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Weak and Strong Bitopological Lindel of Space

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Abstract

Keywords: Topological space, bitopological space, continuous function and Lindelof space. This paper deals with structural properties of weak(strong) Lindelof Bitopological space.

Introduction

J.C. Kelly[1] introduced the concept of bitopological space. S. Balasubramanian and G. Koteswara Rao[3] introduced the concept of strong (weak) Lindelof bitopology space. In this paper, we define strong (weak) continuous function on bitopological space and study the structural properties of Lindelof bitopological space.

Definition

Definition 2.1[1] : A non empty set X together with two topologies Γ_1 & Γ_2, denoted by (X, Γ_1, Γ_2) is called bitopological space.

Definition 2.2[2] : A bitopological space (X, Γ_1, Γ_2) is said to be a compact space if X is Γ_1-compact and Γ_2-compact.

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Definition 2.3[3] : A bitopological space (X, Γ_1, Γ_2) is said to be strong Lindelof bitopological space if both (X, Γ_1) and (X, Γ_2) are Lindelof.

Definition 2.4[3] : A bitopological space (X, Γ_1, Γ_2) is said to be weak Lindelof bitopological space if either (X, Γ_1) or (X, Γ_2) is Lindelof.

Definition 2.5: Let (X, Γ_1, Γ_2) and (Y, σ_1, σ_2) be any two bitopological spaces and $f : \rightarrow Y$ be a single valued function, f is said to be

(a) weak continuous if either

$f : (X, \Gamma_1) \rightarrow (Y, \sigma_1)$ or $f : (X, \Gamma_2) \rightarrow (Y, \sigma_2)$ is continuous.

(b) Strong continuous if both

$f : (X, \Gamma_1) \rightarrow (Y, \sigma_1)$ and $f : (X, \Gamma_2) \rightarrow (Y, \sigma_2)$ are continuous.

Definition 2.6: Let (X, Γ_1, Γ_2) and (Y, σ_1, σ_2) be any two bitopological spaces and $f : x \rightarrow y$ be a single valued function, f is said to be

(c) weak open if either

$f : (X, \Gamma_1) \rightarrow (Y, \sigma_1)$ or $f : (X, \Gamma_2) \rightarrow (Y, \sigma_2)$ is open.

(d) Strong open if both

$f : (X, \Gamma_1) \rightarrow (Y, \sigma_1)$ and $f : (X, \Gamma_2) \rightarrow (Y, \sigma_2)$ are open.

3. Main Results(Structural Properties):

Theorem 3.1 : (Weak) Strong continuous image of (weak) strong Lindelof bitological space is weak (strong) Lindelof.

Proof:

- Let $f: (X, \Gamma_1, \Gamma_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a strong continuous functions, where (X, Γ_1, Γ_2) and $(Y, \sigma_1, \sigma_2)$are two bitopological spaces. Let $y \in Y$ be an element such that $f^{-1}(y) \in X$. Put $x = f^{-1}(y)$, for $^{f-1}(y) \in X$. since X is strong Lindelof there exists $\Gamma 1$ - open set U containing $f^{-1}(y)$ and , Γ_2– open set V containing $f^{-1}(y)$ and hence $y \in f(U)$ and $y \in f(V)$ are open sets in s1 and σ_2 respectively. Since X is strong Lindelof and f - is continuous, we have a countable collection of open sets U and V with respect to, Γ_1 & Γ_2 covering X and countable collection of open sets f(U) and f(V) covering f(X) with respect to σ_1 & σ_2 respectively.

Hence the proof.

Theorem 3.2

If (X, Γ1, Γ2) is weak (strong) Lindelof and (Y, σ1,σ2) is compact then X x Y is weak(strong) Lindelof.

Proof:

From the definitions of Lindelofness, and the compactness, the result is obvious.

Theorem 3.3:

If an arbitrary product of weak(strong) Lindelof bitoplolgical space is Lindelof then

(i) Each component space is weak(strong) Lindelof

(ii) All but finitely many of the compact spaces is weak (strong) Lindelof.

Proof:

Let $x = p\, x_{\alpha\in I}$ be strong Lindelof bitopological space. Since the projection maps are continuous and open,

(i) follows from theorem 3.2. For part (ii), let x Î X then $x = (x)_a$ Î $p_{aÎI} X_a$. Then there exists a finite subset J of I and open set U_b in X_b (bÎJ) such that x Î $p_{bÎJ} U_b$ x $p_{aÎI\text{-}J}$ X_a contained in W from which, again by theorem 3.2, if follows that X_a is Lindelof for aÎI-J

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