcome technique utilizing the suggested algorithms. Subsequently, the finest multi-item with multi-purchaser inventory system is recognised. The results to these examples are achieved through Mat lab software. The suggested fuzzy multi-item with multi-purchaser inventory system can be used in businesses such as vehicles, tires, healthcare products, computer hardware, textiles, home appliances (refrigerators, televisions, air conditioners, and washing machines), massive objects like produced trip panels, cell phones, and so on. The projected integrated multi-item with multi-purchaser inventory system is extra effective aimed at the supply chain business progression of vendor-purchaser administration.

Example. 1

Multi-item with Multi-purchaser Crisp Inventory System

The results demonstrate crisp model with initial inputs taken from (Pan and Yang [16]). The remaining input is made-up related to the problem.

Let us consider the integrated multi-item with multi-purchaser inventory system for four items and four purchasers. That is $U = V = 4$ and identical parameters are FEC_{viq} = \$0.2/shipment, FTC_{viq}=\$0.2/shipment in place of entire i=1, 2, 3, 4 also q=1, 2, 3, 4. In Table 1 and Table 3, certain parameters are specified and it is the same for all purchasers. Table 2 comprises every item based demand for all purchasers and Table 4 comprises the lead time data for all purchasers. To diminish the complication, the entire items are supposed to have same lead time for the $q-$th purchaser, i.e., L_{i1} is the lead time for getting all items for the first purchaser and so on. The summarized lead time data is arranged in Table 4.

Using algorithm 1, the optimal solutions are tabulated in Table 7. From Table 7, it is perceived that for the 1st shipment $m_{i1}=3$ the normal duration of the first purchaser is $L_{i1}=3$ weeks, crashing cost is $R(L_{i1})$=53.2, the resultant optimal order quantities be $(Q_{11}, Q_{21}, Q_{31}, Q_{41})$=(186.14, 188.34, 185.18, 187.19) units, and the crisp integrated total cost of all items of the first purchaser is $ 10918. When the shipment is $m_{i1}=4$ lead time $L_{i1}=4$ weeks, crashing cost is $R(L_{i1})=18.2$, the resultant optimal order quantities be $(Q_{11},Q_{21},Q_{31},Q_{41})$=(138.17, 139.00, 137.81, 138.57) units, and the integrated total cost of all items of the first purchaser is $10139. Similarly, for the shipment $m_{i1}=5$ lead time $L_{i1}=6$ weeks, crashing cost is $R(L_{i1})=1.4$, $m_{i1}=5$, $L_{i1}=8$ weeks, crashing cost is $R(L_{i1})=0$, the optimal order quantities and integrated total costs are $(Q_{11},Q_{21},Q_{31},Q_{41})$=(109.36, 109.42, 109.33, 109.39) units, $9749.3 and $(Q_{11},Q_{21},Q_{31},Q_{41})$=(108.68, 108.68, 108.68, 108.68) units, $9839.1 correspondingly.

Table 1: Common purchaser data for all items

Item i	P_{iq}	C_{viq}	C_{biq}	r_{iq}	S_{iq}	VTC_{viq}	VEC_{viq}	A_{iq}
1	3520	22	27.5	0.22	440	0.55	0.11	24.057
2	3200	20	25	0.2	400	0.5	0.1	21.87
3	3680	23	28.75	0.23	460	0.575	0.115	25.1505
4	3360	21	26.25	0.21	420	0.525	0.105	22.9635

Table 2: Demand of i−th item for the q−th purchaser

D_{iq}	q=1	q=2	q=3	q=4
i=1	1100	1980	1760	1430
i=2	1000	1800	1600	1300
i=3	1150	2070	1840	1495
i=4	1050	1890	1680	1365

Table 3: Common item data for all purchasers

Purchaser q	σ_{iq}	k_{iq}
1	7	2.33
2	8	2.097
3	9	1.864
4	10	1.631

Table 4: Summarized lead time and shipment data

Purchaser q	$(L_{iq}, R(L_{iq}), m_{iq})$ $p=1$	$(L_{iq}, R(L_{iq}), m_{iq})$ $p=2$	$(L_{iq}, R(L_{iq}), m_{iq})$ $p=3$	$(L_{iq}, R(L_{iq}), m_{iq})$ $p=4$
$q=1$	(3, 53.2, 3)	(4, 18.2, 4)	(6, 1.4, 5)	(8, 0, 5)
$q=2$	(3, 53.2, 3)	(4, 18.2, 4)	(6, 1.4, 5)	(8, 0, 5)
$q=3$	(3, 53.2, 3)	(4, 18.2, 4)	(6, 1.4, 5)	(8, 0, 5)
$q=4$	(3, 53.2, 3)	(4, 18.2, 4)	(6, 1.4, 5)	(8, 0, 5)

Bounded by the total costs of the first purchaser $ITCMIB_1 = \in$ {$10918, $10139, $9749.4, $9839.1}, the minimum value is $9749.4, therefore $ITCMIB_1$= $9749.4. In the same manner, $ITCMIB_2$= $11690, $ITCMIB_3$= $11336, and $ITCMIB_4$= $10653 are calculated and the integrated total cost of the system, $ITCMIB$=

\$9749.4+\$11690+\$11336+\$10653 = \$43428.4. This *ITCMIB*=\$43428.4 is the crisp minimum integrated total cost of the system. For the lead time L_{iq} = 3, 4, 6, and 8 weeks, the integrated total cost of the system is \$43428.4. Clearly, the minimum integrated total cost for each purchaser is ($ITCMIB_1$, $ITCMIB_2$, $ITCMIB_3$, $ITCMIB_4$)=(\$9749.4, \$11690, \$11336, \$10653).

Example. 2

Multi-item with Multi-purchaser Fuzzy Inventory System

The inputs are same as in Example 1, except the fuzzy inputs that are given in Table 5 and Table 6.

Table 5: Common $q-$ th purchaser data for all items

	i= 1, 2, 3, 4
P_{iq}	P_{iq1} = (3168, 2880, 3312, 3024), P_{iq2} = (3344, 3040, 3496, 3192), P_{iq3} = (3520, 3200, 3680, 3360), P_{iq4} = (3696, 3360, 3864, 3528), P_{iq5} = (3872, 3520, 4048, 3696).
C_{viq}	C_{viq1} = (19.8, 18, 20.7, 18.9),C_{viq2} = (20.9, 19, 21.85, 19.95), C_{viq3} = (22, 20, 23, 21), C_{viq4} = (23.1, 21, 24.15, 22.05), C_{viq5} = (24.2, 22, 25.3, 23.1).
C_{biq}	C_{biq1} = (24.75, 22.5, 25.875, 23.625), C_{biq2} = (26.125, 23.75, 27.3125, 24.9375), C_{biq3} = (27.5, 25, 28.75, 26.25), C_{biq4} = (28.875, 26.25, 30.1875, 27.5625), C_{biq5} = (30.25, 27.5, 31.625, 28.875).
r_{iq}	r_{iq1} =(0.198, 0.18, 0.207, 0.189), r_{iq2} =(0.209, 0.19, 0.2185, 0.1995), r_{iq3} = (0.22, 0.2, 0.23, 0.21), r_{iq4} =(0.231, 0.21, 0.2415, 0.2205), r_{iq5} = (0.242, 0.22, 0.253, 0.231).
S_{iq}	S_{iq1} =(396, 360, 414, 378), S_{iq2} = (418, 380, 437, 399), S_{iq3} = (440, 400, 460, 420), S_{iq4} = (462, 420, 483, 441), S_{iq5}= (484, 440, 506, 462).
VTC_{viq}	VTC_{viq1}=(0.495, 0.45, 0.5175, 0.4725), VTC_{viq2}=(0.5225, 0.475, 0.54625, 0.49875), VTC_{viq3}= (0.55, 0.5, 0.575, 0.525), VTC_{viq4}=(0.5775, 0.525, 0.60375, 0.55125), VTC_{viq5}=(0.605, 0.55, 0.6325, 0.5775).
VEC_{viq}	VEC_{viq1}=(0.099, 0.09, 0.1035, 0.0945), VEC_{viq2}=(0.1045, 0.095, 0.10925, 0.09975), VEC_{viq3}=(0.11, 0.1, 0.115, 0.105), VEC_{viq4}=(0.1155, 0.105, 0.12075, 0.11025), VEC_{viq5}=(0.121, 0.11, 0.1265, 0.1155).
A_{iq}	A_{iq1}=(21.6513, 19.683, 22.63545, 20.66715), A_{iq2}=(22.85415, 20.7765, 23.892975, 21.815325), A_{iq3}=(24.057, 21.87, 25.1505, 22.9635), A_{iq4}=(25.25985, 22.9635, 26.408025, 24.111675), A_{iq5}=(26.4627, 24.057, 27.66555, 25.25985).

Table 6: Demand of $i-$ th item for the $q-$ th purchaser

$\tilde{D}_{iq}$	i = 1, 2, 3, 4
q = 1	D_{iq1} = (990, 900, 1035, 945), D_{iq2} = (1045, 950, 1092.5, 997.5), D_{iq3} = (1100, 1000, 1150, 1050), D_{iq4} = (1155, 1050, 1207.5, 1102.5), D_{iq5} = (1210, 1100, 1265, 1155).

$q = 2$	D_{iq1} = (1782, 1620, 1863, 1701), D_{iq2} = (1881, 1710, 1966.5, 1795.5), D_{iq3} = (1980, 1800, 2070, 1890), D_{iq4}= (2079, 1890, 2173.5, 1984.5), D_{iq5} = (2178, 1980, 2277, 2079).
$q = 3$	D_{iq1} = (1584, 1440, 1656, 1512), D_{iq2} = (1672, 1520, 1748, 1596), D_{iq3} = (1760, 1600, 1840, 1680), D_{iq4} = (1848, 1680, 1932, 1764), D_{iq5} = (1936, 1760, 2024, 1848).
$q = 4$	D_{iq1} = (1287, 1170, 1345.5, 1228.5), D_{iq2} = (1358.5, 1235, 1420.25, 1296.75), D_{iq3} = (1430, 1300, 1495, 1365), D_{iq4} = (1501.5, 1365, 1569.8, 1433.3), D_{iq5} = (1573, 1430, 1644.5, 1501.5).

Using algorithm 2, the results are depicted in Table 7. It is perceived that for the 1st shipment $m_{i1}=3$ the normal duration of the first purchaser is $L_{i1}=3$ weeks then the crashing cost is R $(L_{i1})= 53.2$ and the resultant optimal order quantities be $(\tilde{Q}_{11},\tilde{Q}_{21},\tilde{Q}_{31},\tilde{Q}_{41})$=(184.83, 186.87, 183.94, 185.80) units, and the fuzzy integrated total cost of all the item of the first purchaser is \$ 10889. When the shipment $m_{i1}=4$, lead time L_{i1}=4 weeks, then the crashing cost is $R(L_{i1})=18.2$, the resultant optimal order quantities be $(\tilde{Q}_{11},\tilde{Q}_{21},\tilde{Q}_{31},\tilde{Q}_{41})$ =(137.50, 138.23, 137.18, 137.85) units, and the integrated total cost of all the items of the first purchaser is \$10110.

Table 7: Crisp and fuzzy multi-item in multi-purchaser optimal solutions

Demand i=1, 2, 3, 4	L_{iq}	$R(L_{iq})$	m_{iq}	Q_{iq}	$ITCMIB_q$	$\tilde{Q}_{iq}$	$I\tilde{T}CMIB_q$
Purchaser -1	3	53.2	3	(186.14, 188.34, 185.18, 187.19)	10918	(184.83, 186.87, 183.94, 185.94)	10889
D_{i1} and $\tilde{D}_{i1}$	4	18.2	4	(138.17, 139.00, 137.81, 138.57)	10139	(137.50, 138.23, 137.18, 137.85)	10110
	6	**1.4**	**5**	**(109.36, 109.42, 108.33, 109.39)**	**9749.4**	**(108.99, 109.00, 108.99, 108.99)**	**9720.5**
	8	0	5	(108.68, 108.68, 108.68, 108.68)	9839.1	(108.32, 108.26, 108.34, 108.29)	9810.7
Purchaser-2 D_{i2} and $\tilde{D}_{i2}$	3	53.2	3	(261.10, 264.18, 259.74, 262.57)	13829.0	(259.30, 262.14, 258.06, 260.66)	13787
	4	18.2	4	(199.66, 200.85, 199.14, 200.23)	12450	(198.81, 199.85, 198.36, 199.31)	12405
	6	**1.4**	**5**	**(161.42, 161.52, 161.38, 161.47)**	**11690**	**(161.07, 161.06, 161.07, 161.07)**	**11644**
	8	0	5	(160.42, 160.42, 160.42, 160.42)	11768	(160.07, 159.97, 160.12, 160.02)	11722

Purchaser-3 D_{i3} and $\tilde{D}_{i3}$	3	53.2	3	(243.35, 246.22, 242.09, 244.72)	13217	(241.67, 244.32, 240.50, 242.93)	13179
	4	18.2	4	(184.59, 185.69, 184.11, 185.11)	11997	(183.77, 184.74, 183.35, 184.23)	11956
	6	**1.4**	**5**	**(148.34, 148.43, 148.30, 148.38)**	**11336**	**(147.96, 147.96, 147.96, 147.96)**	**11294**
	8	0	5	(147.42, 147.42, 147.42, 147.42)	11416	(147.05, 146.96, 147.09, 147.01)	11375
Purchaser-4 D_{i4} and $\tilde{D}_{i4}$	3	53.2	3	(215.71, 218.26, 214.59, 216.92)	12169	(214.20, 216.55, 213.17, 215.32)	12135
	4	18.2	4	(161.78, 162.75, 161.36, 162.24)	11175	(161.03, 161.88, 160.66, 161.43)	11140
	6	**1.4**	**5**	**(128.96, 129.04, 128.93, 129.00)**	**10653**	**(128.58, 128.59, 128.57, 128.58)**	**10618**
	8	0	5	(128.16, 128.16, 128.16, 128.16)	10737	(109.94, 108.80, 107.83, 107.01)	10702

Similarly, for the shipment m_{il} = 5, lead times L_{il} = 6 weeks, then crashing cost is $R(L_{il})$ = 1.4, m_{il} = 5, L_{il} = 8 weeks, then crashing cost is $R(L_{il})$ = 0,, the optimal order quantities and integrated total costs are $(\tilde{Q}_{11}, \tilde{Q}_{21}, \tilde{Q}_{31}, \tilde{Q}_{41})$ = (108.99, 109.00, 108.99, 108.99)units, \$9720.50 and $(\tilde{Q}_{11}, \tilde{Q}_{21}, \tilde{Q}_{31}, \tilde{Q}_{41})$=(108.32, 108.26, 108.34, 108.29) units, \$9810.70 correspondingly. Bounded by the total costs of the first purchaser $I\tilde{T}CMIB_1$ ∈ {\$10889, \$10110, \$9720.50, \$9810.70}, the minimum value is \$9720.50, therefore $I\tilde{T}CMIB_1$=\$9720.50. In the same manner, $I\tilde{T}CMIB_2$= \$11644, $I\tilde{T}CMIB_3$ = \$11294, and $I\tilde{T}CMIB_4$ = \$10618 is acquired, and the total cost of the fuzzy system be $I\tilde{T}CMIB$= \$9720.50+\$11644+\$11294+\$10618= \$43276.5.

This $I\tilde{T}CMIB$=\$43276.5 is the minimum integrated total cost of the fuzzy system. For the lead time L_{iq} = 3, 4, 6, and 8 weeks the integrated total costs of the fuzzy system \$43276.5 is acquired. Clearly, the minimum integrated total cost for each purchaser is ($I\tilde{T}CMIB_1$, $I\tilde{T}CMIB_2$, $I\tilde{T}CMIB_3$, $I\tilde{T}CMIB_4$) = (\$9720.50, \$11644, \$11294, \$1068).

Table 8: Summary of crisp and fuzzy optimal solutions

Demand i = 1, 2, 3, 4	L_i	Savings (%) for each item's Optimal order quantity	Savings (%) of multi-item Integrated total cost for each purchaser
D_{i1} and $\tilde{D}_{i1}$	3	(0.71, 0.78, 0.67, 0.74)	0.27
	4	(0.49, 0.55, 0.46, 0.52)	0.29
	6	(0.34, 0.39, 0.31, 0.36)	0.30
	8	(0.33, 0.39, 0.31, 0.36)	0.29
D_{i2} and $\tilde{D}_{i2}$	3	(0.69, 0.77, 0.65, 0.73)	0.30
	4	(0.42, 0.50, 0.39, 0.46)	0.36
	6	(0.22, 0.28, 0.19, 0.25)	0.39
	8	(0.22, 0.28, 0.19, 0.25)	0.39
D_{i3} and $\tilde{D}_{i3}$	3	(0.69, 0.78, 0.66, 0.73)	0.29
	4	(0.44, 0.51, 0.41, 0.47)	0.34
	6	(0.25, 0.31, 0.23, 0.28)	0.37
	8	(0.25, 0.31, 0.23, 0.28)	0.36
D_{i4} and $\tilde{D}_{i4}$	3	(0.70, 0.78, 0.66, 0.74)	0.28
	4	(0.47, 0.53, 0.44, 0.50)	0.31
	6	(0.30, 0.35, 0.27, 0.32)	0.33
	8	(0.30, 0.35, 0.27, 0.32)	0.33

4.1. Graphical Representations

Each purchaser's integrated total cost for dissimilar values of lead time and demand related mutually in the fuzzy and crisp systems is shown in the graphical representation, Figure (1) to Figure (4). Each purchaser's integrated total cost $ITCMIB_q$ and $I\tilde{T}CMIB_q$ varies while the lead time rises. It is observed that each purchaser's integrated total cost is profitably optimized in the fuzzy model compared to the crisp model.

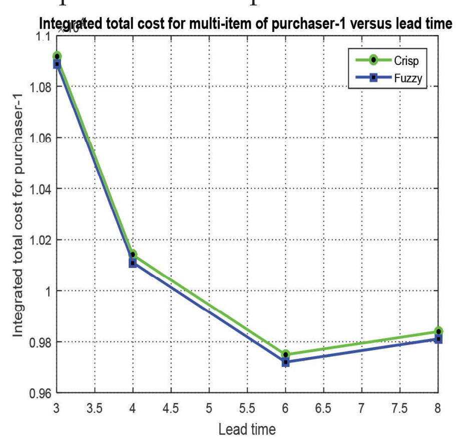

Figure 1: Integrated total cost for multi-item of purchaser-1 versus lead time.

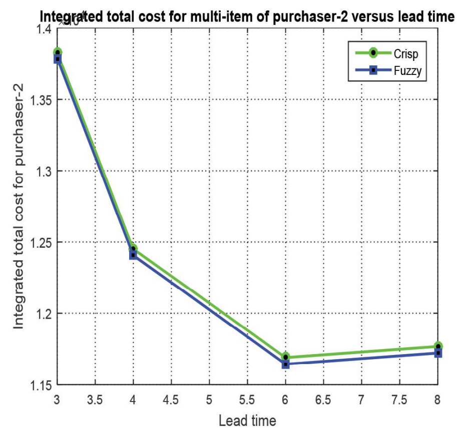

Figure 2: Integrated total cost for multi-item of purchaser-2 versus lead time.

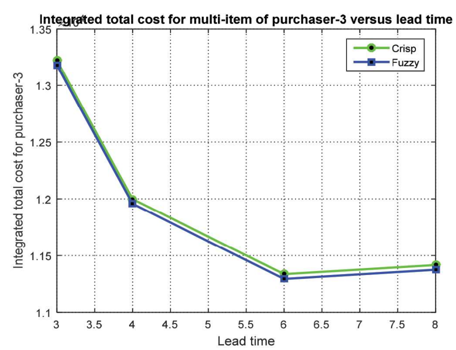

Figure 3: Integrated total cost for multi-item of purchaser-3 versus lead time.

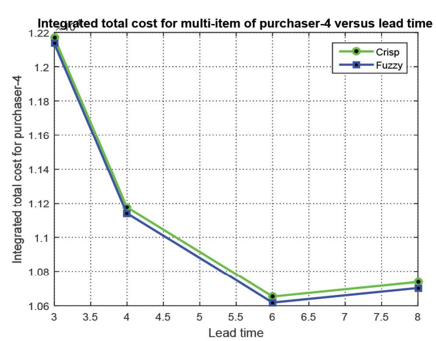

Figure 4: Integrated total cost for multi-item of purchaser-4 versus lead time.

Table 9: Summary of the comparisons

L_{iq}	$R(L_{iq})$	m_{iq}	Comparison	Integrated total cost for multi-item, multi-purchaser system
3	53.2	3	Crisp multi-item multi-purchaser inventory system	43428.4
4	18.2	4		
6	1.4	5	Fuzzy multi-item multi-purchaser inventory system	43276.5
8	0	5		
			Savings (%)	0.35

5. Comparative Study

Table 8 shows savings percentage of multi-item's optimal order quantity and each purchaser's integrated total cost for fuzzy system. In Table 9, the arithmetical outcomes are specified. The optimum standard of crisp multi-item multi-purchaser system's minimized integrated total cost is \$43428.4. The optimum value for fuzzy multi-item multi-purchaser system's minimized integrated total cost is \$43276.5. The relative variations for crisp and fuzzy models in integrated total cost for the systems can be grasped in Table 9. The comparison of crisp and fuzzy multi–item, multi-purchaser inventory model as well as integrated total cost saving percentage is 0.35%.

Fuzzy system supports the businesses manage indeterminate inventory price parameters. Indeterminate cost parameters of inventory management models are found to be optimistic and slightly significant. For this, administrations are capable to find optimum solution in beneficial manner.

6. Conclusion

Multi-item, multi-purchaser integrated supply chain system along lead time with carbon emission cost is established for fuzzy and crisp situations. In the fuzzy situation, wholly interrelated inventory inputs and decision variables are presumed through pentagonal fuzzy quantities. Among defuzzification, the graded mean technique is hired for the estimation of minimum integrated total cost for the multi-item, multi-purchaser system. The addition of Kuhn-Tucker technique is utilized to obtain each item's optimal order quantity for each purchaser. A computational algorithm is made use of for the exploration of special outcomes for fuzzy inputs on minimum multi-item, multi-purchaser integrated total cost. Each item's optimal order quantity for each purchaser is based on suggested inventory system. Graphical representations for the numerical examples are displayed for the suggested fuzzy system. A major quantity of reserves in a multi-item, multi-purchaser integrated supply chain system is found. Subsequently, on comparing the crisp and fuzzy systems, it is perceived that the multi-item, multi-purchaser of the fuzzy inventory system is better than the crisp inventory system.

Further investigations on this system can be made using inventory space limitations, setup cost restrictions, ordering constraints, etc. Moreover, different types of multi-level stream sequence systems can be demonstrated in a crisp situation, fuzzy situation, or together.

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