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A Sustainable Inventory Model for the Electrical Energy Supply Chain Considering Emission Reduction, Setup-cost Optimization, and Green Technology Investment

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Abstract

This article provides an analytical framework for a sustainable power supply chain based on linear, price-dependent customer demand, with setup costs and carbon emissions as choice variables. Customers, transmission and distribution substations, and power generation are all part of the sustainable electrical supply chain. Electricity production rates are determined by consumer demand, which is influenced by the price of electricity generated and by its transfer through several substations. Capital investment in green technologies can help to cut emissions. Additionally, we examined how distribution and transmission costs are affected by transmission rates, power generation, and the distance between stations. Here, we developed a new model based on inventory theory and proposed a strategy for determining the optimal solution. Lastly, the study demonstrates its findings and offers managerial insights through a sensitivity analysis and numerical illustration.

Keywords: sensitive demand, greenhouse gas emissions, transmission and distribution costs, sustainable power supply chain, and green technology investment.

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1. **Introduction**

A supply chain inventory system establishes the operational relationship between retailers and customers, facilitating the efficient movement of products and services. Within such systems, setup costs play a vital role in ensuring the smooth and timely distribution of goods. Although setup activities are not continuous, their effective management is essential for reducing operational inefficiencies, meeting delivery schedules, enhancing productivity, and improving resource allocation. Consequently, organizations seek to minimize setup-related expenditures in order to reduce overall investment costs and improve profitability. In the context of an electrical energy supply chain, the determination of energy capacity is analogous to traditional inventory decisions involving order quantities and profit optimization. Electricity generated at power plants is transmitted through transmission networks and distribution substations before reaching end users. The primary objective of this system is to maximize economic returns while satisfying customer demand under electricity price-driven market conditions.

The consumption of electricity energy has risen dramatically. Global electricity use increased from 1980 to 2015. Electricity use in the industrial, commercial, residential, and transportation sectors has increased since 2015. Climate change has led to an increase in home power consumption by over 100 million kWh/day, primarily due to the necessity for air cooling during summer months. Electricity prices may have affected demand for electricity. The price of power is meant to reduce demand. Commercial and residential clients typically pay the highest prices due to the higher expense of distributing electricity. According to a 2018 United Nations report, global greenhouse gas carbon emissions (GGCE) in all countries are increasing and falling short of the Paris Agreement goals. This comes ahead of a meeting of officials and environmental experts in Katowice, Poland, for climate negotiations. Several years prior, during the environmental negotiations in Paris, all nations committed to reduce greenhouse gas emissions sufficiently to limit global warming to below 2 degrees Celsius, and ideally under 1.5 degrees. A report from the UN’s foremost climate panel published earlier indicated that an additional 0.5 degrees of warming would have significant and detrimental effects on the environment, providing further empirical support for countries advocating for more stringent targets during the COP24 negotiations. To limit temperature increases to 1.5 degrees, global carbon emissions must peak by 2020. Negotiators anticipate that the decade-long trend of increasing carbon emissions will reverse, as global emissions remained relatively flat from 2014 to 2016. After years of stasis, global carbon emissions increased to a record 53.4 gigatons in 2017. To attain the 1.5-degree target, the globe must reduce emissions within the next 12 years. Governments worldwide should implement cap and-trade rules to reduce pollution. Production firms can reduce car bon emissions by monitoring and improving product emission performance throughout the life cycle. Carbon-emission assessments can help companies reduce their emissions. Manufacturers should develop a low-carbon production process.

1.1 Research Questions and Contribution of the Model

It is evident from the literature review section that while some studies attempt to identify sustainable electrical supply chain inventory models, none attempt to identify a model that reduces setup cost and capital investment of green technology. Our study asks the following questions of this model: (1) how does the distribution substation get affected by the electrical power distribution factor, and (2) how does the transmission substation and electrical power generation factor get affected by the electrical power transmission factor; (3) how long does the customer typically use electricity? and (4) what is the selling price of the retailer? In order to address these issues, we created and resolved an inventory model for a sustainable electricity supply chain that takes environmental factors into account while reducing setup costs and carbon emissions. Examining how price-dependent demand affects the sustainable electrical supply chain inventory system while lowering setup costs and carbon emissions is the goal of this study. In order to identify the best option, the energy is sent via distribution networks to consumers whose demand is dictated by the price of electricity.

1.2 Literature Review

A single vendor and single buyer were chosen by Goyal and Nebebe [1] in an attempt to minimize the cost of production. For the manufacturing process of imperfect items, Sana [2] considered an inventory model to find the best parameter for product reliability and variable production rate. To expand an inventory model for imperfect production, Ghosh et al. [3] added the crash of the instrument at an arbitrary time and repair of faulty items into the equation, which resulted in a more optimized total profit. Jain et al. [4] investigated a combined inventory model for fuzzy production, demand, and repair, incorporating time-varying demand under inflationary conditions. Manna et al. [5] developed a production - reproduction model for an inventory system for items made in two separate plants under a common management organization. Khara et al. [6] analysed a production-inventory model in which products were manufactured from raw materials and remanufactured from returned items. Das et al. [7] proposed an inventory model for production when demand depends on replacement time, stock, and price. They assured the buyers that the products would be replaced in a certain amount of time. Manna et al. [8] developed a supply chain model for a manufacturing firm and a group of retailers considering the repair of imperfect items. Modibbo et al. [9] developed an integrated system minimizing the cost associated with production, shipment and delivery time. Kaushik [10] presented an inventory model for perishable items focusing on an optimal replenishment policy and maximizing profit involving two different interest rate charges.

Jaggi and Khanna [11] considered a permissible delay in payment for products with imperfect quality and formulated inventory policies for a retailer. Palanivel and Gowri [12] built up a model with perishable items where the market requirement is a function of the selling value. Shah et al. [13] examined a model for an inventory system of decaying items under trapezoidal demand, with preservation technologies deployed to reduce deterioration. Li et al. [14] conducted research on a problem involving joint pricing, replenishment, and investment in preservation technology for non-instantaneously deteriorating items. Both the duration of the non-deterioration period and the rate of deterioration were affected by the preservation technology considered. Within a broad framework, their developed module included backlog rates that depend on waiting time, time-varying degradation, and price-dependent demand.

Sebatjane [15] used preservation technologies for the three-echelon supply chain of food for growing items which deteriorate over time. Sepehri et al. [16,17] developed a green inventory model for the production of low-quality deteriorating items. They computed how the inventory system's profit was affected by carbon-reduction strategies and conservation technologies. Mahato and Mahata [18] considered a model of non-instantaneous worsening products with price dependent order and time-dependent deterioration under a two-echelon trade credit strategy. Retailers invested in preservation methods to prevent deterioration and slow its rate. Rahman et al. [19] discussed an inventory model for the production of food items with preservation techniques. They saw the need as credit-linked with risk of investment in erratic market conditions, using a parametric approach.

Yang et al. [20] defined the deterioration rate as a controllable variable and proposed a novel indicator for freshness preservation effort (FPE). Additionally, the supervision of perishable inventory was included in their analysis to support decision-making on deterioration control. Mahapatra et al. [21] studied economic order quantity models incorporating promotional effort and full backorder with preservation. Chung et al. [22] developed an inventory system that considers items with varying values over time in realistic environments. They used a time-varying deterioration rate that depends on an item's maximum lifetime and treated items that had exceeded their optimal lifetime as junk and useless. The shop is therefore encouraged to invest in preservation technology to prevent careless deterioration.

A carbon tax and carbon cap model for green, cost-effective production size has been considered by Mishra et al. [23] to reduce carbon emissions through investments in green technology under various conditions of shortage. Tiwari et al. [24] investigated sustainable inventory management that accounts for deteriorating and imperfect-quality items, while also considering carbon emissions. Saga et al. [25] discussed a vendor-buyer inventory model with stochastic demand, accounting for imperfect production and inspection errors. In the model, they accounted for energy impacts, carbon emissions, inspection failures, and flawed production methods. In a carbon-emission scenario, De-La-Cruz-Maquez et al. [26] presented a model for an inventory system with shortages for imperfect-quality items that grow over time when demand is price-dependent. Sepehri [27] studied a model for perishable items with expiry dates, where green technology-controlled carbon emissions. Jauhari [28] gave a model with a single manufacturer and multi-retailer optimising quantity and cost while levying tax on the car bon. The firm invested in environmentally friendly technologies to reduce emissions in compliance with greenhouse gas regulations. Daryanto and Wee [29] proposed a three-level sustainable supply chain model for an inventory system with imperfect quality decaying items. Huang et al. [30] considered a two-echelon inventory model with carbon emission and green investment. There was discussion of greenhouse gas emissions across many stages, including manufacturing, storage, and transportation. Jauhari et al. [31] considered a two-echelon inventory model for a closed-loop supply chain system having a manufacturer and a retailer under a stochastic environment with carbon emission reductions and minimised joint total cost. Hasan et al. [32] proposed a method to derive investment in green technology and optimal order quantity. Companies were able to handle greenhouse gas regulations because to this model. Jauhari et al. [33] made a mathematical model for an inventory system comprising of a single vendor and a single buyer with stochastic demand. Additionally, the vendor employed sustainable green production techniques to lower emissions. Yadav et al. [34] presented a green supply chain inventory system to maximise profit by optimising production, shipment, batch size, and preservation technology to reduce waste. They examined the model with two vendors and a retail store, varying the cross-price elasticities of demand and greenhouse gas emissions across several phases, including shipping, setup, waste management, and stock keeping. Pan et al. [35] proposed an inventory model that maximised the supply chain's total profit, in which the buyer and vendor jointly invested funds to reduce carbon emissions. Jiang et al. [36] investigated two cases: in one, an original equipment manufacturer and a contract manufacturer co-invested in green technology; in the other, they invested independently. Becerra et al. [37] conducted a systematic evaluation of quantitative models for sustainable supply chains, organising the papers by supply chain configuration, mathematical models employed, simulations conducted, and algorithms utilised. Daryanto and Christata [38] considered different demand rates for good and defective products and developed an economic order quantity model that minimises total cost, including emission costs. Sepehri et al. [16,17] presented a demand function for perishable goods that depended on price, accounting for the carbon cap-and-trade regulation. The model also proposed an investment plan for carbon reduction and explored its impact on the inventory management system. Wang and Wu [39] examined how the collection of used products can reduce carbon emissions in a closed-loop supply chain, while accounting for the impact of a carbon cap-and-trade policy. Gautam et al. [40] studied a sustainable production model considering the effect of volume agility with preservation technology under-price-reliant demand. Ji et al. [41] employed a two-stage Stackelberg game to explore production decisions and government cap setting. Their analysis incorporated wholesale price and revenue-sharing contracts under cap-and-trade regulation. The cross-border production inventory model for deteriorating items, incorporating various carbon-emission policies, was proposed by Lu et al. [42]. Gautam et al. [43] developed a production inventory model incorporating price-sensitive demand, carbon emissions and dual options for handling defectives. Sarkar and Bhuniya [44] focused on a sustainable supply chain with flexible production rates and green investment. Chaudhary et al. [45] considered a model incorporating varying demand and environmental expenses and optimised inventory operations. Taheri et al. [46] focused on the green inventory control problem, considering liquidity to aid managers in making optimal decisions in a competitive environment. The authors? The proposed model aimed to simultaneously maximise both profit and greenness. Mehmood et al. [47] analysed 2,392 documents from the Web of Science Core Collection to identify emerging trends in emissions and sustainable development and showed a significant increase in concerns about emissions and the need for cost-effective technologies to reduce them. This study is the first of its kind to focus on emission trends from a sustainable development perspective.

2 Methodology

This section explores a sustainable power system for an electricity supply chain with price-dependent electricity demand. Determining the capacity of a sustainable electrical supply chain system follows a methodology similar to calculating the order quantity (E) in a conventional supply chain inventory system. In a typical supply chain, the vendor and buyer collaborate through an order coordination and freight-forwarding system. The buyer purchases items from the vendor in batches and sells them to customers.

In a sustainable electricity supply chain, electricity generated by a power plant is transmitted via a transmission line to a distribution substation, where it is distributed to customers. Here, customer demand is influenced by the electricity price, and electricity is supplied continuously without interruption. The electricity demand is expressed as $d_0(P)$ kWh/year. The power plant generates electricity in batches of $Et_0\zeta_0\widehat{\eta_0}\rho_0$ kWh, where ζ_0 (a positive integer) represents the distribution factor's impact on the distribution substation, $\widehat{\eta_0}$ (a positive integer) reflects the transmission factor's effect on the transmission substation, and ρ_0 (a positive integer) denotes the power generation factor.

The finite power supply rate is given by $Q = \lambda_0 d_0(p)$ kWh/year, were $\lambda_0 > 1$, along with an associated setup cost. The power generator supplies $Et_0\zeta_0\widehat{\eta_0}$ kWh to the transmission substation, which subsequently transmits $ET_0\zeta_0$ kWh to the distribution substation, and $e = Et_0$ kWh is ultimately consumed by customers. To minimise the total cost of the sustainable electricity supply chain system, various factors are considered, including sales revenue, production costs, setup cost reductions, customer ordering costs, and transmission/distribution costs for substations. The transmission and distribution costs depend on the power plant, transmission substation, and distribution substation, with maximum capacities of α_p^X kWA, α_T^X kWA, and α_D^X kWA, respectively. A comparison between the traditional supply chain inventory system and the sustainable electricity supply chain system is illustrated in Figure 1. The structure of this paper is as follows: Section 2 outlines the assumptions and notation for formulating the electrical supply chain inventory system model. Section 3 presents the model formulation. In Section 4, a numerical example is provided to validate the model. Section 5 explores managerial implications, and Section 6 concludes the study with suggestions for future research.

3 Assumptions and Notations

In this section, we introduce notation that will be used to develop the mathematical model (see Section 3), based on the following assumptions:

3.1 Assumptions

- A fully integrated inventory model is considered. A single customer is regarded for one type of goods. A single buyer is considered for a single item type.
- Power supply blackouts are not accounted for.
- The limited power supply value surpasses the demand volume, resulting in the connection between power availability and demand is given by $Q = \lambda_0 d_0(P)$, where $\lambda_0 > 1$.
- Instead of assuming a fixed demand parameter, demand is represented as a linear function of the cost of sale, which provides a more accurate portrayal of real-world events. The representation of the linear demand equation is $d_0(p) = a_0 - b_0 P_0$ (as shown in Mishra et al. [27]; Mishra et al. [28]) The rate at which con sumers want power is influenced by the selling price, were $a_0 > 0$ serves as a scaling factor, and $b_0 > 1$ represents the elasticity coefficient, ensuring the condition $P_0 < \frac{a_0}{b_0}$ is met."
- The investment price I_0 is considered in order to lower setup costs. The setup cost is expressed as $M_0(I_0) = Se^{-\tau_0 I_0}$, where $S(> 0)$ represents the initial setup cost, and $\tau_0(> 0)$ is a constant parameter. The first and second derivatives, $\frac{dM_0(I_0)}{I_0} = -\tau_0 Se^{-\tau_0 I_0}$ and $\frac{d^2 M_0(I_0)}{dI_0} = \tau_0 Se^{-\tau_0 I_0} > 0$This indicates that a higher investment leads to a lower setup expense. Consequently, the setup cost for each production batch can be minimised through appropriate investment decisions. The investment I_0 directly impacts setup costs, which play a crucial role in production. Refer to Sarkar et al. (2016) for more details.
- The total power used is represented by E in kW.
- The electricity consumption by customers over a given time period t_0 is represented as $e = Et_0 kWh$.
- A total of $Et_0\zeta$ kWh of power is delivered to the substation of supply, where Z_0 (a positive integer) represents the substations of the distribution factor on the substation.
- The power generator supplies $Et_0\zeta\widehat{\eta_0}$ kWh of electricity to the transmission substation, where ζ (a positive integer) reflects the distribution factor's influence on the distribution substation, and $\widehat{\eta_0}$ (a positive integer) represents the transmission factor's effect on the transmission substation.

- In a single batch of power generation, a total of $Et_0\zeta\eta_0\rho_0$ kWh of electricity is produced, where ζ (a positive integer) represents the distribution factor's influence on the distribution substation, $\widehat{\eta_0}$ (a positive integer) de notes the transmission factor's impact on the transmission substation, and ρ_0 (a positive integer) signifies the power generation factor.
- The processes of the production, distribution, and transfer of power operate within defined capacity constraints. The maximum capacity of the power generation plant is represented as α_P^X kWA, while the transmission substation has a maximum capacity of α_T^X kWA. Similarly, the distribution substation operates with a maximum capacity of α_D^X kWA.
- The interrelationship among the highest possible power capacity generation, the distribution substation and the transmission substation is defined by $\alpha_P^X > \alpha_T^X > \alpha_D^X$.
- The entire process of producing power must incorporate the expenses related to with both transmission and distribution.
- The total cost of the low-carbon emission substation consists of three components: energy holding cost and two transmission-related cost elements.
- The energy holding cost is proportional to the amount of electricity stored at the transmission substation during the planning period.
- All cost parameters associated with energy storage and transmission are assumed to be known and constant throughout the study horizon.

4. Notations

The notations used in the model are presented as follows:

O	The price paid for every order placed \$/unit/order
a_0	Factor of scaling
b_0	Coefficient of price elasticity
r	Cost of production
S	Initial stage setup cost
τ_0	The setup cost $M_0(I_0)$ is known.
R_1	Transmission substation's energy holding costs.
R_2	Yearly percentage of power generation's energy holding costs.

Ω_0	Adjustment for power factor.
C_1	Distribution and transmission rates per mile
N_1	The system of mile-by-mile transportation between the power plant and the transmission terminal.
N_2	The system of transport nodes per mile connecting the transmission and distribution substations.
α_P^X	Maximum capacity for producing power.
α_T^X	The maximum capacity of the transmission substation
α_D^X	Maximum Centre of distribution capacity
B	The factor of loss power supply$(0 \leq B \leq 1)$
$\hat{\gamma}_1$	Energy holding's emissions of carbon per unit.
$\hat{\gamma}_2$	The carbon emissions volume distribution and transmission per unit.
$\hat{\Gamma}$	carbon emissions per transmission unit as well as the distribution rates of the emissions of carbon.
$\hat{C}_t$	Tax on carbon emissions calculated per unit, with a monetary charge applied for each unit of carbon released.
I_0	Cost of investment for the establishment of power generation facilities
ζ_0	The distribution substation's reaction to the electrical energy distribution factor (positive integer).
$\hat{\eta}$	A distribution substation's reaction to the electrical energy distribution factor (positive integer).
ρ_0	The electrical power generation factor is a positive integer
t_0	The average electricity consumption of the customer (a positive integer).
P_0	The price at which electricity is sold (a positive integer).
E	The authority of the customer
$d_0(P)$	Customer demand.
y_D^z	The distribution substation's current capacity.
y_T^z	The transmission substation's current capacity.
y_P^z	Current power generation capacity.

5. Model formulation

This section describes how to calculate the overall profit function for power generation, as well as the total cost functions for customers, distribution and transmission substations, and carbon emissions. The following components make up the total cost function:

Ordering cost:

The ordering cost is the total expenditure a customer incurs per unit of time.

$$TC_0 = \frac{Od_0(P)}{Et_0} \tag{1}$$

Distribution cost

The distribution cost is derived from the operation of the distribution substation. The distribution cost for a partial load G_0 can be expressed as $G_0 = \frac{C_1 \alpha_D^X}{y_D^Z}$ The transmission function represents the increase in rate per rVA per mile as y_D^z increases. $G_1 = BG_0 + (1 - B)C_1$, where $0 \leq B \leq 1$ is the coefficient of the modified inverse function, is the distribution cost per rVA/mile. Therefore

$G_1 = BC_1 \left[\frac{\alpha_D^X - y_D^Z}{y_D^Z}\right] + C_1$. The projected overall expense for the distribution substation, contingent upon demand, correction factors, and distance, is derived $G_2 = \left[BC_1 \left[\frac{\alpha_D^X - y_D^Z}{y_D^Z}\right] + C_1\right] d_0(P) N_3 \Omega_0$. The real power supply capacity is represented by

$y_D^z = Et_0 \zeta_0 \Omega_0$; hence, the distribution cost can be expressed by the following equation:

$$\mathrm{TC_D} = \frac{\mathrm{d_0(P)BC_1\alpha_D^X N_3}}{\mathrm{Et_0\zeta_0}} + \mathrm{d_0(P)N_3\Omega_0(1\text{-}B)C_1} \tag{2}$$

Distribution cost of carbon emissions

There is only one component to the cost of distributing carbon emissions. Total distribution rates have an impact on this component of carbon emissions. The following succinctly summaries and represents the component:

$$\mathrm{CE_D} = \hat{\Gamma} \frac{\mathrm{d_0(P)B\widehat{C_t}\alpha_D^X N_3}}{\mathrm{Et_0\zeta_0}} + \mathrm{d_0(P)\widehat{C_t}N_3\Omega_0(1\text{-}B)} \tag{3}$$

Cost of Transmission Substation

Both the transmission and energy holding costs are included in this overall cost. The transmission substation's energy holding costs are displayed by $\frac{R_1\rho_0\zeta_0\hat{\eta}e_0}{2}=\frac{R_1\rho_0\zeta_0\hat{\eta}Et_0}{2}$.

Transmission costs can be computed as $g_{0t}=\frac{d_0(P)BC_1\alpha_T^X N_2}{Et_0\zeta_0\hat{\eta}}+d_0(P)N_2\Omega_0(1-B)C_1$.

The transmission substation's overall cost includes both energy holding and transmission costs. Thus, the overall cost of transmission substations is:

$$t_{CT}=\frac{R_1\rho_0\zeta_0\hat{\eta}Et_0}{2}+\frac{d_0(P)BC_1\alpha_T^X N_2}{Et_0\zeta_0\hat{\eta}}+d_0(P)N_2\Omega_0(1-B)C_1 \tag{4}$$

Substation Cost for Carbon Emission Transmission

There are three parts to the price of a lower-carbon-emission substation. The transmission substation's overall energy holding rates are represented by the first component, and total transmission rates are linked to the second and third components. These elements have been condensed and articulated by

$$CE_T=\frac{\widehat{\gamma_1}\widehat{C}_t P_0\zeta_0\hat{\eta}Et_0}{2}+\left[\frac{d_0(P)BC_1\alpha_T^X N_2}{Et_0\zeta_0\hat{\eta}}+d_0(P)N_2\Omega_0(1-B)\hat{\Gamma}\right] \tag{5}$$

Profitability of power generation

Power generation's gross profit can be expressed as

$$\Phi=\nu-\xi-sc-I_{vc}-g_p-H_c-c_{tr}-c_{pg} \tag{6}$$

Revenue from sales

$$\nu=d_0(P)P_0 \tag{7}$$

cost of production

$$\xi=d_0(P) \tag{8}$$

Power generation generates $(Et_0\zeta_0\hat{\eta}\rho_0)$ kWh of electricity in a single manufacturing run. The following formula can be used to determine the setup cost for electricity generation

$$sc=\frac{d_o(P)Se^{-\tau_0 I_0}}{Et_o\tau_0\hat{\eta}\rho_0} \tag{9}$$

The entire amount spent on setup is

$$I_{vc} = \frac{d_0(P)I_0}{Et_0\tau_0\hat{\eta}\rho_0} \qquad (10)$$

The cost of energy holding H_c is

$$=R_2 r \frac{\left[e_0\zeta_0\hat{\eta}\rho_0\left(\frac{e_0\zeta_0\hat{\eta}}{P_o}+(\rho_0\text{-}1)\text{-}\frac{e_0}{d_0(p)}\right)\text{-}\frac{\rho_0^2[e_0\zeta_0\hat{\eta}]^2}{2P_0}\right]\text{-}\left[\frac{e_0\zeta_0\hat{\eta}}{d_0(P)}\left(1+2+3\ldots+(\rho_0\text{-}1)\right)e_0\right]}{\frac{e_0\zeta_0\hat{\eta}\rho_0}{d_0(P)}},$$

$$H_c = R_2 r\frac{e_0\zeta_0\hat{\eta}}{2}[\rho_0(1-\frac{d_0(P)}{P_0}-1+\frac{2d_0(P)}{P_0}]$$

$P_0 = \lambda_0 d_0(P)$ is entered into H_c. Consequently, the holding expense for is

$$H_c = R_2 r\frac{Et_0\zeta_0\hat{\eta}}{2\lambda_0}[\rho_0(\lambda_0-1)-\lambda_0+2] \qquad (11)$$

Transmission costs can be computed as

$$g_p = \frac{d_0(P)BC_1\alpha_P^X N_1}{Et_0\zeta_0\hat{\eta}} + d_0(P)N_1\Omega_0(1-B) \qquad (12)$$

Total Cost of Transmission Carbon Emissions

There is only one component to the cost of transmitting carbon emissions. The overall transmission rates are connected to this gearbox emission component. The following succinctly summaries and represents the component

$$ce_{tr} = \hat{\Gamma}\left[\frac{d_0(P)\widehat{C_t}B\alpha_P^X N_1}{Et_0\zeta_0\hat{\eta}} + d_0(P)\widehat{C}_t N_1\Omega_0(1-B)\right] \qquad (13)$$

Total Cost of Carbon Emissions from Transmission

There is an energy storage expenses component to carbon emissions. The overall energy collection in a power plant is related to a holding emission component. The brief and succinct component is represented by

$$ce_{pg} = \hat{\gamma}_2\left[r\left(\frac{Et_0\hat{C}_t\zeta_0\hat{z}\eta}{(2\lambda_0)}\right)[\rho_0(\lambda_0-1)-\lambda_0+2]\right] \qquad (14)$$

Because of the K_0 Capital expenditure made in green technologies, this average drops to

$1-\theta_0(1-e^{MK_o})$.

As a result, the overall profit for power generation Φ can be expressed as:

$$\Phi = d_0(P)(P_0 - r) - \frac{d_0(P)Se^{-\tau_0 I_0}}{Et_0\tau_0\hat{\eta}\rho_0} - \frac{d_0(P)I_0}{Et_0\tau_0\hat{\eta}\rho_0} - R_2 r\frac{Et_0\zeta_0\hat{\eta}}{2\lambda_0}[\rho_0(\lambda_0-1) - \lambda_0+2] - [\frac{d_0(P)BC_1\alpha_P^X N_1}{Et_0\zeta_0\hat{\eta}} + d_0(P)N_1\Omega_0(1-B)] - \hat{\Gamma}[\frac{d_0(P)\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)B\alpha_P^X N_1}{Et_0\zeta_0\hat{\eta}} + d_0(P)\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)N_1\Omega_0(1-B)] - \hat{\gamma_2}[r\frac{Et_0\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\zeta_0\hat{\eta}}{2\lambda_0}[\rho_0(\lambda_0-1)-\lambda_0+2]] - K_0 \tag{15}$$

Therefore, the total profit is

$$\Phi_1 = \Phi - TC_0 - TC_D - CE_D - t_{CT} - CE_T \tag{16}$$

$$\Phi_1 = d_0(P)[P_0 - r - (N_1+N_2+N_3)\Omega_0(1-B)(C_1+\hat{\Gamma}) - \frac{d_0(P)}{Et_0\zeta_0\hat{\eta}\rho_0}(O\zeta_0\hat{\eta}\rho_0 + B\left(C_1+\hat{\Gamma}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)\alpha_D^X N_3\hat{\eta}\rho_0 + Se^{-\tau_0 I_0} + I_0 + B\left(C_1+\hat{\Gamma}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)\alpha_P^X N_1) - \frac{Et_0\zeta_0\hat{\eta}}{2\lambda_0}(\lambda_0\left(R_1+\widehat{\gamma_1}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)P_0 + \left(R_2+\widehat{\gamma_2}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)r(\rho_0(\lambda_0-1) - \lambda_0+2))] \tag{17}$$

We use the second order partial derivatives of Equation (16) to the effect of $I_0, \zeta_0, \hat{\eta}, \rho_0, t_0$ and P_0 on Φ_1 fixed E.

$$\frac{\partial\Phi_1}{\partial\zeta_0} = \frac{d_0(P)}{Et_0\zeta_0^2\hat{\eta}\rho_0}(O\hat{\eta}\rho_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)\alpha_D^X N_3\hat{\eta}\rho_0 + Se^{-\tau_0 I_0} + I_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)\alpha_P^X N_1) - \frac{Et_0\hat{\eta}}{2\lambda_0}(\lambda_0\left(R_1 + \widehat{\gamma_1}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)P_0 + \left(R_2 + \widehat{\gamma_2}\widehat{C}_t(1-\theta_0(1-e^{MK_0}))\right)r(\rho_0(\lambda_0-1) - \lambda_0 + 2))] \tag{18}$$

$$\frac{\partial^2\Phi_1}{\partial{\zeta_0}^2}=\frac{d_0(P)}{Et_0\zeta_0^3\hat{\eta}\rho_0}\left(B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0+Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)\alpha_P^X N_1\right)<0 \quad (19)$$

$$\begin{aligned}\frac{\partial\Phi_1}{\partial\hat{\eta}}=\frac{d_0(P)}{Et_0\zeta_0\hat{\eta}^2\rho_0}&(O\zeta_0\rho_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_D^X N_3\rho_0\\&+Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1)\\&-\frac{Et_0\zeta_0}{2\lambda_0}(\lambda_0\left(R_1+\widehat{\gamma_1}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)P_0\\&+\left(R_2+\widehat{\gamma_2}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)r(\rho_0(\lambda_0-1)-\lambda_0+2))]\end{aligned} \quad (20)$$

$$\frac{\partial^2\Phi_1}{\partial\hat{\eta}^2}=\frac{d_0(P)}{Et_0\zeta_0\hat{\eta}^3\rho_0}\left(B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)\alpha_D^X N_3\rho_0+Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)\alpha_P^X N_1\right)<0 \quad (21)$$

$$\frac{\partial\Phi_1}{\partial\rho_0}=\frac{d_0(P)}{Et_0\zeta_0\hat{\eta}\rho_0^2}\left[Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)\alpha_P^X N_1\right]-\frac{Et_0\zeta_0\hat{\eta}\left(R_2+\widehat{\gamma_2\widehat{C_t}}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)r\left(\rho_0(\lambda_0-1)\right)}{2\lambda_0} \quad (22)$$

$$\frac{\partial^2\Phi_1}{\partial\rho_0^2}=\frac{d_0(P)}{Et_0\zeta_0\hat{\eta}\rho_0^3}\left[Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1\right]<0 \quad (23)$$

$$\frac{\partial\Phi_1}{\partial t_0}=\frac{d_0(P)}{Et_0^2\zeta_0\hat{\eta}\rho_0}(O\zeta_0\hat{\eta}\rho_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0+Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1)-\frac{E\zeta_0\hat{\eta}}{2\lambda_0}(\lambda_0\left(R_1+\widehat{\gamma_1}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)P_0+\left(R_2+\widehat{\gamma_2}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)r(\rho_0(\lambda_0-1)-\lambda_0+2))] \quad (24)$$

$$\frac{\partial^2\Phi_1}{\partial t_0^2}=\frac{d_0(P)}{Et_0^3\zeta_0\hat{\eta}\rho_0}(O\zeta_0\hat{\eta}\rho_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0+Se^{-\tau_0 I_0}+I_0+B\left(C_1+\hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1)<0 \quad (25)$$

$$\frac{\partial \Phi_1}{\partial P_0} = a_0 - b_0 P_0 - \ b\_0 \vdash [P_0 - r + (B-1)\Omega_0 \left(C_1 + \Gamma\widehat{\widehat{C}_t}\right)(N_1 + N_2 + N_3) \dashv] - \frac{Et_0\zeta_0\hat{\eta}\left(R_1 + \widehat{\gamma_1}\widehat{C_t}\left(1-\theta_0\left(1-e^{MK_0}\right)\right)\right)}{2} - \frac{b_0}{Et_0\zeta_0\hat{\eta}\rho_0} \vdash [O\zeta_0\hat{\eta}\rho_0 + B\left(C_1 + \Gamma\widehat{\widehat{C}_t}\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_D^X N_3 \hat{\eta}\rho_0 + Se^{-\tau_0 I_0} + I_0 + B\left(C_1 + \Gamma\widehat{\widehat{C}_t}\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_P^X N_1 \dashv] \tag{26}$$

$$\frac{\partial^2 \Phi_1}{\partial P_0^2} = -2b_0 < 0 \tag{27}$$

Equations (18), (20), (22), (24) and (26) show that for every feasible solution of E, the entire profit function Φ_1 represents a concave equation of $I_0, \zeta_0, \hat{\eta}, \rho_0, P_0$ and t_0 .

Theorem 1

The function of objective Φ_1 in equation (16) represents a concavity of I_0 and E for every positive integer $I_0, \zeta_0, \hat{\eta}, \rho_0, t_0$ and P_0. At the points I_0 and E, where $\frac{\partial \Phi_1}{\partial I_0} = 0$ and $\frac{\partial \Phi_1}{\partial I_0} = 0$ the largest value Φ_1 equation (16) is solved.

Proof: Using the derivatives that are partial in relation to I_0 and E, as indicated by the ideal solution for integers with fixed values in the following conditions $I_0, \zeta_0, \hat{\eta}, \rho_0, t_0$ and P_0 was determined. We get,

$$\frac{\partial \Phi_1}{\partial I_0} = -\frac{d_0(P)\left(1 - \tau_0 S e^{-\tau_0 I_0}\right)}{Et_0\zeta_0\hat{\eta}\rho_0} \tag{28}$$

$$\frac{\partial \Phi_1}{\partial E} = \frac{d_0(P)}{Et_0\zeta_0\hat{\eta}}[O\zeta_0\hat{\eta}\rho_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0 + Se^{-\tau_0 I_0} + I_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_P^X N_1] - \frac{t_0\zeta_0\hat{\eta}}{2\lambda_0}[\lambda_0\left(R_1 + \widehat{\gamma_1}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)P_0 + \left(R_2 + \widehat{\gamma_2}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)r(\rho_0(\lambda_0 - 1) - \lambda_0 + 2)] \tag{29}$$

Now, solve for I_0 and E by setting Equations (28) and (29) to zero:

$$-\frac{d_0(P)(1 - \tau_0 S e^{-\tau_0 I_0})}{Et_0\zeta_0\hat{\eta}\,\rho_0} = 0$$

$$E = \frac{1}{\tau_0}\ln(\tau_0 S) \tag{30}$$

Then

$$\frac{d_0(P)E\rho_0}{E^2 t_0\zeta_0\hat{\eta}\rho_0}[O\zeta_0\hat{\eta}\rho_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0 + Se^{-\tau_0 I_0} + I_0 + B\left(C_1 + \hat{\Gamma}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)\alpha_P^X N_1] - \frac{t_0\zeta_0\hat{\eta}}{2\lambda_0}[\lambda_0\left(R_1 + \widehat{\gamma_1}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)P_0 + \left(R_2 + \widehat{\gamma_2}\widehat{C}_t\left(1 - \theta_0(1 - e^{MK_0})\right)\right)r(\rho_0(\lambda_0 - 1) - \lambda_0 + 2)] = 0$$

$$E = \frac{1}{t_0\zeta_0\widehat{\eta}}\sqrt{\frac{2\lambda_0 A_0}{\rho_0 B_0}} \tag{31}$$

$$\begin{aligned} A_0 = d_0(P)[O\zeta_0\hat{\eta}\rho_0 + B\left(C_1 + \hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0 + Se^{-\tau_0 I_0} \\ + I_0 + B\left(C_1 + \hat{\Gamma}\widehat{C_t}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1] \end{aligned}$$

$$\begin{aligned} B_0 = \lambda_0\left(R_1 + \widehat{\gamma_1\widehat{C_t}}\left(1-\theta_0(1-e^{MK_0})\right)\right)P_0 \\ + \left(R_2 + \widehat{\gamma_2\widehat{C_t}}\left(1-\theta_0(1-e^{MK_0})\right)\right)r(\rho_0(\lambda_0-1)-\lambda_0+2) \end{aligned}$$

In order to show the concavity of the necessary objective profit function, Φ_1 can demonstrate that the subsequent conditions. We get,

$$\frac{\partial^2\Phi_1}{\partial I_0^2} = -\frac{d_0(P)\left(1-\tau_0^2 Se^{-\tau_0 I_0}\right)}{Et_0\zeta_0\widehat{\eta}\rho_0} < 0 \tag{32}$$

$$\begin{aligned} \frac{\partial^2\Phi_1}{\partial E^2} = \frac{2d_0(P)}{E^2 t_0\zeta_0\widehat{\eta}\rho_0}[O\zeta_0\hat{\eta}\rho_0 + B\left(C_1 + \widehat{\Gamma\widehat{C_t}}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_D^X N_3\hat{\eta}\rho_0 \\ +Se^{-\tau_0 I_0} + I_0 + B\left(C_1 + \widehat{\Gamma\widehat{C_t}}\left(1-\theta_0(1-e^{MK_0})\right)\right)\alpha_P^X N_1] \end{aligned} \tag{33}$$

$$\frac{\partial^2\Phi_1}{\partial E\partial I_0} = -\frac{\partial^2\Phi_1}{\partial I_0\partial E} = \frac{d_0(P)\left(1-\tau_0 Se^{-\tau_0 I_0}\right)}{Et_0\zeta_0\widehat{\eta}\rho_0} \tag{34}$$

A Hessian matrix will be determined as follows

$$H = \begin{bmatrix} \frac{\partial^2\Phi_1}{\partial I_0^2} & \frac{\partial^2\Phi_1}{\partial I_0\partial E} \\ \frac{\partial^2\Phi_1}{\partial E\partial I_0} & \frac{\partial^2\Phi_1}{\partial E^2} \end{bmatrix} \tag{35}$$

$$|H| = -\frac{[d_0(P)]^2[\tau_0^2 S^2 e^{-2\tau_0 I_0} + 2K - 1]}{Et_0\zeta_0\widehat{\eta}\rho_0} < 0 \tag{36}$$

Algorithm 1:

Step i: Determine I_0 using Equation (28).

Step ii:

a. Set $\zeta_0 = 1$.

b. Set $\hat{\eta} = 1$.

c. Set $\rho_0 = 1$.

d. Set $t_0 = 1$.

e. Set $P_0 = 1$.

Step iii: Use equation (31) to determine the ideal E^*.

Step iv: After completing Step iii, determine the real electrical power capacities.

a. Generation of power:

The electricity generation's real capacity is $y_P^Z = Et_0\zeta_0\hat{\eta}\rho_0\Omega_0$. If $y_P^Z \leq \alpha_P^X$, then find $E = \frac{y_P^Z}{t_0\zeta_0\hat{\eta}\rho_0\Omega_0}$ and go to Step v.

b. Substation for transmission

The transmission substation's real capacity is $y_T^Z = Et_0\zeta_0\hat{\eta}\Omega_0$. If $y_T^Z \leq \alpha_T^X$, then find $E = \frac{y_T^Z}{t_0\zeta_0\hat{\eta}\Omega_0}$ and go to Step v.

c. Substation for distribution

The distribution substation's actual capacity is $y_D^Z = Et_0\zeta_0\Omega_0$. If $y_D^Z \leq \alpha_D^X$,

then find $E = \frac{y_D^Z}{t_0\zeta_0\Omega_0}$ and go to Step v.

Step v: Compute Φ_1 from Equation (16).

Step vi: Set $\zeta_0 = 1 + 1$ and repeat Step iii to Step v.

Step vii: If

$\Phi_1\left(E_{\zeta_0}^*, \zeta_0, \widehat{\eta_{\zeta_0}}, \rho_{0\zeta_0}, t_{0\zeta_0}, P_{0\zeta_0}\right) \geq \Phi_1\left(E_{\zeta_0-1}^*, \zeta_0 - 1, \widehat{\eta_{\zeta_0-1}}, \rho_{0(\zeta_0-1)}, t_{0(\zeta_0-1)}, P_{0(\zeta_0-1)}\right)$

then go to Step viii. Otherwise, go to Step vi.

Step viii: Set $\hat{\eta} = 1 + 1$ and repeat Step ii b to Step vii.

Step ix: If

$\Phi_1\left(E_{\hat{\eta}}^*, \zeta_0\hat{\eta}, \hat{\eta}, \rho_{0\hat{\eta}}, t_{0\hat{\eta}}, P_{0\hat{\eta}}\right) \leq \Phi_1\left(E_{\hat{\eta}-1}^*, \zeta_0 - 1\hat{\eta}, \hat{\eta} - 1, \rho_{0(\hat{\eta}-1)}, t_{0(\hat{\eta}-1)}, P_{0(\hat{\eta}-1)}\right)$ then go to Step ix. Otherwise, go to Step viii.

Step x: Set $\rho_0 = 1 + 1$ and repeat Step ii c to Step ix.

Step xi: If

$\Phi_1\left(E_{\rho_0}^*, \zeta_0\rho_0, \hat{\eta}\rho_0, \rho_0, t_{0\rho_0}, P_{0\rho_0}\right) \geq \Phi_1\left(E_{\rho_0-1}^*, \zeta_0 - 1\rho_0, \hat{\eta} - 1\rho_0, \rho_0, t_{0(\rho_0-1)}, P_{0(\rho_0-1)}\right)$ then go to Step xii. Otherwise, go to Step x.

Step xii: Set $t_0 = 1 + 1$ and repeat Step ii c to Step x.

Step xiii: If

$\Phi_1\left(E_{t_0}^*, \zeta_0 t_0, \hat{\eta} t_0, \rho_0 t_0, t_0, P_{0t_0}\right) \geq \Phi_1\left(E_{t_0-1}^*, \zeta_0 - 1t_0, \hat{\eta} - 1t_0, \rho_{0t_0}, t_0 - 1, P_{0(t_0-1)}\right)$ then go to Step xiii. Otherwise, go to Step xv.

Step xiv: Set $P_0 = 1 + 1$ and repeat Step ii c to Step xi.

Step xv: If

$$\Phi_1\left(E^*_{P_0},\zeta_0 P_0,\hat{\eta} P_0,\rho_0 P_0,t_0 P_0,P_0\right)\geq\Phi_1\left(E^*_{P_0\text{-}1},\zeta_0\text{-}1P_0,\hat{\eta}\text{-}1P_0,\rho_{0P_0},t_0,P_{0(P_0\text{-}1)}\right)$$

Table1: Table of sensitivity analysis of key parameters of the model

Parameter	Changes	ζ₀*	η̂*	ρ₀*	t₀*	P₀*	d₀(P)*	E*	I₀*	α_ P^{X*}	α_ T^{X*}	α_ D^{X*}	Φ₁*
a_0	60	2	1	1	1	20	20	131.133	1.68236	314.719	314.719	314.719	287.189
	70	2	1	1	1	20	30	160.605	1.68236	385.452	385.452	385.452	439.814
	80	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	90	2	1	1	1	24	42	206.829	1.68236	496.146	496.146	496.146	770.623
	100	2	1	1	1	27	46	228.775	1.68236	549.060	549.060	549.060	997.886
b_0	1.8	3	1	1	1	24	36.8	353.749	1.68236	1273.49	1273.49	1273.49	643.645
	1.9	3	1	1	1	22	36.3	188.50	1.68236	441.300	441.300	441.300	601.653
	2.0	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	2.1	2	1	1	1	22	36	180.345	1.68236	421.241	421.241	421.241	561.897
	2.2	2	1	1	1	22	36	175.943	1.68236	421.241	421.241	421.241	531.978
r	2	2	1	1	1	22	38	181.449	1.68236	435.477	435.477	435.477	676.583
	3	2	1	1	1	22	37	183.254	1.68236	441.300	441.300	441.300	611.653
	4	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	5	2	1	1	1	22	34	183.965	1.68236	441.146	441.146	441.146	565.209
	6	2	1	1	1	23	34	185.482	1.68236	445.166	445.166	445.166	570.873

Parameter	Changes	ζ_0^*	η^*	ρ_0^*	t_0^*	P_0^*	$d_0(P)^*$	E*	I_0^*	α_P^{X*}	α_T^{X*}	α_D^{X*}	Φ_1^*
λ_0	1.8	2	1	1	1	22	36	220.455	1.68236	529.092	529.092	529.092	575.565
	1.9	2	1	1	1	22	36	194.300	1.68236	490.125	490.125	490.125	590.137
	2.0	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	2.1	2	1	1	1	22	36	157.798	1.68236	390.875	390.875	390.875	609.451
	2.2	2	1	1	1	22	36	122.397	1.68236	293.752	293.752	293.752	611.235
S	5	2	1	1	1	22	36	183.810	1.53742	441.144	441.144	441.144	601.668
	6	2	1	1	1	22	36	183.810	1.61042	441.144	441.144	441.144	601.654
	7	2	1	1	1	22	36	183.875	1.68236	441.144	441.144	441.144	601.653
	8	2	1	1	1	22	36	183.906	1.75328	441.374	441.374	441.374	601.644
	9	2	1	1	1	22	36	183.937	1.82322	441.448	441.448	441.448	601.636
τ_0	0.18	2	1	1	1	22	36	183.945	1.82395	441.300	441.300	441.300	601.672
	0.19	2	1	1	1	22	36	183.911	1.50994	441.386	441.386	441.386	601.643
	0.20	2	1	1	1	22	36	183.837	1.68236	441.300	441.300	441.300	601.653
	0.22	2	1	1	1	22	36	183.797	1.96265	441.112	441.112	441.112	601.672
	0.18	2	1	1	1	22	36	183.945	1.82395	441.300	441.300	441.300	601.672
O	80	2	1	1	1	22	36	165.126	1.68236	396.302	396.302	396.302	606.378
	90	2	1	1	1	22	36	174.752	1.68236	419.404	419.404	419.404	604.366
	100	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	110	2	1	1	1	22	36	192.566	1.68236	460.900	460.900	460.900	599.462
	120	2	1	1	1	22	36	200.881	1.68236	482.114	482.114	482.114	597.366

ĉ₁	**0.00012**	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	**0.00012**
	0.00020	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	**0.00020**
	0.00030	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	**0.00030**
	0.00040	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	**0.00040**
	0.00050	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	**0.00050**
γ₁	0.002	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	0.002
	0.003	2	1	1	1	22	36	197.606	1.68236	474.254	474.254	474.254	0.003
	0.004	2	1	1	1	22	36	210.443	1.68236	503.063	503.063	503.063	0.004
	0.005	2	1	1	1	22	36	222.541	1.68236	534.099	534.099	534.099	0.005
	0.006	2	1	1	1	22	36	234.014	1.68236	561.634	561.634	561.634	0.006
B	0.008	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	0.008
	0.009	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	0.009
	0.010	2	1	1	1	22	36	186.447	1.68236	441.300	441.300	441.300	0.010
	0.011	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	0.011
	0.012	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	0.012

6. Numerical Analysis:

To demonstrate the model, we present a numerical example based on a data set obtained from earlier research.

Let $a_0 = 80units,\ b_0 = 2units,\ O = \$100/$unit/order, $R_1 = \$0.004/$unit/year, $\widehat{\gamma_1} = 0.002$ ton/year, $R_2 = \$0.002/$unit/year, $\widehat{\gamma_2} = 0.003$ ton/year,

$B = 0.01$ unit, $\Omega_0 = 1.2$ kVA/kWh, $S = 7$/setup, $\widehat{C_t} = 1$/ton/year, $\tau_0 = 0.2$ unit, $r = 4$/kWh, $\lambda_0 = 2$ units, $C_1 = 0.00011$/kVA/mile, $\alpha_P^X = 6$ kVA, $\alpha_T^X = 5$ kVA, $\alpha_D^X = 2$ kVA,

$N_1 = 0.2$ mile, $N_2 = 0.15$ mile, $N_3 = 0.1$ mile.

Utilizing the input data alongside Algorithm and MATLAB, the optimal solutions are

$\zeta_0^* = 1, \widehat{\eta}^* = 1, \rho_0^* = 1, t_0^* = 1, P_0^* = 1, d_0(P)^* = 32, I_0^* = 34, E^* = 1.58236,$

$\alpha_P^{X*} = 183.875,$ $\alpha_T^{X*} = 431.300, \alpha_D^{X*} = 417.832,$ and $\Phi_1 = 511.653.$ Furthermore, Figure 1 shows that the total profit function's graphical representation has a concave shape with respect to the possible optimal values.

7. Managerial Implication

This section summarizes the findings of the sensitivity evaluation and examines the impact of changes in the primary parameters (see Table 2)

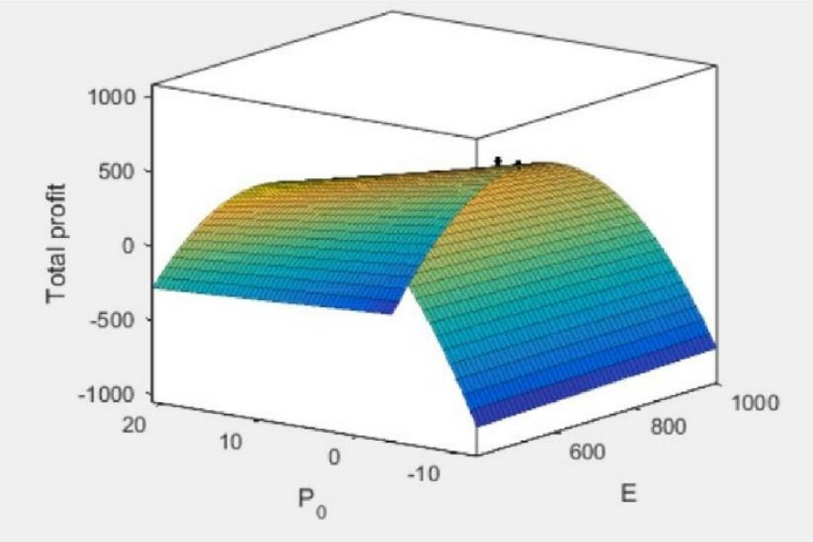

Figure 1: Concavity of Optimal Solution

7.1 Influence on Demand Variables

An increase in the parameter a_0 leads to a rise in demand, compelling the retailer to increase the selling price P_0 while keeping $\zeta_0, \hat{\eta}, \rho_0, t_0$ and I_0 fixed. Consequently, the retailer lowers the selling price P_0 while maintaining fixed values for $\hat{\eta}, \rho_0, t_0$ and I_0. This reduction decreases the client's power usage as well as the supply, transmission, and production sub-stations ' real capacity. Although a higher value of b0 may help sustain demand at an elevated level, it ultimately leads to a decline in the electrical supply chain's profit.

7.2 Effect on Manufacturing Variables:

A rise in the amount of λ_0 leads to higher production while maintaining a fixed demand rate. Consequently, the retailer retains the purchase price P_0 and other parameters, such as $\zeta_0, \hat{\eta}, \rho_0, t_0 and I_0$ unchanged. This, in turn, lowers the real capacity utilization of the distribution as well as transmission networks, power producing substations, and consumer electricity consumption, ultimately augmenting the revenue of the power distribution system Conversely, an augmentation in the worth of r reduces demand, prompting the seller wants to increase the cost of selling P_0 while keeping $\zeta_0, \hat{\eta}, \rho_0, t_0 and I_0$ constant. This adjustment results in higher capacity utilization of the supply and production system, electrical power stations, and consumer electricity usage. However, despite the ability to sustain demand at a lower level, the increased energy consumption and capacity usage lead to a decline in the overall profitability of the electrical supply chain.

7.3 Effects on Parameters for Reducing Setup Costs

An elevation in the value of S leads to an augmentation in setup costs, given constant demand rates, compelling the retailer to stabilize $\zeta_0, \hat{\eta}, \rho_0, t_0 and I_0$ This increases the capacity of transportation, delivery, and electric power stations along with client use of electricity. However, it also increases construction costs and reduces earnings in the power supply chain. Due to increases in customers' power consumption and actual capacity combined with fixed selling prices, the electrical supply chain system's profit dropped. Conversely, a higher value of τ_0 leads to a lower initial expense with a fixed requirement prices, forcing the vendor to correct the $\zeta_0, \hat{\eta}, \rho_0, t_0 and I_0$. This reduces customer electricity usage and transformer capacity, increases installation costs, and increases revenue for the energy distribution network. In this instance, the energy transportation system profit is raised since the market demand value kept constant at a high value of τ_0Customers utilise less power, and all actual capacity is reduced.

7.4 Effect on the parameters of carbon emissions

With fixed demand rates, an increase in the value of all greenhouse gas emissions parameters $\hat{\Gamma}, \widehat{\gamma_1}$, and $\widehat{\gamma_2}$ causes the customer's power consumption rate to increase. The price of sale increases supply, communication, and electricity generation at the substations' actual capacity. Increasing real capacity and client energy use rates at a fixed selling price reduced profit in the electric supply chain.

8. Conclusion

This study examines a mathematical framework to represent a sustainable energy supply chain, assuming that customer requests are linearly price-dependent and that price is a deciding factor in reducing setup costs under carbon emissions. This model was created using inventory management theory as a basis, examining how key elements affect all optimal choice variables and the overall profitability of a sustainable electrical supply chain. Our computational findings provide managerial insights to marketing and production system managers to aid in the advancement of a field electrical energy logistics network that is both sustainable and successful. Researchers may enhance this study in future years by incorporating price reduction strategies and examining the effects of pursuing green technology on greenhouse gas emissions through a combined transmission and supply substation

References

[1] A. Sepahri, D. Mishra, M.-L. Tseng, B. Sarkar, Joint replenishment and inventory model for deteriorating items with maximum life time and controllable carbon emissions under permissible delay in payments, Mathematics 9 (5) (2021) 127.

[2] A. Sepahri, U. Mishra, B. Sarkar, A sustainable production inventory model with imperfect quality under preservation technology and quality improvement investment, J. Clean. Prod. 310 (2021) 127332,

[3] A. Sepharjee, Controllable carbon emissions in an inventory model for perishable items under cap-and-trade credit policy for credit-risk customers, Carbon Capt. Sci. Technol. 1 (2021) 100004,

[4] A.K. Manna, A.K. Ghosh, An integrated green production? inventory model with deteriorating and imperfect quality items considering carbon emission, J. Prod. Eng. 21 (2018) 281-292.

[5] A.K. Manna, J.K. Dey, S.K. Mondal, Controlling GHG emission from industrial perusal production inventory model with fuzzy pollution parameters, Int. J. Syst. Sci. Oper. Logist. 6 (4) (2019) 368-388.

[6] A.K. Manna, R. Mondal, A.A. Shaikh, H. Ali, A.B. Bhuniya, Single-manufacturer and multi-retailer supply chain model with pre-payment based partial free transportation, RAIROOper. Res. 55 (2021) 1063-1076,

[7] A.S. Mahapatra, M.S. Mahaswar, B. Sarkar, S. Majumder, Benefit of preservation technology with promotion and trade-credit policy under fuzzy learning, Expert Syst. Appl. 169 (2021).

[8] B. Khara, J. Deo, S. Mondal, Sustainable recycling in an imperfect production system with acceptance quality level dependent development cost and demand, Comput. Ind. Eng. 147 (2020).

[9] C. Mahto, G.C. Mahtta, Optimal replenishment, pricing and preservation technology investment policies for non instantaneous deteriorating items under two-level trade credit

policy, J. Ind. Manag. Optim. 18 (5) (2022) 3499?3517,

[10] C.G.De-La-Cruz-Marquez, J.E. Cardenas- Barron, B.Mandal, An inventory model for growing items with imperfect quality when demand is price sensitive under emission and purchasing tax obligations, Mathematics 9(5)(2021).

[11] C.-J. Li, M. Gu, T.-S. Lee, C.-T. Yang, Impact of carbon emission policy combinations on the optimal production? inventory decisions for deteriorating items, Expert Syst. Appl. 198 (2022).

[12] C.K. Jaggi, A. Khanna, Credit financing for deteriorating imperfect quality items under inflationary conditions, Int. J. Serv. Oper. Inf. 6 (4) (2011) 292-309.

[13] D. Yadav, N. Kumar, M. Kumar, B. Sarkar, Redefining smart inventory model through the selection of items with cross-price elasticity of demand to form a sustainable supply chain with preservation technology, J. Clean. Prod. 312 (2021) 126298.

[14] G. Li, Y. He, J. Zhou, H.Wu, Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items, Omega 84 (2019) 114-126,

[15] H. Barari, S. Bhuiyan, A sustainable flexible manufacturing? remanufacturing model with improved service and green investment under variable demand, Expert Syst. Appl. 202-215 (2022).

[16] J. Kushak, An inventory model with permissible delay in payment and different interest rate charges, Decis. Anal. J. 6 (2023)

[17] J. Pan, C.Y. Chiu, K.S. Wu, H.F. Wen, Y.W. Wang, Sustainable inventory model in technical cooperation on investment to reduce carbon emissions, Processes 8 (11) (2020).

[18] K. Menkovi, Q.K. Saifullah, M.M. Abur, Unearthing research trends in emissions and sustainable development: Potential implications for future directions, Gondwana Res. 119 (2023) 227-245.

[19] K.J. Chung, S.F. Hse, H.M. Sivatsava, S.D. Lin, S.L. Kang, J.L. Liao, Optimal ordering policy and preservation technology for deteriorating

items with maximum lifetime under inflation and time discounting, J. Ind. Manag. Optim. 20 (7) (2023) 5533-5379,

[20] M. Jainlewal, S. Gowar, A production inventory model with delayed deteriorating items with quadratic and price dependent demand, Int. J. Pure Appl. Math. 119 (15) (2018) 3271-3283.

[21] M. Sepharjee, The impact of preservation technology investments on lot-sizing and shipment strategies in a three-echelon food supply chain involving growing and deteriorating items, Oper. Res. Perspect. 9 (2022) 100241,

[22] M.I. Taher, M.S. Amanlick, A.A. Taleizadeh, E. Mardan, Investigating the green inventory control problem considering liquidity risk: Application in the dairy industry, Sustainable Cities Soc. 92 (2023).

[23] M.R. Hasan, T.C. Roy, Y. Daryanto, H.M. Wee, optimizing inventory and technology investment under a carbon tax, cap and- trade and strict carbon limit regulations, Sustain. Prod. Consum. 25 (2021) 604-621.

[24] M.S. Hasan, A.K. Manna, A.A. Shaikh, A.B. Bhuniya, An application of internal deterioration effect on a production inventory model with interval-valued demand via resizing and optimization technique under partial prepayment promotion, Int. J. Intell. Syst. 35 (8) (2020) 1280-1326.

[25] N.B. Shah, D.B. Shah, D.G. Patel, Optimal preservation investment, pricing and ordering policies for deteriorating inventory with trapezoidal demand, Int. J. Oper. Res. 26 (3) (2016)367-381.

[26] P. Becerra, J.M. R. Christata, Green supply chain quantitative models for sustainable inventory management: A review, J. Clean. Prod. 328 (2021).

[27] P. Gautam, K.M. Kamna, C.K. Jaggi, Sustainable production policies under the effect of volume agility, preservation technology and price-reliant demand, Yugoslav J. Oper. Res. 30 (3) (2020) 307-324.

[28] P. Gautam, S. Maheshwari, C.K. Jaggi, Sustainable inventory model with greening degree and dual determinants of defective items, J. Clean. Prod. 367 (2022).

[29] P.K. Ghosh, A.K. Manna, J.K. Dey, Deteriorating manufacturing system with selling price discount under random machine breakdown, Int. J. Comput. Eng. Manag. 20 (2017) 8-17.

[30] R. Chaudhary, R. Mittal, M. Jayaswal, A sustainable inventory model for three-levels under fuzzy environment, Decis. Anal. J. 7 (2023).

[31] S. Das, A.K. Manna, E.E. Mahmoud, K.M. Abulnasr, A.H. Abdel, A.A. Shaikh, Product replacement policy in a production inventory model with replacement period-stock, and price dependent demand. Hindawi, J. Adv. Math. (2020) 8,

[32] S. Huang, C.C. Fang, Y. Lin, Inventory management in carbon supply chains with consideration of logistics, green investment and different carbon emissions policies, Comput. Ind. Eng. 139 (2020).

[33] S. Jain, S. Tiwari, L.F. Cardenas-Barron, A.A. Shaikh, S.R. Singh, A fuzzy imperfect production and repair inventory model with time-dependent demand, production and repairs rates under inflationary conditions, RAIRO Oper. Res. 52 (1) (2018) 217-239.

[34] S. Li,W. He, Inventory strategies in a green two-echelon supply chain with green investment and emissions reduction, Expert Syst. Appl. 202 (2022) 117250,

[35] S.J. Ye, Q. Lin, Managing green innovation investment in a cooperative supply chain under capital constraint, J. Clean. Prod. 290 (2021).

[36] S.K. Goyal, F. Nebebe, Determination of economic production shipment policy for a single vendor single buyer system, Eur. J. Oper. Res. 121 (2000) 175-178.

[37] S.S. Sana, A production inventory model in an imperfect production process, Eur. J. Oper. Res. 200 (2) (2010) 451-464.

[38] T. Ji, X. Xu, X. Yan, Y. Yu, The production decisions and the design of emission price and revenue sharing contracts under cap-and-trade regulation, Int. J. Prod. Res. 58 (2020)128-147.

[39] U. Mishra, J.Wu, B. Sarkar, A sustainable production-inventory model for a controllable carbon emission rate under shortages, J. Clean. Prod. 256 (2020) 120268,

[40] U.M. Modibbo, S. Gupta, A. Ali I. Ahmed, An integrated multi objective multi-product inventory managed production planning problem under uncertain environment, Ann. Oper. Res. (2022).

[41] W.A. Jauhari, I.N. Pujawan, M. Suel, A closed-loop supply chain inventory model with stochastic demand, hybrid production, carbon emissions, and take-back incentives, J. Clean.Prod. 320 (2021).

[42] W.A. Jauhari, N. Pujawan, M. Suel, K. Govindan, Low-carbon inventory model for vendor-buyer system with hybrid production and adjustable production rate under stochastic demand, Appl. Math. Model. 102 (2022) 840-868,

[43] W.A. Jauhari, R.W.P. Laksono, A.R. Dwicahyani, Investigating carbon emission regulation in a production-inventory model under imperfect production, inspection errors and carbon tax, hybrid policy, Int. J. Logist. Syst. Manag. 39 (4) (2021) 502-520,

[44] Y. Daryanto, B.R. Christata, Optimal order quantity considering carbon emissions costs, defective items, and partial backorder, Uncertain Supply Chain Manag. 9 (2021) 307-316,

[45] Y. Yang, H. Zhou, W. Zhou, T. Tan, S. Pramutha, Deterioration control decision support for perishable inventory management, Decis. Support Syst. (2020) 117.

[46] Z. Wang, Q. Wu, Carbon emission reduction cost and product selection decisions in the closed-loop supply chain with cap-and-trade regulation, Int. J. Prod. Res. 59 (14) (2021) 4395-4398.

[47] Z. Wang, Y. Hong, H. Wei, A multi-echelon green inventory chain system for imperfect quality deteriorating items, Oper. Supply Chain Manag. 14 (1) (2021) 26-38.

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Conflict of Interest

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